A quarterback lost 8 yards on the first play, gained 12 yards on the second play, gained 23 yards on the next play, and lost 4 yards on the fourth play.

Show the quarterback's total gain or loss of yards over the four plays as an addition and solve.

The debate club raised $912 at its last fund-raiser. The twelve members of the club share the money equally between them. They use it to pay travel expenses for the next two debate tournaments.

How much money will each member have for each debate tournament?

Evaluate the following:

-25 ÷ 5

15 - (–17)

Rewrite the following additions as subtractions.

Example

Rewrite 32 ÷ 8 as an addition problem.

Solution

Rewrite 5 + 17 as a multiplication problem. Remember that dividing is the same as multiplying by a reciprocal.

1. Rewrite the following subtractions as additions.
   a. –6 – (–7)
   b. 18 – 9
   c. 28 – (–14)

2. Rewrite the following additions as subtractions.
   a. 5 + (–17)
   b. 24 + (–2)

Rewrite the following as a multiplication problem.

6
8

5
3

Remember that dividing is the same as multiplying by a reciprocal.
California Algebra I

Homework Book
California Standards-Driven Program
This book covers each of the requirements of the California Algebra I Standards.

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**California Algebra I Standards**

The following table lists all the California Mathematics Content Standards for Algebra I with cross references to where each Standard is covered in this Program. This will enable you to measure your progression against the California Algebra I Standards as you work your way through the Program.

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<th>California Standard</th>
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<td>1.0</td>
<td>Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.</td>
<td>Chapter 1</td>
</tr>
<tr>
<td>1.1</td>
<td>Students use properties of numbers to demonstrate whether assertions are true or false.</td>
<td>Chapter 1</td>
</tr>
<tr>
<td>2.0</td>
<td>Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.</td>
<td>Chapters 1, 6</td>
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<tr>
<td>3.0</td>
<td>Students solve equations and inequalities involving absolute values.</td>
<td>Chapters 2, 3</td>
</tr>
<tr>
<td>4.0</td>
<td>Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.</td>
<td>Chapters 2, 3</td>
</tr>
<tr>
<td>5.0</td>
<td>Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.</td>
<td>Chapters 2, 3</td>
</tr>
<tr>
<td>6.0</td>
<td>Students graph a linear equation and compute the x- and y-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequalities (e.g., they sketch the region defined by $2x + 6y &lt; 4$).</td>
<td>Chapter 4</td>
</tr>
<tr>
<td>7.0</td>
<td>Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.</td>
<td>Chapter 4</td>
</tr>
<tr>
<td>8.0</td>
<td>Students understand the concepts of parallel lines and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.</td>
<td>Chapter 4</td>
</tr>
<tr>
<td>9.0</td>
<td>Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.</td>
<td>Chapters 4, 5</td>
</tr>
<tr>
<td>10.0</td>
<td>Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.</td>
<td>Chapter 6</td>
</tr>
<tr>
<td>11.0</td>
<td>Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.</td>
<td>Chapters 6, 7</td>
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<td>12.0</td>
<td>Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.</td>
<td>Chapter 8</td>
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<tr>
<td>13.0</td>
<td>Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.</td>
<td>Chapter 8</td>
</tr>
<tr>
<td>California Standard</td>
<td>Standard Description</td>
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<tr>
<td>14.0</td>
<td>Students solve a quadratic equation by factoring or completing the square.</td>
<td></td>
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<tr>
<td>15.0</td>
<td>Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.</td>
<td></td>
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<tr>
<td>16.0</td>
<td>Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.</td>
<td></td>
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<tr>
<td>17.0</td>
<td>Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.</td>
<td></td>
</tr>
<tr>
<td>18.0</td>
<td>Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.</td>
<td></td>
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<tr>
<td>19.0</td>
<td>Students know the quadratic formula and are familiar with its proof by completing the square.</td>
<td></td>
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<tr>
<td>20.0</td>
<td>Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.</td>
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<tr>
<td>21.0</td>
<td>Students graph quadratic functions and know that their roots are the $x$-intercepts.</td>
<td></td>
</tr>
<tr>
<td>22.0</td>
<td>Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the $x$-axis in zero, one, or two points.</td>
<td></td>
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<tr>
<td>23.0</td>
<td>Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.</td>
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<tr>
<td>24.0</td>
<td>Students use and know simple aspects of a logical argument:</td>
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<td>24.1</td>
<td>Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.</td>
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<td>24.2</td>
<td>Students identify the hypothesis and conclusion in logical deduction.</td>
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<td>24.3</td>
<td>Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.</td>
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<tr>
<td>25.0</td>
<td>Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:</td>
<td></td>
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<tr>
<td>25.1</td>
<td>Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.</td>
<td></td>
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<tr>
<td>25.2</td>
<td>Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.</td>
<td></td>
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<tr>
<td>25.3</td>
<td>Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.</td>
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Show key standards
Advice for Parents and Guardians

What the Homework Book is For

Homework helps students improve their thinking skills and develop learning outside the classroom. This Homework book is an integral part of the CGP California Standards-Driven Algebra I Program.

- It focuses purely on the California Content Standards for Algebra with no extraneous content.
- It has been written to match the California Algebra I Content Standards, using the Mathematics Framework for California Public Schools (2005) as a guide. The Standards are listed on pages iv–v of this book.
- It has a clear and simple structure which is the same in each component of the Program.
- It is a flexible program which caters for a diverse student body.

This Homework Book follows exactly the same structure and order of teaching as the Student Textbook.

- The Algebra I Program is broken down into eight Chapters (see pages viii–xi for more detail)
- These Chapters are in turn divided into smaller Sections which cover broad areas of the Algebra I course.
- These Sections are then broken down into smaller, manageable Topics, which are designed to be worked through in a typical 50-minute math class.
- Each Topic in the Student Textbook starts with the relevant California standard in full. This is then linked to a clear learning objective written in everyday language so that your child can understand what the Lesson is about and how it fits in with the overall California Algebra I Standards.

At the end of each Lesson, the teacher will assign homework from this book, which contains one worksheet for each Topic of the Algebra I Program. That means that there is always additional work for your child to practice the skills learned in the math Lesson.

Each worksheet is perforated and hole punched — so your child can hand each sheet in to the teacher, and then store the marked and corrected sheets in a separate file at home or at school.

Using the Homework Book

The worksheets in this book have been designed to be straightforward to use. The worksheets have lots of common features:

- The relevant California Mathematics Standard is always stated at the start of each homework sheet.
- Each homework sheet covers several difficulty levels. The teacher has information about which problems are suitable for each student so will set specific problems for your child.
- The pages are perforated so that each homework sheet can be pulled out and handed in to the teacher.
- There’s one homework sheet for every Topic in the book — so it’s easy to refer back to the relevant part of the Textbook.
- Useful hints help students to understand each problem.
- Worked examples help students to answer problems and show them how to write their own solutions.

At the back of the book you will find worked solutions to the first problem from each worksheet. Together with the worked example on the worksheet itself, this will allow you to check that your child understands the key concepts for each worksheet.

This book contains more than just questions. Pages viii–xi give Chapter Overviews with lists of quick-reference Key Notation and Terminology, and pages xii–xiii contain advice on Question Technique.
By getting actively involved in your child’s education, you can make a real difference to his or her success. Even if you are less confident with the math yourself, you can still provide help — both on a practical level and in less direct ways. Just by showing an interest in your child’s work, you can help improve their perception of the value of math. Here is some practical, day-to-day advice on how to help your child get the most out of their Algebra I course.

**Provide a Suitable Working Environment**

It is important that your child has an appropriate place to work in. It is very easy to get distracted at home, so each of the measures suggested below is designed to keep your child as focused as possible.

- He or she should work at an uncluttered table or desk — a kitchen table is fine for this.

- The environment should contain as few distractions as possible — for example, if there is a TV in the room, make sure it is turned off and your child is facing away from it (even a turned off TV can be distracting).

- Try to set aside a regular time each day for working. This can be difficult to fit around other commitments, but it is worth making the effort. Having a homework slot as part of your family’s regular schedule can help your child to get into the right frame of mind to work.

- Encourage your child to work solidly for twenty minutes, then take a five minute break. Even with the best intentions, many children find it hard to focus for long periods of time — so working in short, intensive bursts is often most effective. The short break can also be used as an incentive — working hard for twenty minutes straight earns five minutes off.

**Try to Identify Problem Areas**

If your child is struggling with a piece of work, there are several possible root causes. Try to identify which aspects of a question your child is having difficulty with, so that he or she can take steps to solve the problem.

- Ask your child to explain in words how the problem would be solved. Struggling with this may suggest that the basic concept behind the question has not been grasped. (You do not have to understand the math yourself to do this, you just need to judge whether your child can explain the concept clearly.) See page xii, Concept Questions. It is useful to go through this process even if your child is doing well. Students often learn procedures for answering questions without understanding the underlying math. Although this can be sufficient for simple problems, they may run into difficulties later if earlier concepts are missed.

- If your child understands the concept, but still cannot answer the question correctly, the problem may be with a particular component skill (for example, one step in the work). Read through a worked example with your child (you will find these in every Topic of the Homework Book). Then copy out the question onto a separate piece of paper and ask your child to try to answer it. Compare each step of the work with that given in the book to see where the problem is.

- The difficulty may lie with the type of question. On pages xii and xiii, Guidance on Question Technique, there are lists of measures that can be taken to deal with particular question types.

**Keep in Contact with the School**

You ought to receive regular reports on your child’s progress from the school, but remember that communication between home and school can be in two directions. If you are concerned about any aspect of your child’s progress, it may help to discuss the issue with his or her teacher.
Overviews of Chapter Content

Algebra I is not a course that can be treated in isolation. In each Chapter, you learn new concepts that are part of the larger picture of mathematics, and everything you learn builds on your knowledge from previous grades.

Chapter One — Working with Real Numbers

Chapter 1 is about the absolute fundamentals of the real number system.

The following concepts are covered in this Chapter:

- the real numbers, plus some important subsets — integers, whole numbers, rational numbers and irrational numbers
- the axioms of the real number system
- exponent laws
- properties of roots
- using fractions
- simplifying and evaluating mathematical expressions
- mathematical arguments and proofs

How the Chapter follows on from previous study:

You have been working with real numbers and the basic operations of addition, subtraction, multiplication, and division since very early grades. This Chapter formalizes your previous work using the real number axioms.

Section 1.3 provides a comprehensive review of the manipulation of exponents, roots, and fractions, before introducing new concepts in these areas.

The Section on mathematical argument and proof follows on from grade 7 work, where you made and tested hypotheses using both inductive and deductive reasoning.

Chapter Two — Single Variable Linear Equations

Chapter 2 is about solving equations — mostly linear equations.

The following concepts are covered in this Chapter:

- rearranging and solving linear equations, including equations with fractional and decimal coefficients
- applications of linear equations, including coin tasks, consecutive integer tasks, age-related tasks, time and rate tasks, mixture tasks, and work-related tasks
- absolute value equations

How the Chapter follows on from previous study:

In grades 6 and 7, you learned to simplify and solve simple linear equations. In this Chapter, these skills are formalized in the properties of equality, extended to more complicated equations, and applied to real-world situations.

At the end of this Chapter, absolute value equations are introduced. This leads on from the definition of the absolute value of a real number that you met in grade 7.
Chapter Three — Single Variable Linear Inequalities

Chapter 3 is about solving inequalities.

The following concepts are covered in this Chapter:

- rearranging and solving linear inequalities, including graphing solutions on a number line and using interval notation
- applications of linear inequalities
- compound inequalities
- absolute value inequalities

How the Chapter follows on from previous study:

You should be familiar with the four different inequality symbols and their meanings from grade 7, where you solved simple linear inequalities in one variable.

In Chapter 2, you learned various techniques for solving linear equations in one variable. These skills are now applied to inequalities, and formalized in the properties of inequalities.

In Sections 3.3 and 3.4, your experience of inequalities is extended to include compound and absolute value inequalities. The absolute value Section leads on from Chapter 2, where you solved absolute value equations.

Chapter Four — Linear Equations and Their Graphs

Chapter 4 is about lines and the coordinate plane.

The following concepts are covered in this Chapter:

- the coordinate plane
- the slope of a line, and its intercepts on the x- and y-axes
- lines and equations: standard form, slope-intercept form, and point-slope form
- perpendicular and parallel lines
- regions defined by equations and inequalities

How the Chapter follows on from previous study:

Section 4.1 reviews and reinforces your knowledge of the coordinate plane, that you have been working with since grade 4.

Where previously you have graphed linear relationships that pass through the origin (0, 0), in this Chapter you develop various strategies for plotting lines that cross the x- and y-axes at different points.

This Chapter also provides formal definitions of parallel and perpendicular lines — the concepts of which you have been familiar with since grade 4.

Finally, you build on the inequalities work from Chapter 3 by sketching regions defined by linear inequalities on the coordinate plane.

Key Notation and Terminology:

Symbols

- <: is less than
- : is less than or equal to
- >: is greater than
- : is greater than or equal to

Properties of Inequalities

- Addition property of inequalities: \( a + c > b + c \)
- Subtraction property of inequalities: \( a - c > b - c \)
- Multiplication property of inequalities: if \( c > 0 \), then \( ac > bc \)
- Division property of inequalities: if \( c > 0 \), then \( \frac{a}{c} < \frac{b}{c} \)

Compound Inequalities

- Conjunction: two inequalities combined using “and”
- Disjunction: two inequalities combined using “or”

Interval Notation

- \((a, b)\): open interval — all real numbers between, but not including, \(a\) and \(b\). Corresponds to \(a < x < b\).
- \([a, b)\): closed interval — all real numbers between, and including, \(a\) and \(b\). Corresponds to \(a \leq x < b\).
- \([a, b]\): half-open interval — all real numbers between \(a\) and \(b\), including \(a\), but not \(b\). Corresponds to \(a \leq x < b\).

Key Notation and Terminology:

The Coordinate Plane

- Quadrants: the plane is split into four quadrants (I–IV)
- Origin: the point (0, 0)
- x- and y-axes

Graphing Lines

- \(x\)-intercept: point at which a line crosses the \(x\)-axis (0, \(y\))
- \(y\)-intercept: point at which a line crosses the \(y\)-axis (\(x\), 0)
- Slope: change in \(y\) / change in \(x\)

Equations of Lines

- Standard form: \(Ax + By = C\) [\(A\), \(B\), and \(C\) are all constants]
- Point-slope formula: \(y - y_1 = m(x - x_1)\)
- Slope-intercept form: \(y = mx + b\)

Parallel and Perpendicular Lines

- \(l_1 \parallel l_2\): \(l_1\) is parallel to \(l_2\) if their slopes \((m_1\) and \(m_2))\) are equal
- \(l_1 \perp l_2\): \(l_1\) is perpendicular to \(l_2\) if the product of their slopes \((m_1 \times m_2)\) is equal to –1
Chapter 5 — Systems of Equations

Chapter 5 is about solving systems of linear equations.

The following concepts are covered in this Chapter:

- solving a system of equations by graphing, substitution, and elimination methods
- independent, dependent, and inconsistent systems of equations
- applications of systems of equations, including percent mix problems and rate problems

How the Chapter follows on from previous study:

In grades 6 and 7, you learned to simplify and solve simple linear equations.

In Chapter 2, these skills were extended to more complicated equations, and applied to real-world situations.

Then in Chapter 4, you learned how to graph linear equations of the form $Ax + By = C$ or $y = mx + b$.

This Chapter extends your previous understanding of linear equations to systems of linear equations in two variables.

You build on Chapter 4 work with the graphing method of solving systems of equations, and use algebraic methods that are extensions of Chapter 2 work.

The final Section of this Chapter deals with applications. It introduces an alternative method for solving problems of the type that you met in Sections 2.4, 2.5, 2.6, and 2.7.

Chapter Six — Manipulating Polynomials

Chapter 6 is about polynomials.

The following concepts are covered in this Chapter:

- polynomial basics: adding, subtracting, multiplying, and dividing
- factoring polynomials (including quadratics) — finding a common factor for all terms, recognizing the difference of two squares, and recognizing perfect squares of binomials

How the Chapter follows on from previous study:

Since grade 6, you have used the properties of rational numbers to simplify expressions — work that was formalized in Chapter 1 with the real number axioms, and extended in Chapter 2 by collecting like terms.

This Chapter draws on these skills to add, subtract, and simplify polynomials.

In grade 7 you multiplied and divided by monomials.

This is extended in Chapter 6 to multiplying and dividing by polynomials. Polynomial multiplication and division requires application of the rules of exponents (reviewed in Section 1.3), long division (grade 5), and canceling common factors (grade 6).

Leading on from work on factoring real numbers and monomials in grades 5 and 6, you learn in this Chapter to factor quadratic expressions and simple third-degree polynomials.
Chapter Seven — Quadratic Equations and Their Applications

Chapter 7 is about solving and graphing quadratic equations.

The following concepts are covered in this Chapter:
- solving quadratic equations by factoring and by taking square roots
- completing the square
- the quadratic formula and the discriminant
- graphs of quadratic functions
- applications of quadratic functions

How the Chapter follows on from previous study:
In grade 7, you found square roots of perfect square integers and monomials, and in Chapter 1 you learned that all expressions have both a positive and a negative square root. In Chapter 7, you apply this to solve quadratic equations by taking roots.

Chapter 1 also introduced the zero product rule which, together with the factoring skills you learned in Chapter 6, allows you to solve quadratic equations by factoring.

Later in the Chapter, these techniques are extended to include completing the square — a method for which you need to be able to recognize perfect square binomials (Chapter 6). Completing the square is used to derive the quadratic formula, which can be used to solve any quadratic with real solutions.

The Sections on graphing quadratics towards the end of the Chapter follow on from grade 7, where you graphed equations of the form $y = nx^2$.

Chapter Eight — Rational Expressions and Functions

Chapter 8 is about rational expressions, plus relations and functions.

The following concepts are covered in this Chapter:
- rational expressions (or algebraic fractions)
- basic operations with rational expressions: adding, subtracting, multiplying, and dividing
- solving fractional equations
- relations and functions

How the Chapter follows on from previous study:
You learned to add, subtract, multiply and divide numerical fractions in grade 7 (and this was reviewed in Chapter 1).

In this Chapter, your experience of rational expressions is extended to include those containing variables.

In Chapter 6, you developed techniques for factoring polynomials. These techniques are applied here to simplify rational expressions.

The work from Section 2.3 on solving equations with fractional coefficients is extended in this Chapter to solving fractional equations.

The final Section formalizes the work you have done on functions — writing ordered pairs, constructing tables of values, and plotting graphs. You may not have been aware of it at the time, but you have been plotting functions and using them to solve problems since grade 5.
Guidance on Question Technique

There are a number of different abilities required in order to be successful in Algebra I — concepts have to be understood, skills have to be learned, and those skills should be applied to new situations. So math questions are not all the same.

This Course uses a Variety of Question Types

The five main types of question covered are:
- **Concept Questions**
- **Skills Practice Questions**
- **Applying Skills to New Situations**
- **Interesting/Challenging Questions**
- **Proof Questions**

In addition to this, many questions will be a mixture of types — for example, an application of learned skills to a new situation may also be an interesting mathematical problem.

It’s Important to Understand These Differences

All of these types of questions can be tackled in different ways. If you struggle with one particular type, there are specific measures you can take, as outlined below.

**Concept Questions**

Concept Questions are those that help you to understand the topic, and help to check your understanding.

These may be probing questions asked by your teacher as part of the teaching process, such as: "Dividing by ½ is the same as multiplying by which number? Why?"

Or exercises that reinforce and check understanding, such as: "For each of the polynomials a) – f), state whether it is a monomial, a binomial, or a trinomial."

Concept Questions are fundamental to the learning of mathematics, as they are based on understanding rather than skills. If you do not understand a concept, you may struggle to learn the necessary skills. At the same time, if you have a firm grasp of the concepts, the skills you learn will make much more sense.

If you are struggling with Concept Questions:

1) Go back to earlier work that you are comfortable with. This will give you a useful starting point.
2) Then gradually work through the concepts, one by one. Try looking at worked examples, making sure that you follow the reasoning behind every step.
3) This will give you a better understanding of where the math comes from than if you had merely "rote-learned" the facts.
4) If you can, return to the original questions that you were struggling with to check that you now understand the concepts.

**Skills Practice Questions**

These are drill-type questions that let you practice the skills you have just learned, and check that you have learned those skills properly, for example. "For each of parts a) to v), simplify the algebraic expression."

Skills Practice Questions are repetitive, and are designed this way to help you learn — it is generally easier to remember something that you’ve done 10 times, than something you’ve only done once.

These questions do not go beyond the scope of what you have learned in class; they simply provide lots of practice at using the same skills, over and over again.

If you are making mistakes in Skills Practice Questions, there are a few possible root problems:

1) It could be that you have not fully understood the concept, and so are not applying the method correctly. In this case, see the advice for Concept Questions.
2) If you do understand the concept, it may simply be that you need to brush up on one or more of the component skills. Try doing a worked example and comparing each of your steps with the steps in the book. That way you can see exactly where you are going wrong and get extra help with those topics if necessary.
3) To check that you have learned the skills required, practice them again and again until you consistently perform well.

There are plenty of questions in this book and in the Textbook, and your teacher may be able to provide you with extra questions.
Problem Solving — Applying Skills to New Situations

These kinds of questions give you more practice at using learned skills, but they also require problem-solving abilities. They are often real-life applications of theoretical problems, for example, *On Monday, a distribution company shipped a load of oranges in crates, with a total weight of 124 lb. On Tuesday it shipped another load of oranges, also with a total weight of 124 lb. However, on Tuesday there was one crate fewer than on Monday, so each crate was 1/8 lb heavier. How many crates were shipped on Monday?* You need two distinct abilities here: the skills required to solve the equations, and the ability to translate the real-life problem into math. If you are struggling with this type of question, it is very important to pinpoint the cause of the difficulty.

1) If you have difficulty even starting these questions, then you need practice at problem solving, and translating real-world problems into math. Try to get your teacher (or other students) to work through some real-world examples with you. Ask them to start with simple examples, and move on only when you have understood each one.

2) If you can translate the problem into math, but then you solve it incorrectly, you may need to review and relearn the necessary skills. Have a look at the advice on Skills Practice Questions.

3) You may be able to solve real-world problems “in your head,” without the need to write down your method. While this is an equally valid way of solving the problem, you should realize that it is important to explain all your steps — if only for the purposes of checking any mistakes later on. If you find this difficult, you could start by explaining your reasoning to someone, and ask them to help you to write that “in math.”

Not all Applying Skills Questions will be real-life applications. Some will be Challenge Questions (see below), where you will be asked to apply your skills to different kinds of theoretical problems.

Interesting/Challenging Questions

These are questions designed to interest or challenge you, particularly once you have already mastered the basics. They are usually Applying Skills Questions, but are generally more difficult and often involve several different skills in one question, for example, *Solve using substitution: \(3x + y = 10\)
\(4x - z = 7\)
\(7y + 2z = 17\)*

This is a system of equations in three variables, whereas you may have only so far experienced two. The skills are the same, but you need to figure out how to apply those skills to a more difficult problem.

These questions are designed to stretch and challenge you, and are the kind that will usually only be set in class, where you can get help from your teacher or other students to work through the problem. In these questions it is not always the algebra that is more difficult — the questions sometimes involve different ways of thinking. So, even if you struggle with some areas of math, you may still be able to make good headway with some of the Challenge Questions. The most important thing is to not get fazed by them — try applying what you know and see what happens.

Proof Questions

Some questions ask you to use logical arguments to show or prove that a mathematical statement is true or false, or to justify steps in a given proof, for example, *Show that \(2(5x - 4) - 5(2x - 2) = 2,\) or *Given the real number \(x,\) then \(-1 \cdot x = -x,\) as shown below. Fill in the missing properties to support each step in the following proof.*

\[-1 \cdot x = -1 \cdot x + 0\]
\[= -1 \cdot x + \{x + (-x)\}\]
\[= (-1 \cdot x + x) + (-x)\]
\[= (-1 \cdot 1 \cdot x + 1 \cdot x) + (-x)\]
\[= (-1 + 1) x + (-x)\]
\[= 0x + (-x)\]
\[= 0 + (-x)\]
\[= -x\]

Many people struggle with this idea, and are more comfortable with questions where you need to “find an answer.” However, the processes are the same whether you are finding an answer or proving that a given answer is true. Each step should be justifiable, or it may be incorrect.

It is useful to have a list of axioms next to you when you are trying to prove something. If you find yourself unsure of the next step in the proof, look down through the list and see if you can apply any of them. If you find an axiom that you can apply, try it and see what happens. If you struggle with this, try to get your teacher (or other students) to work through plenty of examples with you.
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1. Write the following descriptions in set notation.
   a. The set $D$ contains the elements 3, 6, and 9.  
   b. 8 is an element of the set $N$. 

2. Given that set $P = \{z, 3, 7, w\}$, determine whether each of the following statements is true or false.
   a. $z \notin P$ 
   b. $w \in P$ 
   c. $9 \notin P$ 
   d. $3 \in P$ 

3. Given that set $K = \{\text{factors of } 36\}$, determine whether each of the following statements is true or false.
   a. $72 \in K$ 
   b. $13 \in K$ 
   c. $15 \notin K$ 
   d. $1 \in K$ 
   e. $18 \in K$ 

4. Write down the set $J = \{\text{all odd numbers greater than } 5 \text{ but less than } 16\}$. 

5. Write down the set $L = \{\text{all factors of } 9 \text{ greater than } 4 \text{ but less than } 9\}$. 

6. Let $C = \{\text{multiples of } 3\}$ and $D = \{30, 60, 90\}$.
   a. Explain why $D \subseteq C$. 
   b. Let set $E = \{1, 3, 6, 9, 12\}$. Explain whether $E \subseteq C$. 

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Example

Determine and list the subsets of $B = \{e, 3\}$.

Solution

Subsets of $B = \{e, 3\}$ are: $\emptyset, \{e\}, \{3\}, \{e, 3\}$

The empty set is a subset of every set.

Every set is a subset of itself.

Remember, the factors of a number are all the numbers that divide into it.

Remember, the multiples of a number are all the numbers that divide into it.
7. List all the subsets of the set \( A = \{2, m, 9\} \).

8. Let \( F = \{2, 6, 10, 14\} \) and \( G = \{\text{all even numbers}\} \).
Complete each of the statements below with either \( \in \), \( \notin \), or \( \subseteq \) to make each a true statement.

a. \( 10 \quad \in \quad F \)

b. \( 5 \quad \notin \quad G \)

c. \( F \quad \subseteq \quad G \)

d. \( 18 \quad \in \quad G \)

9. Let the set \( T = \{\text{all factors of 72 less than 36, but greater than or equal to 12}\} \).
List set \( T \) and all its subsets.

10. Let \( H = \{\text{all real numbers } k \text{ such that } k = 4x + 1\} \).
List all the members of set \( H \) if:

   a. \( x \in \{2, 4, 6\} \)

   b. \( x \in \{0, 3, 5\} \)

   c. \( x \in \{1, 2, 7, 8\} \)

   d. List all the subsets of set \( H \) if \( x \in \{2, 4, 6\} \).

11. Let \( A = \{24, 28, 22, 26\} \) and \( B = \{\text{all even numbers less than 30 but greater than or equal to 20}\} \).
Explain whether \( A = B \).

12. Let \( F = \{2x + 3\} \) and \( G = \{3x\} \).

   For what value of \( x \) are the sets \( F \) and \( G \) equal? 

13. Let \( S = \{\text{all odd numbers greater than or equal to 3 but less than 10}\} \).

   Set \( T \) is a subset of set \( S \), containing 4 elements. Explain whether sets \( S \) and \( T \) are equal.
Topic 1.1.2 Subsets of the Real Numbers

California Standards: 1.0

1. Let $M = \{-7, -1, 0, \sqrt{3}, \frac{3}{4}, \pi, 11\}$.
   a. List all the natural numbers in $M$. 
   b. List all the whole numbers in $M$. 
   c. List all the integers in $M$. 

2. Explain whether each of the following statements is true or false.
   a. $N \subset R$ 
   b. $R \subset W$ 
   c. $Z \not\subset R$ 

Example

Determine the subset of $Z$ whose elements are odd and greater than 1.

Solution

\{3, 5, 7, 9, 11,...\} 

You can’t write down all the odd numbers, so just use dots to show that the set continues.

3. Determine the subset of $W$ whose elements are less than 5.

4. Determine the subset of $Z$ whose elements are prime numbers greater than 5 but less than 17.

Remember, prime numbers only divide by themselves and 1.
5. Let \( M = \{-7, -1, 0, \sqrt{3}, \frac{3}{4}, \pi, 11\} \).

   a. List all the rational numbers in \( M \). 
      
   b. List all the irrational numbers in \( M \). 
      
6. Let \( \mathbb{R} = \{\text{real numbers}\} \), \( \mathbb{Q} = \{\text{rational numbers}\} \), and \( \mathbb{I} = \{\text{irrational numbers}\} \).
   Classify each of the following statements as true or false.

   a. \( \mathbb{I} \subseteq \mathbb{R} \) 
   b. \( \mathbb{Q} \subseteq \mathbb{R} \) 
   c. \( \mathbb{Q} \subseteq \mathbb{I} \) 
   d. \( \mathbb{I} \subseteq \mathbb{Q} \) 
   e. \( \mathbb{R} \subseteq \mathbb{I} \) 

7. Determine the subset of \( \mathbb{N} \) whose elements are also members of \( \mathbb{I} \). 

8. Explain whether each of the following statements is true or false.

   a. \( \mathbb{N} \subseteq \mathbb{W} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \) 
   
   b. \( \mathbb{N} \subseteq \mathbb{W} \subseteq \mathbb{Z} \subseteq \mathbb{I} \subseteq \mathbb{R} \) 

9. For each of the following sets, write down which of \( \mathbb{R}, \mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \) and \( \mathbb{I} \) the set is a subset of.

   a. \( A = \{4, 6, 8, 10, 12, 14\} \) 
   
   b. \( B = \{-5, 7, 12, 15, 22\} \) 
   
   c. \( C = \{-\frac{3}{4}, \frac{2}{3}, \frac{1}{5}, 9, 20\} \) 
   
   d. \( D = \{-\sqrt{5}, \sqrt{6}, \sqrt{8}, \sqrt{11}\} \) 

Don't be confused by this notation — it just means that \( \mathbb{N} \subseteq \mathbb{W}, \mathbb{W} \subseteq \mathbb{Z}, \) etc.
**Topic 1.1.3**  
**Unions and Intersections**

**California Standards: 1.0**

**Example**

Let \( X = \{3, k, 7, 3z\} \) and \( Y = \{-5, 3, k, 12\} \).

a. Find \( X \cup Y \) \( \quad \) \( X \cup Y \) means the set of elements that are in \( X \) or \( Y \).

b. Find \( X \cap Y \) \( \quad \) \( X \cap Y \) means the set of elements that are in \( X \) and \( Y \).

**Solution**

a. \( X \cup Y = \{-5, 3, k, 7, 12, 3z\} \)

b. \( X \cap Y = \{3, k\} \)

1. Let \( A = \{1, 0, -5, \frac{1}{2}\} \), \( B = \{-5, 1, e\} \), and \( C = \{0, 1, 2\} \)

Find:

a. \( A \cup B \)

b. \( A \cap C \)

c. \( B \cup C \)

2. Let \( F = \{4, 6, 8, 10\} \), \( G = \{\text{all even numbers}\} \) and \( H = \{3, 5, 7, 9\} \).

Find:

a. \( F \cap G \)

b. \( F \cap H \)

3. Let \( M = \{\text{all natural numbers larger than 7 but less than 11}\} \) and \( C = \{\text{multiples of 2 less than 12 but greater than 4}\} \).

Find:

a. \( M \cap C \)

b. \( M \cup C \)

4. Let \( B = \{2, 4, 6, 8, 10, 12\} \) and \( C = \{3, 6, 9, 12\} \).

Find:

a. \( B \cap C \)

b. \( B \cup C \)

c. the number of subsets of \( B \cap C \) \( \quad \) \( \text{Remember — the empty set is a subset of every set.} \)
5. Let \( A = \{2, t, -4, 3\} \) and \( B = \{2, t, 12, 3, -1\} \).

   a. Find \( A \cap B \).

   b. Show that the number of subsets of \( A \cap B \) is 8.

   c. Explain why \( B \not\subset A \).

6. Let \( A = \{12, 15, 18, 22, 36\} \) and \( B = \{12, 13, 14, 15, 16\} \). Determine the subsets of \( A \cap B \).

7. Let \( K = \{6, 7, 8, 9, 10, 11, 12\} \), \( L = \{12, 14, 16, 18, 20, 22\} \), 
   \( M = \{10, 13, 16, 19, 22\} \), and \( P = \{1, 7, 12, 20, 25, 32\} \).
   Determine:
   a. \( (K \cap P) \cap M \)
   b. \( (L \cap M) \cup P \)
   c. \( (K \cap L) \cap P \)

8. When will the union of two sets result in an empty set?

9. Let \( S = \{-5, -4, -3, -2, -1, 0\} \), \( T = \{-2, 2, 4, 6\} \), and \( V = \{2, 4, 5, 7\} \).
   Determine the number of subsets for each of the following:
   a. \( S \cap T \)
   b. \( S \cap V \)
   c. \( V \cap T \)

10. Let \( C = \{\text{set of natural numbers greater than or equal to 10}\} \), \( D = \{\text{multiples of 10}\} \),
    \( E = \{\text{factors of 60}\} \), and \( F = \{\text{all odd numbers}\} \).
    Determine:
    a. \( (C \cup D) \cap E \)
    b. \( (E \cap D) \cup F \)
    c. \( (F \cap C) \cap D \)
    d. \( E \cap (C \cap D) \)
1. Calculate the value of each numeric expression.
   a. \(834 \div 6\)
   b. \(17(3)\)
   c. \(456 + 44\)
   d. \(400 - 220\)

2. Identify each of the following as a numeric or algebraic expression.
   a. \(5(6)\)
   b. \(16 - y\)
   c. \(y + 12\)
   d. \(64 - 15\)

Example
Write an algebraic expression for each of the following.
   a. The difference between \(y\) and 8
   b. 18 more than the product of 4 and \(z\)
   c. \(m\) divided by 3 minus 30

Solution
   a. \(y - 8\) (or \(8 - y\))
   b. \(18 + 4z\)
   c. \(\frac{m}{3} - 30\)

3. Write an algebraic expression for each of the following phrases.
   a. 6 increased by \(e\)
   b. \(w\) decreased by 12
   c. three times \(h\)
   d. \(5\) divided by \(p\)
   e. three times \(k\) minus twice \(x\)
   f. five more than the product of 2 and \(x\)
4. Write an algebraic expression for each of the following.
   a. The product of 6 more than a number, \( n \), and 25. 
   b. A number, \( n \), divided by 10.

5. Write an algebraic or numeric expression for each of the following.
   a. The cost of \( p \) pencils at $0.20 each
   b. The cost of one marker if 12 markers cost $3.

6. Write an algebraic expression for each phrase:
   a. The cost in dollars of \( k \) books at $5 each.
   b. My age in 5 years if my current age is 2\( k \).
   c. Seven less than five times Jason’s age (\( p \)).
   d. The cost in cents of seven bananas and three apples, if bananas cost \( b \)¢ each and apples cost \( a \)¢ each.

7. Write an algebraic expression for each of the following.
   a. The diameter of a ball, in \( d \) inches, deflated by 1/8th of an inch.
   b. The amount of food eaten by an Asian elephant who eats 130 pounds of food a day over \( x \) days.
   c. The donation to each of five charities when the money collected, \( m \), was distributed evenly.
   d. The total number of hours a 7 day old baby has slept, if he sleeps \( t \) hours each night and \( d \) hours each day.
   e. The price of a dozen roses if each rose costs $\( r \) and there is a $0.75 discount for buying a dozen.

8. Write an algebraic expression for each of the following.
   a. The sum of \( x \) times itself and 4.
   b. \((4x - 3)\) less than \( m \).
   c. Twice \( y \) divided by 3 less than \( y \).
California Standards: 1.1

1. Find the coefficient of the variable in each of these expressions.
   a. $5 + 3v$
   b. $y - 7$
   c. $14 - 2r$
   d. $10m$

2. Find the coefficient of the variable in each of these expressions.
   a. $6(3 + y)$
   b. $9(x - 3)$
   c. $7(2x + 5)$
   d. $8(4 - 3x)$

Example

Simplify the following expressions.
   a. $16 ÷ 8 + 7$
   b. $2(4a + 6a)$

Solution
   a. $16 ÷ 8 + 7 = 2 + 7 = 9$
   b. $2(4a + 6a) = 2(10a) = 20a$

3. Simplify each expression.
   a. $10 + 65$
   b. $72 ÷ 8$
   c. $77 - 11$
   d. $3(4)(5)$
4. Simplify each of the following expressions.
   
a. \(-3(6a)\)  
b. \((-4b)(-3b)\)  
c. \(-14c \div (-2)\)  
d. \(3(-2d)(-4d)\)  

5. If \(r = 4\), \(w = 9\), \(y = 12\), and \(z = 1\) then evaluate each expression.
   
a. \(rz\)  
b. \(w + y\)  
c. \(y - z + 8\)  
d. \(\frac{y}{r}\)  

6. If \(d = -1\), \(e = 9\), \(f = -6\), and \(g = 2\), evaluate each expression.
   
a. \(d + g\)  
b. \(\frac{f}{g}\)  
c. \(de + f\)  
d. \(\frac{eg}{f}\)  
e. \(e - ge\)  
f. \(g(d - f)\)  

7. A jumbo bag of peanuts contains twice as many peanuts as a regular bag and costs $0.60 more.
   
a. The cost of a jumbo bag can be expressed as \($(r + 0.60)\). Evaluate this expression when \(r = 2.7\).  
   
b. Write an algebraic expression for the number of peanuts in a jumbo bag in terms of \(n\), the number of peanuts in a regular bag.  
   
c. If there are 45 peanuts in a regular bag, how many peanuts are there in a jumbo bag?  
   
d. The cost of each peanut in a regular bag can be expressed as \($(r \div n)\). Write an algebraic expression for the cost of each peanut in a jumbo bag.
Identify the property of equality for each of the following.

a. If \( m = 4 \) and \( m = t \) then \( 4 = t \).
b. If \( y = 9 \) then \( 9 = y \).
c. \( 123 = 123 \)

**Solution**

a. Transitive property
b. Symmetric property
c. Reflexive property

1. Name the property of equality being used in each statement.

a. \( 5v = 5v \)

b. If \( 9 + z = 17 \) then \( 17 = 9 + z \)

c. If \( 4r = a \) and \( 28 = a \) then \( 4r = 28 \)

2. Finish each statement using the property of equality given.

a. Reflexive property: \( 4x = \)

b. Transitive property: \( w = t \) and \( t = z \) then

c. Symmetric property: If \( 4 = q \) then

d. Substitution property: If \( x = 3 \) then \( x + 9 = \)

3. Which statement represents the symmetric property of equality?

a. If \( 4 = g \) and \( h = g \) then \( 4 = h \)
b. \( 5c = 5c \)
c. If \( 45 = y \) then \( y = 45 \)

4. Complete each statement to demonstrate the transitive property of equality.

a. If \( 4 + x = y \) and \( y = 26 \) then

b. If \( \frac{r}{8} = 12 \) and \( 12 = 15y \) then

c. If \( 2d = f \) and \( f = 40 \) then
5. Identify each statement as true or false.
   a. The input numbers for multiplication are called addends.
   b. The binary operations are addition and subtraction.
   c. For any real numbers, the sum of two real numbers is a real number.

6. Use the fact that $7 \in \mathbb{R}$ and $8 \in \mathbb{R}$ to explain why $15 \in \mathbb{R}$ and $56 \in \mathbb{R}$.

7. Kenzie buys a CD that costs $11.99. He also has to pay tax on the CD of $1.15. Is the total cost of the CD (including tax) an element of the real numbers? Explain your answer.

8. $A = \{\text{all odd numbers}\}$ and $B = \{\text{factors of 24}\}$. Decide whether the following statements are true or false.
   a. $A$ is closed under addition and multiplication for the numbers 5 and 13.
   b. $B$ is closed under addition and multiplication for the numbers 2 and 6.

9. Show that the sets of numbers, $\mathbb{W}$, $\mathbb{Z}$, and $\mathbb{R}$, are closed under the binary operations of addition and multiplication using the numbers given.
   a. $\mathbb{W}$ using 0 and 9
   b. $\mathbb{Z}$ using $-5$ and 8
   c. $\mathbb{R}$ using $\frac{3}{4}$ and $\frac{1}{3}$

10. If $\sqrt{6} \in \mathbb{I}$ is true, show that $\left(\sqrt{6}\right)\left(\sqrt{6}\right) \in \mathbb{I}$ is not true.
1. Find the value of \( x \) in the following equations.
   a. \((3 + 4) \times 7 = 60 - 11\) and \(x[(3 + 4) \times 7] = 60 - 11\)
   b. \(2f + 4 = 24\) and \(x(2f + 4) = 24\)
   c. \(16 + x = 16\)
   d. \(h + x = h\)

Example

Complete the table.

<table>
<thead>
<tr>
<th>Number</th>
<th>Additive Inverse</th>
<th>Multiplicative Inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-6</td>
<td>1/6</td>
</tr>
<tr>
<td>-7</td>
<td>7</td>
<td>-1/7</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.5</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Find the additive inverse of each of the following:
   a. 12
   b. -48
   c. 5m
   d. \(-\frac{2}{5}\)
   e. 4 + t
   f. 3w - 6

3. Using the additive inverse, find the value of \( x \).
   a. \(6.3 + x = 0\)
   b. \(x + (-4) = 0\)
   c. \(\frac{3}{4} + \left(-\frac{3}{4}\right) = x\)
4. Find the reciprocal of each number.
   a. 78  -------------------------------  b. 3  -------------------------------
   c. \( \frac{3}{8} \)  -------------------------------  d. -16  -------------------------------

5. Find the multiplicative inverse of each of the following.
   a. \( g \)  -------------------------------  b. \( r^{-1} \)  -------------------------------
   c. \( -\frac{1}{5} \)  -------------------------------  d. 1.5  -------------------------------

6. Complete the table.

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<tbody>
<tr>
<td>a. 10y</td>
<td></td>
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</tr>
<tr>
<td>b. -x</td>
<td></td>
<td></td>
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<tr>
<td>c. 2/7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. -0.8</td>
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</tbody>
</table>

7. Using the multiplicative inverse find the value of \( x \).
   a. \( 19x = 1 \)  -------------------------------  b. \( \frac{5}{6}x = 1 \)  -------------------------------
   c. \( -1x = 1 \)  -------------------------------  d. \( r^{-1}x = 1 \)  -------------------------------

8. Rewrite each number using positive exponents.
   a. \( 5^{-1} \)  -------------------------------  b. \( (-17)^{-1} \)  -------------------------------
   c. \( \left(\frac{1}{2}\right)^{-1} \)  -------------------------------  d. \( (0.6)^{-1} \)  -------------------------------

9. Willard bought a coat for $63 on Monday and returned the coat on Wednesday. Write an equation using the additive inverse property to express Willard’s purchase and return of the coat.

   ---------------------------------------------------------------
The Number Line and Absolute Values

California Standards: 1.1

Example

Plot the following numbers on the number line below.
A = –6
B = 9
C = 0
D = –3

Solution

There's a one-to-one correspondence between the real numbers and the points on a number line.

1. Identify the corresponding real numbers for each point on the number line below.

   a. ........................................
   b. ........................................
   c. ........................................
   d. ........................................
   e. ........................................

2. Plot the following numbers on the number line below. Label each point with the corresponding letter.
   a. –3
   b. –9
   c. 10
   d. 2

3. Plot the following numbers on the number line below. Label each point with the corresponding letter.
   a. 3.5
   b. –5.9
   c. –9.1
   d. 5.7
4. Plot the following numbers on the number line below. Label each point with the corresponding letter.
   a. \( \frac{3}{4} \) 
   b. \( \frac{5}{2} \) 
   c. \(-0.25\) 
   d. \(-\frac{2}{3}\)

5. Plot the following numbers on the number line below. Label each point with the corresponding letter.
   a. \(-\sqrt{2}\) 
   b. \(\pi\) 
   c. \(\sqrt{4}\) 
   d. \(-\sqrt{5}\)

6. Use the number line to solve the following problems.
   a. \(5 - 4\) 
   b. \(-8 + 10\) 
   c. \(8 - 1 + 3\) 
   d. \(-5 - (-2)\)

7. Simplify the following expressions.
   a. \(|24|\) 
   b. \(|-13|\) 
   c. \(\left|\frac{-3}{10}\right|\) 
   d. \(|0.04|\)

8. Simplify the following expressions.
   a. \(|-4 + 2|\) 
   b. \(|-\sqrt{7}|\) 
   c. \(|6 - 4|\) 
   d. \(|-25|\) 
   e. \(|-25|\) 
   f. \(|-25|\)

9. Solve the following equations to find the value of \(x\).
   a. \(|47| = x\) 
   b. \(|-8| = x\) 
   c. \(|x| = 13\) 
   d. \(|x| = \frac{2}{5}\)
Topic 1.2.4  Addition and Multiplication

California Standards: 1.0

Example

State with a reason the sign of each problem.

a. \((-18) + 20\)
b. \(9(-7)\)
c. \((-7)^2\)
d. \(20 + 7\)

Solution

a. The sum is positive since \(|20| > |-18|\).
b. There is an odd number of negative factors, so the product is negative.
c. There is an even number of negative factors, so the product is positive.
d. The sum is positive since both numbers are positive.

1. State with a reason the sign of each sum, then find the sum.

a. \(5 + 18\)
b. \(-5 + (-16)\)
c. \((-4) + 2\)
d. \(9 + (-1)\)

2. Simplify the following expressions.

a. \(x + (-11)\)
b. \(-6 + (-x)\)
c. \(x + 19\)
d. \(15 + (-x) + y\)
e. \((-x) + (-y) + z\)

3. State with a reason the sign of each product, then find the product.

a. \((-4)(-11)\)
b. \(10(8)\)
c. \((-12)(3)\)
d. \((-9)^3\)
4. Simplify the following expressions.
   a. \((-20)(-x)\)  
   b. \((y)(-11)\)  
   c. \(8(x)\)  
   d. \((-x)(-y)(2)\)  
   e. \((-x)^4\)

5. Given: \(x = y^{27}\)
   a. For what values of \(y\) will \(x\) be positive?  
   b. For what values of \(y\) will \(x\) be negative?

6. Given: \(x = -h^4\)
   a. For what values of \(h\) will \(x\) be positive?  
   b. For what values of \(h\) will \(x\) be negative?

7. Evaluate the following numerical expressions.
   a. \(5 + (-3)\)  
   b. \(5(-6) + 24\)  
   c. \(14 - 12 - (-5)\)  
   d. \(4(-5)^2\)  
   e. \(5^2 + (-5)^2\)  
   f. \(-2[-3 + 4(-5)]\)  
   g. \(6^3 + (-3)^3\)

8. State with a reason the sign of each product.
   a. \(t^8\) where \(t < 0\)  
   b. \(t^{13}\) where \(t > 0\)  
   c. \(4t^2v^3\) where \(t < 0\) and \(v > 0\)  
   d. \(-4t^2v^3\) where \(t < 0\) and \(v > 0\)
Topic 1.2.5

Subtraction and Division

California Standards: 1.0, 2.0

Example

a. Rewrite $5 - (-17)$ as an addition problem.
b. Rewrite $32 \div 8$ as a multiplication problem. Remember that dividing is the same as multiplying by a reciprocal

Solution

a. $5 + 17$
b. $32 \frac{1}{8}$

1. Rewrite the following subtractions as additions.
   a. $-6 - (-7)$
   b. $18 - 9$
   c. $25 - (-14)$

2. Rewrite the following additions as subtractions.
   a. $5 + 17$
   b. $24 + (-2)$

3. Evaluate the following:
   a. $-9 - (-16)$
   b. $16 - 13$
   c. $15 - (-17)$
   d. $-\frac{2}{3} - \frac{1}{2}$

4. Rewrite each of the following as a division:
   a. $-15(4)$
   b. $\frac{6}{11} \left( -\frac{2}{3} \right)$
   c. $\frac{a}{b} (c)$
5. Rewrite the following divisions as multiplications.
   a. \(-25 \div 5^{-1}\)  
   b. \(-\frac{2}{5} \div \left(-\frac{1}{3}\right)\)  
   c. \(18 \div (-6)\)

6. Simplify the following:
   a. \(45 \div (-9)\)
   b. \(-45 \div 5^{-1}\)
   c. \(-45 \div \frac{1}{3}\)
   d. \(45^{-1} \div 3^{-1}\)

7. Using the numbers 8 and \(-5\) show that:
   a. addition is commutative
   b. multiplication is commutative
   c. subtraction is not commutative
   d. division is not commutative

8. A quarterback lost 8 yards on the first play, gained 12 yards on the second play, gained 23 yards on the next play, and lost 4 yards on the fourth play. Show the quarterback’s total gain or loss of yards over the four plays as an addition and solve.

9. The debate club raised $912 at its last fund-raiser. The twelve members of the club share the money equally between them. They use it to pay travel expenses for the next two debate tournaments. How much money will each member have for each debate tournament?
Topic 1.2.6

Order of Operations

California Standards: 1.1

1. Simplify the following:
   a. \((4 + 7) - (5 + 6 - 2)\)
   b. \(8 - 12 + (3 - 7)\)
   c. \(4 - (5 + 5)\)
   d. \(8 + (2 - 3) - 12\)
   e. \(7 + [2 + (1 - 3) - 5]\)
   f. \([(17 - 5) + 2] - (4 - 2)\)
   g. \(16 - \{7 - [4 - (–5)] + (7 + 11)\}\)

Example

Simplify the following numeric expression: \(8 - 9 \times 2 - 24 \div 4 + 7\)

Solution

\[
\text{Evaluate expressions in the order: grouping symbols, exponents, multiplications and divisions (from left to right), additions and subtractions (from left to right).}
\]

Solution

\[
8 - 9 \times 2 - 24 \div 4 + 7
\]
\[
= 8 - 18 - 6 + 7
\]
\[
= -10 - 6 + 7
\]
\[
= -16 + 7
\]
\[
= -9
\]

2. Evaluate the following:
   a. \(32 \div 4 + 2 \times 3\)
   b. \(3 \times (2 - 3) + 18 \div 3\)
   c. \([5 \times 2 \div (12 \div 6)] + 3 \times 7\)
   d. \(11 - 3^2 + 4 \cdot 3 - 17\)
   e. \(3 - [17 - (2 + 5) \cdot 2^3 - 7^2]\)
   f. \(\frac{36 - 5 \cdot 2^2}{2(3 + 5)}\)
   g. \(-3^2(-2)(-4)\)
   h. \((-4)^2 - 3 \cdot 2^3 + (-2)^3\)
   i. \([-4^2 \cdot 3 - 3^2 \cdot (-2)] \div (-3^2 - 1)\)
3. Evaluate the following:
   a. \(-3(-4)\left(-\frac{1}{6}\right)\)  
   b. \(\frac{4 - 7^3}{(4 - 7)^2}\)

4. Simplify the following:
   a. \((18 - 6 \cdot 3) + (2 - 6 \cdot 3)\)  
   b. \(3(14 \cdot 2 - 7) - (4 + 9 \cdot 5)\)  
   c. \(\{(5 \cdot 3) - (7 \cdot 8) - [-1 \cdot 4 + (-3)(9)]\} - 14\)  
   d. \((-3)^2 \cdot 2^3 - [-1 \cdot 4 + (-3)(9)] + 4^2 - 18\)

5. Insert grouping symbols to make each statement true.
   a. \(5 \times 3 + 2 - 7 \div 2 = -5\)  
   b. \(-8 \times 3 - 4^2 \div 2 + 9 \times 3 - 2 = 49\)

6. Evaluate each expression in exercises a. – f., using the given values.
   a. \(x^2 - x\), when \(x = 3\)  
   b. \(3a^2 - x\), when \(a = 2, x = -3\)  
   c. \(A = \pi r^2\), when \(r = 4\)
   d. \(5a^3 - 2a + b\), when \(a = -1, b = 7\)  
   e. \(\frac{k^2 + 2k - 3}{t^2 - 5}\), when \(k = 3, t = 4\)  
   f. \(2pl\), when \(r = 4, l = 6\)

7. Evaluate the following expressions for \(x = -2, y = -3\), and \(m = 4\).
   a. \(-xmy^2\)  
   b. \(x^2 - 2xy - y^2\)  
   c. \(\frac{m^2 - 3m}{x^2 - 2y}\)
1. Identify the property that makes each of the following statements true.
   a. \( xy = yx \)
   b. \( m + (6 + p) = (m + 6) + p \)
   c. \( (r + 9)(-3) = -3r - 27 \)
   d. \( -19 + -12 = -12 + -19 \)

2. Which of the following is not a property of real numbers?
   a. Associative property of addition
   b. Commutative property of multiplication
   c. Distributive property of addition over multiplication

3. Complete each equation to reflect the property listed.
   a. Associative Property of Addition, \( \quad = (-8 + w) + r \)
   b. Distributive Property, \( \quad = 3a + 24 \)
   c. Commutative Property of Addition, \( p + 22 = \quad \)
   d. Associative Property of Multiplication, \( (-8w)(7) = \quad \)
   e. Distributive Property, \( \quad = x^2 + 3x \)
4. Decide whether each of the following statements is true or false for real numbers.
   a. \( a \times b = b \times a \)
   b. \( a \times (b + c) = (a + b) \times c \)
   c. \( a \times (b \times c) = (a \times b) \times c \)
   d. \( -a \times -(b + c) = -(a + b) \times -c \)

5. Expand:
   a. \(-9(x - 6)\)
   b. \(3(2x + 4y)\)
   c. \(-(c - 2d + 3e)\)
   d. \((13 - 3r)(-2)\)
   e. \(12[(a + b) - c]\)
   f. \(-6[(3w + x) + (2y - z)]\)

6. Complete the statements below.
   a. \(4(3x + \ldots) = 12x - 36\)
   b. \(\ldots(-5m - 7) = 15m + 21\)
   c. \(\ldots(y + 8z) = -4y - 32z\)
   d. \(\ldots-2 = -12p + 4\)

7. Show that the associative property does not hold for subtraction.

8. Charlie has fourteen more pretzels than his brother, Sam. Charlie’s mom decides to double his number of pretzels. Write an expression using the distributive law to represent the number of pretzels Charlie has, \(c\), compared to Sam, \(s\).
Topic 1.2.8
More on Multiplication

California Standards: 2.0

Example

Find the opposite of each expression.

a. $9t$
b. $-3x + 14$

Solution

a. $-9t$
b. $-(3x + 14) = 3x - 14$

1. Find the opposite for each expression.

a. $-6$

b. $-27y$

c. $18 + u$

d. $-39 - 8h$

e. $12h + 27$

f. $-x + 2y$

2. Solve for $x$.

a. $x + 7 = 0$

b. $-9 + x = 0$

c. $x(x - 6) = 0$

d. $(5 + x)(3) = 0$

3. Josie solves the equation $4x(x + 2) = 0$ and writes down her answer as “$x = 0$ and $x = -2$”. Why is she wrong?
4. Explain why the statement below is false.
If \( x, y \in \mathbb{N} \), and \( xy = 0 \), then either \( x = 0 \) or \( y = 0 \).

5. True or false?
   a. The opposite of \( t - 7 \) is \( -t - 7 \).
   b. The opposite of \( -7m + 8 \) is \( 7m - 8 \).
   c. \( 7 + 0 = 7 \times 0 \)
   d. If \( 8 - x = 0 \) then \( x = 8 \).
   e. If \( (8 - x)(x + 2) = 0 \), then \( x = 2 \) or \( x = 8 \).

6. Why is it not possible to divide any number by zero?

7. Solve.
   a. \( x(x + 10)(x - 10) = 0 \)
   b. \( 4x(5 - x) = 0 \)
   c. \( (x - 3)(2x - 5) = 0 \)
   d. \( (26 - x)(-8x) = 0 \)
   e. \( (5x - 10)(25x) = 0 \)
   f. \( (18 - 3x)(-2 - x)(4x + 24) = 0 \)
   g. \( (-2 - 3x)(-x)(-5x - 45) = 0 \)
1. In exercise a. – l., state a real number property that justifies each statement.

   a. \( a(7 + 3) = 7a + 3a \)  
   b. \( k \times (5 \times c) = (k \times 5) \times c \)  
   c. \( m \cdot m^{-1} = 1 \)  
   d. \( 12 + 7 = 7 + 12 \)  
   e. \( 7 - 3 = 7 + (-3) \)  
   f. \( 23 \cdot 4 = 4 \cdot 23 \)  
   g. \( 4 + (-1) = (-1) + 4 \)  
   h. \( (m - 5)c = mc - 5c \)  
   i. \( k + 0 = k \)  
   j. \( (6 + 2) + 13 = 6 + (2 + 13) \)  
   k. \( -3 + 3 = 0 \)  
   l. \( 17 \times 1 = 17 \)  

2. Which property of real numbers is illustrated by \( (h - t) + [-(-h - t)] = 0 \)?

3. Which property of real numbers is illustrated by the expression \( 9 \cdot (3 \cdot 5) = (3 \cdot 5) \cdot 9 \)?
4. Given real numbers 3, r, and t, show that \(3(r - t) = 3r - 3t\). Support your argument.

5. Show that \(8(4u) = 8u(4)\). Support your argument using real number properties.

6. The following is a proof showing \((6 + (-6)) + 4x - 8 = 4x - 8\).
   Fill in the missing properties to support each step in the proof.
   \[
   (6 + (-6)) + 4x - 8 = 0 + 4x - 8 \\
   = 0 + 4x + (-8) \\
   = 0 + (-8) + 4x \\
   = -8 + 4x \\
   = 4x + (-8) \\
   = 4x - 8
   \]
   Definition of subtraction

7. Show that \(8(x - 2) + 16(1) = 8x\). Justify each step in your proof.

8. State the real-number property that justifies each step in the following procedure.
   \[
   a + b(c + 1) = a + [b \times c + (b \times 1)] \\
   = a + [(b \times c) + b] \\
   = a + [b + (b \times c)] \\
   = (a + b) + b \times c
   \]

9. Show that \(-3(c + 6) + (3 + \frac{1}{2}c)(6) = 0\). Justify each step in your proof.
Exponent Laws

California Standards: 2.0

1. Write each expression using exponents.
   a. \(5 \times 5 \times 5 \times 5 \times 5\)  
   b. \(2 \times 2 \times 2 \times h \times h\)  
   c. \(7 \times p \times 7 \times p \times 7 \times p\)  
   d. \(8 \times w \times y \times 8 \times y \times y \times y \times y\)  
   e. \(6 \times r \times 6 \times 6 \times t \times r \times t\)  
   f. \(10 \times m \times n \times m \times 10 \times m\)

2. Solve:
   a. Find the area of a square patio that has a side length of 14 feet.

   b. Find the volume of a cube with a side length of 14 inches.

3. Expand each expression and evaluate.
   a. \(6^3\)
   b. \(9^{-2}\)
   c. \(2^4 \cdot 2^3\)
   d. \((3^3)^2\)
   e. \((-1)^4\)
4. Show that \((x^3)^2 = x^6\).

5. Is \((-3)^4\) equivalent to \(3^4\)? Explain.

6. Simplify each expression in exercises a. – f.
   - a. \(6x^0\)
   - b. \((3a)^2 \cdot 2a\)
   - c. \(\frac{3^3r^8h}{(rh)^2}\)
   - d. \(2^2v \cdot (2v)^3\)
   - e. \((x^4 \cdot x^{-3})^2 \div x^5\)
   - f. \((9x)^\frac{3}{2} \cdot x^3\)

7. Simplify \(\{3(6 - 4)^2 - 2^1\} \cdot 3 - 3^2\).

8. Simplify \(\frac{12p^3r^2t}{6p^3r^3t}\).

   - a. \(\frac{(2x)^3(-1)^5}{(2x)^3}\)
   - b. \((x^5yz^8)(x^{-3}yz^4)\)
   - c. \(\left(\sqrt{36x^4}\right)(-2x^2)^3\)

---

**Topic 1.3.1**
## Square Roots

**California Standards: 2.0**

1. Complete the following.
   a. \( \sqrt[2]{w} \) is read the ______ root of ______.
   b. The radicand of \( \sqrt[2]{x + y} \) is ______.
   c. \( (\sqrt{15})^2 \) equals ______.
   d. \( \sqrt[2]{m} \) can be written \( m^{\text{______}} \).

### Example

Use the "±" symbol to give the principal and minor square root for each of the following.

a. \( 361 \)  
   b. \( h^2 \)  
   c. \( (rt)^2 \)

**Solution**

a. \( ±7 \)  
   b. \( ±h \)  
   c. \( ±rt \)

### Example

Find the principal and minor square root of \( \frac{16a^2}{45b^2c^2} \).

**Solution**

\[
±\sqrt[2]{\frac{16a^2}{45b^2c^2}} = ±\frac{\sqrt[2]{16a^2}}{\sqrt[2]{45b^2c^2}} = ±\frac{4a}{3\sqrt[2]{5bc}}
\]

Every expression has both a **positive** (principal) and a **negative** (minor) square root.

2. Use the "±" symbol to give the principal and minor square root for each of the following.

a. \( 361 \)  
   b. \( (5r)^2 \)

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3. Evaluate the following. Express your answers using only the principal square root.
   a. $121^{\frac{1}{2}}$                          b. $(16 \times 16)^{\frac{1}{2}}$
   c. $(d^2)^{\frac{1}{2}}$                      d. $(t^4)^{\frac{1}{2}}$

4. Simplify the following. Express your answers using only the principal square root.
   a. $\sqrt{5 \cdot 5 \cdot t}$                b. $\sqrt{(3h + k)^2}$
   c. $\sqrt{(2g)^2}$                          d. $\sqrt{3(m + n)^2}$

5. Simplify the following expressions. Express your answers using only the principal square root.
   a. $(t^8)^{\frac{1}{2}}$                     b. $(t^8)^{\frac{1}{2}}$
   c. $\sqrt{x^{16}}$                          d. $(w^{26})^{\frac{1}{2}}$
   e. $\sqrt{(x + y)^3 \cdot (x + y)^5}$      f. $\sqrt{(3h + 2)^{12}}$

6. Show that $\sqrt{(a + b)^2} \neq \sqrt{a} + \sqrt{b}$ by using $a = 16$ and $b = 9.$

7. Given: $\sqrt{j^2} = |j|,$ explain why the answer includes absolute value bars.

8. Determine the values of $x$ that make the radicand positive.
   a. $\sqrt{x + 4}$                            b. $\sqrt{x - 7}$
Properties of Roots

California Standards:  2.0

Example

Find the following.

a. $\sqrt{32}$
   
   b. $\sqrt{18}$
   
   c. $\sqrt{16a^2} \div \sqrt{5b^2c^2}$

Solution

a. $\sqrt{16} \sqrt{2} = 4\sqrt{2}$
   
   b. $\frac{\sqrt{9} \sqrt{2}}{\sqrt{25}} = \frac{3\sqrt{2}}{5}$
   
   c. $\frac{\sqrt{16} \sqrt{a^2}}{\sqrt{5}\sqrt{b^2} \sqrt{c^2}} = \frac{4|a|}{3\sqrt{5|bc|}}$

Both 4 and 16 are factors of 32 and perfect square roots. Choose the larger perfect square root.

1. Find the following.
   
   a. $\sqrt{72}$
   
   b. $\sqrt{80}$
   
   c. $\sqrt{50}$
   
   d. $\sqrt{92}$
   
   e. $\sqrt{108}$

2. Find the following.
   
   a. $\sqrt{36x^2}$
   
   b. $\sqrt{81x^4}$
   
   c. $\sqrt{56x^2y^2}$
   
   d. $\sqrt{63(mn)^2}$
   
   e. $\sqrt{12x^2}$
   
   f. $\sqrt{72j^6k^6}$

3. Find the following.
   
   a. $\sqrt{9(x + 9)^2}$
   
   b. $\sqrt{100(x - 5)^2}$
   
   c. $\sqrt{8(2x + 3)^2}$
4. Find the following.

a. \(\sqrt{\frac{169}{100}}\)  
   b. \(\sqrt{\frac{400}{49}}\)

\[= \frac{13}{10}, \frac{20}{7}\]

c. \(\sqrt{\frac{24}{50}}\)  
   d. \(\sqrt{\frac{28}{45}}\)

\[\neq \frac{6}{5}, \frac{2\sqrt{7}}{3}\]

e. \(\sqrt{\frac{12}{49}}\)  
   f. \(\sqrt{\frac{48}{63}}\)

\[\neq \frac{2}{7}, \frac{4\sqrt{3}}{3}\]

5. Find the following.

a. \(\sqrt{\frac{w^2}{144}}\)  
   b. \(\sqrt{\frac{8f^2g^2}{9}}\)  
   c. \(\sqrt{\frac{12a^2}{64}}\)

\[= \frac{w}{12}, \frac{4fg}{3}, \frac{3a}{8}\]

d. \(\sqrt{\frac{36p^2}{4d^2}}, d \neq 0\)  
   e. \(\sqrt{\frac{20k^2}{6l^2}}, t \neq 0\)  
   f. \(\sqrt{\frac{40r^6}{l^2}}, l \neq 0\)

\[= \frac{3p}{2d}, \frac{2k}{\sqrt{6l}}, \frac{2\sqrt{10r^3}}{l}\]

6. Find the following.

a. \(\sqrt{\frac{(m-n)^2}{(3x)^2}}, x \neq 0\)  
   b. \(\sqrt{\frac{(4-z)^2}{(y+5)^2}}, y \neq -5\)  
   c. \(\sqrt{\frac{16(a+b)^2}{25(x-7)^2}}, x \neq 7\)

\[= \frac{m-n}{3x}, \frac{4-z}{y+5}, \frac{4(a+b)}{5(x-7)}\]
**Equivalent Fractions**

**California Standards: 2.0**

1. a. Identify the numerator in the fraction \( \frac{24}{37} \).  

b. Identify the denominator in the fraction \( \frac{5}{12} \).

c. Identify the greatest common factor of the fraction \( \frac{12}{28} \).

2. Convert each fraction to \( \frac{1}{2} \).
   a. \( \frac{24}{48} \)  
   b. \( \frac{9}{18} \)

3. Convert each fraction to \( \frac{4}{7} \).
   a. \( \frac{8}{14} \)  
   b. \( \frac{24}{42} \)

4. Show that \( \frac{64}{72} \) and \( \frac{8}{9} \) are equivalent fractions.

---

**Example**

Convert the fractions to \( \frac{4}{5} \).

a. \( \frac{20}{25} \)  
b. \( \frac{12}{15} \)

**Solution**

a. \( \frac{20}{25} = \frac{4}{5} \)  
b. \( \frac{12}{15} = \frac{4}{5} \)

---

CGP Education Algebra I — Homework Book
5. Find four fractions equivalent to \( \frac{2}{9} \).

6. Nicole cleans six of the ten windows in her house. Her mom said she needs to wash \( \frac{3}{5} \) of the windows. Has Nicole cleaned enough windows? Explain your answer.

7. Find the greatest common factor for each of the following.
   a. 18 and 36
   b. 24 and 30
   c. 45 and 81
   d. 32, 48, and 64

8. Seamus has done 18 homework problems out of the 26 assigned.
   a. Write a ratio in its simplest form to express the number of homework problems completed to the number of homework problems assigned.
   b. Write a ratio in its simplest form to express the number of homework problems completed to the number of homework problems left to finish.

9. Simplify each fraction.
   a. \( \frac{25}{35} \)
   b. \( \frac{30}{60} \)
   c. \( \frac{16}{24} \)
   d. \( \frac{4}{10} \)
   e. \( \frac{6}{24} \)
   f. \( \frac{9}{24} \)

10. Simplify.
    a. \( \frac{15x^2}{18x} \)
    b. \( \frac{35y^3}{49y^5} \)
    c. \( \frac{28z^4}{32z} \)
Multiplying and Dividing Fractions

California Standards: 2.0

Example

Simplify.

\[
a. \quad \frac{8}{9} \div \frac{12}{20} \\
b. \quad \frac{10}{16} \div \frac{3}{8}
\]

Solution

\[
a \cdot c \div b \cdot d = \frac{ac}{bd} \\
a \cdot c \div b \cdot d = \frac{ad}{bc}
\]

\[
a. \quad \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}{\cancel{9} \cdot \cancel{3} \cdot \cancel{2} \cdot 5} = \frac{8}{15} \\
b. \quad \frac{10 \cdot 8}{16 \cdot 3} = \frac{1 \cdot 5 \cdot 2}{1 \cdot 3 \cdot 2} = \frac{5}{3}
\]

1. Simplify each expression below.

\[
a. \quad \frac{16}{39} \div \frac{26}{32} \\
b. \quad \frac{9}{24} \div \frac{20}{27} \\
c. \quad \frac{5}{16} \div \frac{42}{55}
\]

\[
d. \quad \frac{21}{36} \div \frac{4}{7} \\
e. \quad \frac{9}{48} \div \frac{22}{45} \\
f. \quad \frac{42}{81} \div \frac{63}{20}
\]

\[
g. \quad \frac{8}{21} \div \frac{26}{35} \\
h. \quad \frac{15}{28} \div \frac{27}{49} \\
i. \quad \frac{42}{25} \div \frac{36}{65}
\]

\[
j. \quad \frac{12}{91} \div \frac{48}{39} \\
k. \quad \frac{14}{45} \div \frac{35}{54} \\
l. \quad \frac{17}{40} \div \frac{34}{75}
\]
2. A cookie recipe needs $2/3$ cup of sugar. If Mrs. Scheibel plans to halve the recipe then how much sugar does she need for this cookie recipe?

   
   a. \( \frac{3}{4} \cdot \frac{8}{15} \cdot \frac{10}{32} \)
   
   b. \( \frac{2}{3} \cdot \frac{-9}{10} \cdot \frac{14}{21} \)
   
   c. \( \frac{4}{10} \cdot \frac{8}{15} \cdot \frac{10}{12} \)

4. Isaiah earned $140 last week at work. If he plans to deposit three-fourths of his weekly earnings in his savings account then what is the amount of the deposit?

5. a. Mrs. Kaster and Mrs. Gill walk $3/4$ of a mile in $1/3$ of an hour. What is their speed in miles per hour?

   b. Jessica runs $5/6$ of a mile in $1/6$ of an hour. What is Jessica’s speed in miles per hour?

   
   a. \( \frac{2}{3} \div \frac{3}{10} \div \frac{6}{11} \)
   
   b. \( \left( \frac{-15}{22} \div \frac{-8}{5} \right) \div \left( \frac{3}{4} \div \frac{9}{11} \right) \)
   
   c. \( \left( \frac{1}{2} \div \frac{3}{4} \right) \div \left( \frac{3}{7} \div \frac{6}{14} \right) \)

7. Simplify.
   
   a. \( \frac{14x^3}{15y} \cdot \frac{35y^4}{21x} \)
   
   b. \( \frac{5c^3}{8d} \cdot \frac{12d}{15c^2} \)
   
   c. \( \frac{9f}{11h} \div \frac{21f}{33h^2} \)
Adding and Subtracting Fractions

California Standards:  2.0

Example

Simplify.

a. $\frac{9}{10} + \frac{7}{10}$

b. $\frac{3}{7} - \frac{1}{5}$

Solution

a. $\frac{9+7}{10} = \frac{16}{10} = \frac{8}{5}$

b. $\frac{3\cdot5 - 1\cdot7}{7\cdot5 - 5\cdot7} = \frac{15 - 7}{35 - 35} = \frac{8}{35}$

1. Simplify each expression in exercises a. – l.

a. $\frac{3}{14} + \frac{9}{14}$

b. $\frac{4}{9} + \frac{8}{9}$

c. $\frac{18}{27} - \frac{25}{27}$

d. $\frac{43}{40} - \frac{31}{40}$

e. $\frac{5}{7} + \frac{13}{14}$

f. $\frac{7}{6} + \frac{9}{4}$

g. $\frac{17}{21} + \frac{19}{28}$

h. $\frac{26}{19} + \frac{45}{38}$

i. $\frac{43}{16} - \frac{21}{24}$

j. $\frac{11}{30} - \frac{19}{20}$

k. $\frac{8}{9} - \frac{13}{12}$

l. $\frac{11}{12} - \frac{7}{8}$

First, convert the fractions into equivalent fractions with the same denominator.
2. Find the least common multiple of each set of numbers.
   a. 24 and 36  
   b. 18 and 48  
   c. 34 and 27  
   d. 12, 36, and 48  
   e. 15, 30, and 45

3. Sophie’s grandma gave her ½ of a dollar on Monday and ¾ of a dollar on Saturday. Express the total amount of money Sophie’s grandma gave her as a fraction.

4. A dump truck of soil deposits 2/5 of the truck’s capacity at construction site A before lunch. Half of the truck’s capacity is deposited at construction site B after lunch.
   a. What part of the total truck’s capacity was deposited at the two construction sites?
   b. How much soil remains in the truck after construction site B?

5. Simplify.
   a. $\frac{2}{3} - 4 \cdot \frac{1}{2} + \frac{2}{5}$
   b. $\frac{5}{12} + \left(\frac{2}{3}\right)^2 \div 4 - \frac{1}{3}$
   c. $\frac{-2}{3} \cdot \frac{15}{17} + \frac{3}{17} \cdot \frac{1}{4}$
   
   d. $\sqrt{\frac{1}{16}} + \sqrt{\frac{9}{36}}$
   e. $\sqrt{\frac{8^2}{12^2}} - \sqrt{\frac{3^2}{5^2}}$
   f. $\sqrt{\frac{49}{225}} - \sqrt{\frac{9}{25}}$
### Mathematical Proofs

**Example**

Complete the following proof by adding either the next step to the left-hand column, or the justification to the right-hand column.

\[
x - 2 = 10
\]
\[
(x - 2) + 2 = 10 + 2
\]
\[
[x + (-2)] + 2 = 10 + 2
\]
\[
[x + (-2)] + 2 = 12
\]
\[
x + [(-2) + 2] = 12
\]
\[
x + 0 = 12
\]
\[
x = 12
\]

Use real number axioms to justify each step.

- **given equation**
- **addition property of equality**
- **definition of subtraction**
- **associative property of addition**
- **inverse property of addition**
- **identity property of addition**

1. Complete the following proofs by adding either the next step to the left-hand column, or the justification to the right-hand column.

   **a.** Given: \(-9m = 27\); Prove: \(m = -3\)

   \[
   -9m = 27
   \]
   \[
   \left(\frac{-1}{9} \times (-9)\right) m = \frac{-1}{9} \times 27
   \]
   \[
   \left(\frac{-1}{9} \times (-9)\right) m = -3
   \]
   \[
   m = -3
   \]
   - **Multiplication property of equality**
   - **Inverse property of multiplication**

   **b.** Given: \(n - 15 = 22\); Prove: \(n = 37\)

   \[
   n - 15 + 15 = 22 + 15
   \]
   \[
   n + 0 = 37
   \]
   - **Given equation**
   - **Inverse property of addition**
   - **Identity property of addition**
2. Use a two-column proof to show that \( x = 48 \) given \( x \div 2 = 24 \).

3. Solve the following equations.
   a. \( 4x + 9 = 29 \)
   b. \( (x - 3) \div 5 = -6 \)
   c. \( 4(x + 2) + 3 = 27 \)
   d. \( 2x + 6 = 4(x + 1) \)

4. Rewrite the following in “If… then...” format.
   a. Hypothesis: \( 3c + 2 = 17 \); Conclusion: \( c = 5 \)
   b. Hypothesis: \( d \div 12 = -3 \); Conclusion: \( d = -36 \)
   c. \( 14 - x = 7 \) means that \( x = 7 \)
   d. A sixteen year old teenager is eligible to drive in Missouri.

5. Identify the hypothesis and conclusion in each sentence.
   a. If \(-4y = -28\) then \( y = 7 \).
   b. If \( 8x + z = 10 \) then \( z = -8x + 10 \).
   c. If an animal is gray then it is an elephant.
   d. If today is December 5\(^{th}\) then tomorrow is December 6\(^{th}\).
**Topic 1.4.2 Inductive and Deductive Reasoning**

**California Standards:** 24.1, 24.3

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**Example**

Joey arrives 10 minutes early to work on Monday, 5 minutes early on Tuesday, on time on Wednesday, and 5 minutes late on Thursday.

a. How many minutes early/late would you expect him to arrive on Friday? Explain why.

b. What sort of reasoning have you used to come up with this rule?

c. Provide a counterexample that would show that this assertion is false.

---

**Solution**

a. He will arrive 10 minutes late on Friday because he arrives 5 minutes later each day.

b. Inductive reasoning.

c. He arrives early on the Friday.

---

1. Use inductive reasoning to work out an expression for the \( n^{\text{th}} \) term \((x_n)\) of these sequences.

   a. 3, 6, 9, 12,...

   b. 8, 9, 10, 11,...

   c. 3, 8, 13, 18,...

2. Write an expression for the \( n^{\text{th}} \) term \((x_n)\) of these sequences.

   a. 5, 9, 13, 17, 21, 25,...

   b. 1, 4, 9, 16, 25, 36,...

   c. \(-9, -7, -5, -3, -1,\)...

   d. \(-\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3,\)...

3. Sean is able to do 2 pull-ups on Saturday, 4 pull-ups on Sunday, 6 pull-ups on Monday, and 8 pull-ups on Tuesday. He decides he will therefore be able to do 12 pull-ups on Thursday. What sort of logic does he use to come to this conclusion?
4. Predict the next two numbers in each sequence.
   a. 0, 3, 8, 15, 24,...
   b. 5, 7, 9, 11,...
   c. -4, 8, -12, 16, -20,...
   d. -6, -3, $\frac{-3}{2}$, $\frac{-3}{4}$, $\frac{-3}{8}$,...

5. Give a counterexample to disprove each of the following statements.
   a. All even numbers are divisible by both 2 and 4.
   b. All multiples of 3 are odd.
   c. If a date is written as $m/d/y$, then $d \geq m$.
   d. The sum of the digits of all odd numbers over 10 is even.
   e. $x^2 > x$ for all natural numbers.

6. Use deductive reasoning to work out the 5th term of these sequences.
   a. $x_n = 6n - 2$
   b. $x_n = n^2 + 10$
   c. $x_n = n(n + 3)$

7. Find the first three terms of each sequence using deductive reasoning.
   a. $x_n = n^3 - 14$
   b. $x_n = \frac{n}{2} + 11$
   c. $x_n = -n + 9$

8. A number sequence begins 3, 5, 7, 9, 11...
   a. Use inductive reasoning to write an expression for the $n^{th}$ term ($x_n$) of this sequence.
   b. Use deductive reasoning to calculate the 24th term of the sequence.
Topic 1.4.3

Algebraic Statements

California Standards: 25.3

Example

Find \( x \) given that \( x^2 + 1 = 17 \).

Solution

\[
x^2 + 1 - 1 = 17 - 1
\]

\[
x^2 = 16
\]

\[
x = 4 \text{ or } x = -4
\]

There are two values satisfying the equation.

1. Find the value(s) of \( x \) that satisfy each equation.
   
   a. \( x(x) = 81 \)
   
   b. \( x + (-5) = 2 \)
   
   c. \( x + x = 18 \)
   
   d. \( x^4 = 16 \)

2. Say whether the following statements are always, sometimes, or never true.
   
   a. If \( x \) is a real number, then \( x^2 < 0 \).
   
   b. \( x^2 - 4x - 5 = 0 \)
   
   c. \( |3x| = -12 \)
   
   d. The sum of any two odd numbers is even.

3. Determine whether the following equations are always, sometimes, or never true by finding the value(s) of \( x \) that satisfy them.
   
   a. \( |x| = 29 \)
   
   b. \( -|x| = 14 \)
   
   c. \( |x + 3| > 0 \)
   
   d. \( |x - 7| = 12 \)
   
   e. \( |x| - 5 = 15 \)
   
   f. \( |x + 3| = 0 \)
4. Find the values of $x$ that satisfy each equation.
   a. $4x^2 - 5 = 31$
   b. $25 - x^2 = 0$
   c. $x^2 + 10 > 0$
   d. $x^2 + 4 < 0$

5. Determine whether the following statements are true. Explain your answers.
   a. If $|a| = |b|$, then $a = b$.
   b. If $|a + b| = |a| + |b|$, then $a = b$.
   c. If $y = 2x + 6$, then $x = \frac{y}{2} - 3$
   d. If $y = x^2 + 6$, then $y > 0$.

6. A set of rectangles have length $l$ and width $w$, where $w = 2l$.
   a. Calculate the perimeter of a rectangle with $l = 4$.
   b. Calculate the perimeter of a rectangle with $l = 5$.
   c. Show that the perimeters of any two rectangles in this set with lengths $l$ and $l + 1$ will have a difference of 6.

7. Prove that the following statements are not true.
   a. If $n$ is a factor of 24 then $n$ is even.
   b. If $n$ is a square number, then $n + 1$ is prime.
   c. If $x$ is a real number, then $4x^2 + 9 < 12x$.

8. Use the statement ‘If the digits of a number sum to give 9, the number is a multiple of 9’ to say whether the following hypotheses are true.
   a. 63 is divisible by 9.
   b. 3257 is divisible by 9.

9. Use the statement ‘If $x_n = \frac{n(n+1)}{2}$, then $x_n$ is a triangle number’ to say whether the following is true:
   ‘The difference between a triangle number $x_n$ and the preceding triangle number $x_{(n-1)}$ is equal to $n$.’
**Topic 2.1.1 Simplifying Algebraic Expressions**

**California Standards: 4.0**

1. Write the number of terms in each expression.
   
   a. \(7v + 3w - 8u^2 + 6\)  
   b. \(pq + p - q - 11 + p^2\)  
   c. 8  
   d. \(-9 + k^3\)

2. Given: \(6xy - 4x^2 + 8y - 5xy^2 - x + w^3 + 3z\).
   
   a. Identify the term with a coefficient of 8.  
   b. Identify the term with a coefficient of \(-1\)  
   c. Identify the term with an exponent of 3.

**Example**

Simplify the following expressions.

a. \(8y^3 - 3y^3\)  
   b. \(5m^2 + 8m - m^2 + 2m\)

**Solution**

a. \((8 - 3)y^3 = 5y^3\)
   
   b. \(5m^2 + 8m - m^2 + 2m = 5m^2 - m^2 + 8m + 2m\)
   
   \(= (5 - 1)m^2 + (8 + 2)m\)
   
   \(= 4m^2 + 10m\)

3. Simplify the following expressions.
   
   a. \(-9r - 8r\)  
   b. \(-16x + 9x\)  
   c. \(-g + 17g - 3g + 11g\)

4. Simplify the following expressions.
   
   a. \((4x + 8) + (7 + 6x)\)  
   b. \((2x - 9 + 3y) + (10 - 6x - 4y)\)  
   c. \((3d^2 + 8d - 6) + (4d - 9d^2 + 15)\)
5. Simplify the following expressions.
   a. \(4 \times 3g - 2 \times (-6g) + 8g \div 2\)
   b. \(-11 \times (-k) + 8k \times 2 - 36k \div 9\)
   c. \(3 \times 4m - 3m \times 2^2 - 3^3 \times 2\)

6. Simplify the following expressions.
   a. \(3x - (2y - x)\)
   b. \(3xy - ky - 5xy + 2ky\)
   c. \(5m - [7x + (3m - 4x)]\)
   d. \((x^2 - 3x + 6) - (2x^2 + 5x + 7)\)

   Remember:
   \(a - (b - c) = a - b + c\)

7. Simplify the following expressions.
   a. \(\frac{4}{5}p^2 - \frac{1}{3}q + \frac{1}{2}q - \frac{1}{2}p^2\)
   b. \(\frac{1}{6}m + \frac{2}{3}n - \frac{1}{4}m - \frac{3}{8}n\)

8. A square has a side length of \((x + 7)\) ft.
   a. Write an expression for the perimeter of the square in feet.
   b. Find the perimeter of the square when \(x = 5\).
   c. A rectangle has a width of \((2x + 13)\) ft and a length of \((3x - 7)\) ft.
      Write an expression to represent the perimeter of the rectangle in feet.

9. Ray blocked \(x\) shots at his first volleyball game. At each of his next two volleyball games, he blocked \(x + 5\) shots. At the fourth game he blocked \(x - 1\) shots.
   a. Write an expression to represent the number of shots Ray blocked at the four games.
   b. If Ray blocked 9 shots at the first game, calculate his total number of blocks for the four games?

10. Roseanna deposited \(x + 25\) dollars in her savings account in June. Her next deposit was for \(2x + 50\) dollars in August. Finally, in October Roseanna deposited \(x + 30\) dollars. Write an expression to represent Roseanna’s total deposits in her savings account between June and October.
Topic 2.1.2  Getting Rid of Grouping Symbols

California Standards:  4.0

Example

Simplify $3(x - 3) - 5(x - 2) + 1$. Justify each step.

Solution

$3(x - 3) - 5(x - 2) + 1 = 6x - 9 - 5x + 10 + 1$  Distributive property of multiplication over addition

$= 6x - 5x - 9 + 10 + 1$  Commutative property of addition

$= x + 2$  Adding and subtracting

1. Simplify the following expressions.

   a. $2(x + 3)$

   b. $5(x + 3) + 2(6 + x)$

   c. $4(x - 2) + (x + 6)$

2. Simplify the following expressions.

   a. $-4(x + 1)$

   b. $2(x - 1) - 3(2 + x)$

   c. $-2(1 - x) - 5(x + 2) + 3(3x - 1)$

   d. $7(x - 3(2x - 2[3 - 4x] - 1)$  Simplify the innermost parentheses first.

3. Simplify the following expressions. In exercises a and b, justify each step.

   a. $2(6x - 1)$

   b. $3(m + 2) + 6(2m - 2)$

   c. $4(5a + 2) - 2(a - 1)$

   d. $2(12x - 10) - 2(4x - 7)$

   e. $2(5b + 2) + 6(3a + b - 2)$
4. Evaluate the following expressions for \( x = -4 \).
   a. \( 4(x + 1) \)
   b. \( 7(3 - 5x) \)
   c. \( 2(x + 4) - (2x - 1) \)
   d. \( 4(3 - 2x) - (x - 2) \)

5. Simplify \( 7 - [2(1 - x) - 3(x + 2) - 3] \).

6. Show that \( 5(3 - x) - 7(x + 2) - 3(1 - 2x) \) is equivalent to \(-2 - 6x\).

7. Simplify \( \frac{5(2x-1)}{3} - \frac{3(x-5)}{4} - \frac{5}{6}x \).

8. Write an expression in its simplest form for the perimeter of each shape.
   a. \[
   \text{These little lines show equal side lengths. So you know that this shape has two pairs of equal sides.}
   \]
   b. \[
   \]
   c. \[
   \]

9. Simplify the following equation and solve to find \( x \).
   \[
   \frac{2}{5}(20x+5) - \frac{1}{2}(8-4x) = 38
   \]
More Simplifying
and Checking Answers

California Standards: 4.0

1. Simplify the following expressions.
   a. \(x(x + 3)\)  
   b. \(-4x(x + 1)\)  
   c. \(5x(2 + x) - x(7 + x)\)  
   d. \(x(5 + x) - 2x(x - 3)\)  
   e. \(2(x - 4) + 2x(5 - x)\)  
   f. \(-3x(-4x) + x(x - 1)\)

2. Simplify the following expressions.
   a. \(4a(b + 3a)\)  
   b. \(6a(a^2 + 6b) + b(a + 2)\)  
   c. \(a^2(a + b) - b^2(a + b)\)  
   d. \(ab(2 + a) + a(b - 3a)\)  
   e. \(a(5ab + b) - 2ab(a - 3)\)  
   f. \(a^2(a + 5) - 2(a - b)\)

3. Show that \(-3x(2 - 3) - x(7 - 3) - 3(1 + x)\) is equivalent to \(-4x^2 + 13x - 3\).

4. Show that \(7 - 5x(1 + x) - 2x(x - 3) + 5(x^2 - 1)\) is equivalent to \(-2x^2 + x + 2\).

5. Show that \(2[(m - c) - (3m + c)] - [(2m + c) - (m + 2c)]\) is equivalent to \(-5m - 3c\).
6. Show that \( x = 3 \) is a solution of \( 2(x + 7) = 4(3x - 4) \).

Solution
\[
\begin{align*}
3(-21 + 2) - 1 &= 4(-21 - 4) - 2(-21) \\
3(-19) - 1 &= 4(-25) + 42 \\
-57 - 1 &= -100 + 42 \\
-58 &= -58
\end{align*}
\]

7. Show that \( 4(x - 3) \) is equivalent to \( 2(x + 1) \) when \( x = 7 \).

8. Evaluate the expression \( ab(4 - 2a) - b(3a + b) + 4(b - 5) \), for \( a = 3 \) and \( b = -1 \).

Take care when you are substituting negative numbers. Make sure you write down all your steps, so it is easier to spot mistakes.

9. Given \( p = 2 \) and \( q = -1 \), find the value of \( 2(p + 3q - 4) + 5q(2p - q) \).

10. Given \( a = 3 \) and \( b = 2 \), find the value of \( 9a(a + 5b) + 2(3b - 11) - (12 + ab) \).

11. The formula \( C = r(1.6m) + 3 \) is used to calculate the cost of a taxi ride in San Diego, where \( C \) is the cost of the journey in dollars, \( r \) is the rate code (\( r = 1 \) in the daytime, \( r = 2 \) at night and \( r = 3 \) on public holidays) and \( m \) is the number of miles traveled.

a. Calculate the cost of a journey of 4.3 miles taken in the daytime.

b. Show that an 8.1 mile journey taken on a public holiday would cost $41.88.

c. Calculate the minimum cost of a taxi ride in San Diego.
Properties of Equality

California Standards: 4.0

Example

Solve \(2x - 9 = -1\).

Solution

\[
\begin{align*}
2x - 9 &= -1 \\
2x - 9 + 9 &= -1 + 9 \\
2x &= 8 \\
\frac{2x}{2} &= \frac{8}{2} \\
x &= 4
\end{align*}
\]

Example

Solve \(\frac{x}{2} - 3 = 2\).

Solution

\[
\begin{align*}
\frac{x}{2} - 3 &= 2 \\
\frac{x}{2} - 3 + 3 &= 2 + 3 \\
\frac{x}{2} &= 5 \\
x &= 2(5) = 10
\end{align*}
\]

Example

Solve \(4(2x - 1) - 5(2x - 3) = 21\). Justify each step.

Solution

\[
\begin{align*}
4(2x - 1) - 5(2x - 3) &= 21 & \text{Given equation} \\
8x - 4 - 10x + 15 &= 21 & \text{Distributive property of multiplication} \\
8x - 10x - 4 + 15 &= 21 & \text{Commutative property of addition} \\
-2x + 11 &= 21 & \text{Adding} \\
-2x + 11 - 11 &= 21 - 11 & \text{Subtraction property of equality} \\
-2x &= 10 & \text{Subtracting} \\
\frac{-2x}{-2} &= \frac{10}{-2} & \text{Division property of equality} \\
x &= -5 & \text{Dividing}
\end{align*}
\]

1. Solve the following equations.
   a. \(a + 6 = 12\)
   b. \(a - 17 = 4 + 2\)
   c. \(7 - a = 16\)
   d. \(\frac{1}{4} = a - \frac{2}{4}\)

2. Solve the following equations.
   a. \(4b = 24\)
   b. \(6 \times b = 15\)
   c. \(\frac{3}{4}b = 15\)
   d. \(\frac{2}{7}b = \frac{8}{4}\)
3. Solve the following equations.
   a. \(3x = -21\)  
   b. \(12 - x = -3\)  
   c. \(5x - 7 = -17\)  
   d. \(3 + 2x = x + 13\)  
   e. \(x - 4 = 4x + 11\)  
   f. \(4(x - 2) = 7(1 + 2x)\)

4. Solve the following equations.
   a. \(5(x - 1) - 3x = -(x - 4)\)  
   b. \(2(x - 4) + 8 = 3x + 10\)  
   c. \(7x - 2(x - 5) = 3(x - 2) - 5x\)  
   d. \(7 - 2(1 - x) - 3(x - 2) = 3x - 5\)  
   e. \(4(3x + 2) - 6x = 2(3 + x) + 6\)  
   f. \(14 - 4(x - 3) - 3(2x - 4) = 15 - 5x - 2\)

5. Solve the following equations.
   a. \(\frac{x}{3} = 12\)  
   b. \(\frac{2x}{7} = 6\)  
   c. \(\frac{x + 5}{2} = 17\)  
   d. \(\frac{20 - 3x}{x} = 2\)  
   e. \(\frac{3x - 1}{5} = \frac{x + 3}{3}\)  
   f. \(\frac{7x - 4}{10} = \frac{2x - 1}{2}\)

6. Solve the following equations and justify each step.
   a. \(9 - 5x = -6\)  
   b. \(5x - 9 = 3x - 11\)  
   c. \(-3(1 - x) + 2(-2x - 3) = -2(x - 4)\)  
   d. \(3y - 2[y - 3(2 - 3y) + 5] = 18\)

Remember:
\[-a(b - c) = -ab + ac\]

Use the real number axioms and the properties of equality to justify your steps.
Removing Fractions

California Standards: 4.0

1. Find the least common multiple of the denominators for the following equations.
   a. \( \frac{5}{6}x + \frac{1}{2}x = \frac{1}{3} \)
   b. \(-9 + \frac{3}{7}x = \frac{1}{3}x - \frac{2}{3}\)
   c. \(\frac{5}{9}x - \frac{2}{3} = \frac{4}{5}x + \frac{1}{9}\)
   d. \(\frac{1}{2}x + \frac{1}{7} = \frac{2}{6} + \frac{1}{4}x\)

Example

a. Solve \(\frac{4}{5}x + 1 = 21\)
   Solution
   \(4x + 5 = 105\)  \(\text{Multiply every term in the equation by the LCM of the denominators — 5}\)
   \(4x = 100\)
   \(x = 25\)

b. Solve \(\frac{2}{3}x - \frac{1}{4}x = 1\)  \(\text{Removing the fractions from an equation can make it easier to solve.}\)
   \(\frac{8}{12}x - \frac{3}{12}x = 1\)
   \(\frac{5}{12}x = 1\)
   \(x = \frac{12}{5}\)

2. Solve the following equations.
   a. \(\frac{x}{3} + 2 = 11\)
   b. \(\frac{x}{5} - 7 = 9\)  \(\text{Removing the fractions from an equation can make it easier to solve.}\)
   c. \(\frac{1}{6}x + 8 = 4\)
   d. \(\frac{2}{7}x - 1 = 3\)
   e. \(\frac{2}{5}x + 15 = 2\)
   f. \(-\frac{5}{6}x + 3 = -12\)
3. Solve the following equations.  

a. \( \frac{3x}{2} + \frac{6x}{3} = 7 \)  
b. \( \frac{4x}{5} + \frac{x}{2} = 13 \)  
c. \( -\frac{x}{5} - \frac{6x}{3} = 11 \)  
d. \( \frac{10x}{9} - \frac{7x}{6} = 3 \)  
e. \( \frac{3x}{2} - \frac{6x}{8} = -3 \)  
f. \( \frac{7x}{3} + \frac{2x}{4} = -17 \)  
g. \( \frac{3x - 7}{8} - \frac{x + 2}{4} = -1 \)  
h. \( -\frac{3}{4}x + \frac{2}{5}x = \frac{7}{20} \)

4. Solve the following equations.  

a. \( \frac{\frac{1}{2}x + \frac{3}{5}x}{\frac{3}{4}} = -\frac{\frac{3}{2}x}{\frac{5}{4}} \)  
b. \( -\frac{\frac{3}{8}x + \frac{1}{2}x}{\frac{5}{8}} = -\frac{\frac{5}{8}x + \frac{3}{4}}{\frac{5}{8}} \)  
c. \( \frac{\frac{2}{7}x - \frac{1}{3}}{\frac{7}{4}} = -\frac{\frac{4}{7}x + 3}{\frac{7}{4}} \)

5. A right angle measures \( (\frac{2}{3}x + 18)^{\circ} \).  
a. Find the value of \( x \).

b. The sum of the angles in a quadrilateral is \( 360^{\circ} \). The four angles in one quadrilateral measure \( \left[ \frac{1}{6}x \right]^{\circ}, \left[ \frac{2}{3}(x - 2) \right]^{\circ}, \left[ \frac{1}{2}(x - 5) \right]^{\circ}, \) and \( \left[ \frac{1}{3}(x + 29) \right]^{\circ} \). Find the value of \( x \).

6. The height of a lake falls \( \frac{4}{5}x + 7 \) inches during the summer, rises \( \frac{2}{3}x \) inches in the fall, and rises \( \frac{1}{3}x + 2 \) inches in the winter. The total change in the lake’s height is 2 inches.  
a. Determine the value of \( x \).

b. Calculate how many inches the height of the lake rose in the fall and winter.
Topic 2.3.2  Fractional Coefficients in Algebraic Expressions

California Standards:  4.0

Example

Solve \(-\frac{1}{3}x + \frac{4}{9} = -\frac{5}{6}\)

Solution

\(-6x + 8 = -15x \\
9x + 8 = 0 \\
9x = -8 \\
x = -\frac{8}{9}\)

Multiply every term in the equation by the LCM (18)

1. Show that the following solutions are correct.
   a. If \(\frac{2}{3}x - \frac{1}{7} = \frac{3}{7}x + \frac{1}{3}\), then \(x = 2\).
   
   b. If \(\frac{x+7}{4} - \frac{x-3}{3} = \frac{1}{4}\), then \(x = 30\).
   
   c. If \(-\frac{6x+1}{6} + \frac{x+5}{3} = \frac{9-x}{4}\), then \(x = -1\).

2. Solve the following equations.
   a. \(-2 + \frac{5}{9}x = -\frac{1}{3}\)
   
   b. \(-\frac{y}{3} - \frac{1}{5} = \frac{7}{15}\)
   
   c. \(\frac{4}{5}x - \frac{7}{10} = \frac{5}{6}x\)

3. Solve the following equations.
   a. \(\frac{5}{12} - \frac{1}{2}x = \frac{2}{3}x - \frac{1}{6}\)
   
   b. \(\frac{1}{5}x - \frac{3}{5} = \frac{1}{3}x - \frac{4}{15}x\)
4. Solve the following equations.
   a. \[ \frac{2x - 1}{2} - \frac{3x - 1}{3} = 4x - 9 \]
   b. \[ \frac{2x - 3}{2} - \frac{x + 7}{9} = \frac{3x - 11}{9} \]
   c. \[ \frac{x + 3}{5} + \frac{2x + 1}{2} = \frac{1}{10} \]
   d. \[ \frac{3x + 4}{4} + \frac{2x - 7}{5} = \frac{7x}{20} \]

5. Solve the following equations.
   a. \[ -\frac{2}{5}k - 2(3 + k) = \frac{6}{5} \]
   b. \[ \frac{2x - 1}{3} - \frac{x + 3}{4} = \frac{3x - 7}{12} \]
   c. \[ \frac{4(2x - 7)}{3} = \frac{2(3x - 5)}{4} - \frac{5x}{8} + \frac{1}{3} \]
   d. \[ \frac{2}{5}(x - 2) - \frac{3}{5} \left(5 - x\right) = -\frac{7}{10} \left(\frac{5x}{7} + 2\right) \]
   e. \[ \frac{2}{5}(x - 1) - \frac{1}{3}(3 + 2x) = \frac{7}{15}(1 - x) \]
   f. \[ \frac{3k - 5}{3} - \frac{2k + 3}{4} = \frac{5(4 - k)}{6} \]

6. Solve the following equations.
   a. \[ \frac{2x - 3}{4} - \frac{3}{7}x = 1 \]
   b. \[ \frac{3x - 2}{5} - \frac{x - 3}{2} = \frac{-7 - 2x}{10} \]

7. Solve the following equations.
   a. \[ \frac{4x - 1}{11} + 3 - \frac{x - 7}{4} = 9 \]
   b. \[ \frac{5x + 7}{8} - \frac{5x - 9}{4} + \frac{x - 3}{3} = \frac{1}{12} \]

8. Solve the following equation: \[ \frac{5 - 2x}{6} + \frac{6x - 6}{12} - \frac{3 + 2x}{9} = x - 2 - \frac{7x}{9} \]
Topic 2.3.3 Eliminating Decimal Coefficients

California Standards: 4.0

Example

Solve the following equation.
0.04x − 0.06 = 0.10

Solution
100(0.04x − 0.06) = 100(0.10)
4x − 6 = 10
4x = 16
x = 4

It's 100 because each decimal
has 2 decimal places.

1. Give the power of 10 that the following equations should be multiplied by to eliminate the decimals.

   a. 8.1x + 1.2 = −1.5
   b. 0.09x − 0.07(6 − 3x) = 0.48
   c. 8.32x − 0.056 = 1.25

2. Solve the following equations.

   a. 0.3x = 21
   b. 0.6x + 8 = 1.4
   c. 4.5 − 3.54x = 8.04

3. Solve the following equations.

   a. 2.2x + 4.14 = 8.34 − 2x
   b. 0.9x − 6.2 = 2.8x + 5.2
   c. 3.5x − 6.5 = 4.5x + 12.5

4. Solve the following equations.

   a. 0.8(6 − 8x) = 11.2
   b. 1.74x = 1.8(x − 7)
   c. 3.2(2x − 6) = 1.6(4 − x)
   d. 9.4(3x − 8) − 4.9 = 4.5
   e. 6.7x = 8.1(2x − 3) − 23.2
   f. 6.52 + 2.31x = 0.08(3x − 22)
5. Solve the following equations.
   a. \[6.2(3x - 8) - 4.3(5 - 2x) = 37.7\]
   b. \[0.05(0.4x - 2.6) - 0.25(0.8 - 0.2x) = 2.26\]

6. Solve the following equations.
   a. \[0.004(x - 89) + 0.007(12 + x) = 0.004(-12 - x) - 0.001x\]
   b. \[0.013(x + 4) - 0.010(2x - 8) + 0.006 = 0.016(x - 11) - 0.042(x + 7)\]
   c. \[0.198(x - 14) + 0.074(x - 9) = 0.203(37 - 4x) - 0.015(-4x - 42) + 0.709\]

7. The perimeter of the rectangle in the diagram below is 31.2 inches. Determine the value of \(x\).

8. A gallon of gasoline costs $1.95 on a Monday and $2.05 on a Friday. Enrique spent $37.65 on gasoline this week.
   a. Write an equation for how much Enrique spent on gasoline in terms of the number of gallons he bought on Monday, \(m\), and the number of gallons he bought on Friday, \(f\).
   b. Enrique bought 6 gallons of gasoline on Friday. Calculate how many gallons of gasoline he bought on Monday.

9. Movie rentals cost $1.80 each plus a monthly fee of $11.80. The Yang’s spent $28 during May. Calculate how many movies they rented that month.
**Topic 2.4.1 Applications of Linear Equations**

**Example**

The sum of 4 times the number \( x \) and 7 is equal to 23. Write and solve an equation to find \( x \).

**Solution**

\[
4x + 7 = 23 \\
4x + 7 - 7 = 23 - 7 \\
4x = 16 \\
x = 4
\]

1. The sum of 4 times the number \( x \) and 11 is equal to –17. Write and solve an equation to find \( x \).

2. Twelve less than the product of 6 and \( x \) is 18. Write and solve an equation to find \( x \).

3. Four times the sum of 6 and the number \( x \) is equal to 16. Write and solve an equation to find \( x \).

4. The sum of 9 and \(-x\), all divided by 7, is equal to 2. Write and solve an equation to find \( x \).

5. 9 less than the product of five and \( c \) is 16. Find the number \( c \).

These are examples of problem-solving questions. See page xiii of this book for advice on how to tackle them.
6. The sum of two numbers is equal to 12. Write and solve an equation to find the value of each number, if the larger number is twice the value of the smaller number. 

\[ \text{Let } x = \text{ the smaller number.} \]

7. A number is 20 more than four times another number. The sum of the two numbers is 75. Find the two numbers.

8. Willard’s age is eight less than three times Frank’s age. The sum of their ages is 48. Find the age of each person.

9. Point M is the midpoint of segment AB.

\[ \frac{1}{2}x + \frac{1}{3} \quad M \quad \frac{2}{3}x - 2 \]

a. Find the value of x. 

b. Find the length of AM. 

c. Find the length of AB.

10. The area of a rectangle with dimensions 17 cm by \((3x - 17)\) cm is 323 cm². Find the value of x.

\[ \text{Area} = \text{length} \times \text{width} \]

\[ 17 \text{ cm} \quad (3x - 17) \text{ cm} \]

\[ 323 \text{ cm}^2 \]

11. The larger of two numbers is 8 less than 3 times the smaller number. Write and solve an equation to find the value of each number if 5 times the larger number, less 10 times the sum of the smaller number and 2 is equal to 35.
Topic 2.4.2

Coin Tasks

California Standards: 4.0, 5.0

Example

Mr. Pautler bought 7 fewer cheese sandwiches than salami sandwiches for the soccer team. The salami sandwiches cost $2.60 and the cheese sandwiches cost $3.10. Mr. Pautler spent $41 on sandwiches for the soccer team. How many cheese sandwiches did he buy?

Solution

Let \( c \) be the number of cheese sandwiches bought, and \( s \) be the number of salami sandwiches bought.

\[
2.60s + 3.10c = 41 \quad \text{Mr. Pautler spent$41 in total.}
\]

\[
s = c + 7 \quad \text{Mr. Pautler bought 7 fewer cheese sandwiches than salami sandwiches.}
\]

\[
2.60(c + 7) + 3.10c = 41 \quad \text{Substitute (c + 7) for } s
\]

\[
26c + 182 + 31c = 410 \quad \text{Multiply by 10 to simplify}
\]

\[
57c + 182 = 410 \quad \text{Solve for } c
\]

\[
c = 4
\]

So Mr. Pautler bought 4 cheese sandwiches.

Check with the other equation

\[
2.60 \times 11 + 3.10 \times 4 = 41, 28.6 + 12.4 = 41, 41 = 41
\]

1. Andres has 23 coins in his pocket. The coins consist of nickels and dimes. If the value of the coins is $1.65, how many coins of each type does Andres have in his pocket?

2. Enrico has three more dimes than quarters and five times as many nickels as dimes in his piggy bank. The value of these coins is $5.25. How many coins of each type does Enrico have?

3. A shopkeeper deposited five more quarters than dimes, and eight times as many half dollars as quarters. The total deposit was $86.50, how many coins of each type did the shopkeeper deposit.

4. Rosetta was paid $11.45 for babysitting the O’Connell’s. Mrs. O’Connell paid Rosetta in quarters and dimes. The number of quarters she received was five more than twice the number of dimes. How many of each coin did Rosetta receive?
5. Lauren’s grandpa has her pick a handful of change. She picks only nickels, dimes and quarters. Lauren has 5 more quarters than dimes and half as many nickels as quarters. If her total amount is $4.75, then how many of each coin does Lauren pick?

6. Tickets for a school dance cost $7 in advance or $11 at the door. If 109 students attended the dance and the total spent was $887, how many tickets were bought in advance and how many at the door?

7. Twenty-one raffle tickets were sold at a charity event raising a total of $16.65. One stall sold each raffle ticket for $0.75, but the other stall sold the tickets for $0.85 each. How many tickets were sold at each stall?

8. Brad is paid time-and-a-half if he works over 40 hours in a week. One week he worked 55 hours and earned $697.50. Write and solve an equation that can be used to find Brad’s regular hourly rate.

9. An international call to Zimbabwe costs $2.50 per minute for the first 3 minutes and then $0.85 for each additional minute. If Sarah paid $17.70 for a call to Zimbabwe, how many additional minutes did she spend on the phone?

10. A cell phone plan charges $25 per month for 250 minutes plus $0.55 per additional minute of airtime. Pedro’s phone bill for the month was $67.35. How many additional minutes did Pedro use?
Consecutive Integer Tasks

California Standards: 4.0, 5.0

1. Find the unknown integers in each of the following cases.
   a. Two consecutive integers have a sum of 91.
   b. Three consecutive integers have a sum of –51.
   c. Four consecutive integers have a sum of 162.
   d. The sum of three consecutive integers is 0.

Example

Three consecutive even integers have a sum of 342. Find the integers.

Solution

Let \( f \) = 1st (smallest) even integer
\( f + 2 \) = 2nd even integer
\( f + 4 \) = 3rd even integer

Write the second and third integers in terms of the first — here, by adding 2 each time

Set up your equation

\[
\begin{align*}
\text{Equation: } & f + (f + 2) + (f + 4) = 342 \\
& f + f + 2 + 4 = 342 \\
& 3f + 6 = 342 \\
& 3f + 6 - 6 = 342 - 6 \\
& 3f = 336 \\
& \frac{3f}{3} = \frac{336}{3} \\
& f = 112 \\
\end{align*}
\]

Answer the question fully

The consecutive even integers are 112, 114, 116.

Check your answer by adding the three integers together: 112 + 114 + 116 = 342

2. Find the integers described in each case.
   a. Two consecutive odd integers have a sum of 116.
   b. Two consecutive even integers have a sum of 74.
   c. Two consecutive even integers have a sum of –42.
   d. Two consecutive odd integers have a sum of 0.
3. Three consecutive odd integers have a sum of 111. Find the product of the smallest and the largest of the three integers.

4. Four consecutive odd integers have a sum of 272. Find the four odd integers.

5. Four consecutive even integers have a sum of 508. Find the difference between the largest of the four even integers and 117.

6. There is a sequence of three integers with a common difference of five. Four times the largest integer minus twice the second largest integer is 108. Find the three integers.

7. Find a sequence of three integers with a common difference of three, such that three times the sum of the first two integers minus five times the largest integer is 61.

8. Effie received a raise at work which meant she was paid $0.75 more per hour this week than last week. Effie worked 18 hours last week and 14 hours this week. She was paid $218.50 for the two weeks. Find Effie's hourly rate:
   a. last week. 
   b. this week.

9. The county fair lasted three days — Friday, Saturday and Sunday. On Saturday, twice as many visitors came as on Friday. On Sunday there were twenty more than three times the number of visitors on Friday. 1,700 visitors attended the fair in total. How many visitors were there on:
   a. Friday?
   b. Saturday?
   c. Sunday?
**Age-Related Tasks**

**California Standards:** 4.0, 5.0

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**Example**

Carmen is four years older than twice her daughter Rosario’s age. How old will each be in five years if the sum of their current ages is 58 years?

**Solution**

Let \( r \) = Rosario’s age  
\( 2r + 4 \) = Carmen’s age

**Equation:**  
\[ r + (2r + 4) = 58 \]

\[ 3r + 4 = 58 \]

\[ 3r = 54 \]

\[ r = 18 \]

Rosario is 18 years old.  
Carmen is \( 2(18) + 4 = 40 \) years old.

In five years, **Rosario will be 23 years old** (18 + 5), **and Carmen will be 45 years old** (40 + 5).

---

1. Raymond is eight years older than Jeremiah. In 15 years, the sum of their ages will be 96 years. How old is each now?

   Raymond  
   Jeremiah

2. Janessa is half Kendra’s age. In 24 years, the sum of their ages will be 135. How old is Janessa now?

   

3. Fiza is three years older than half her mother’s age. The sum of their current ages is 75. How old will Fiza be in 10 years?

   

4. Emily is 26 years older than her daughter Maria. In 10 years, Emily will be three times as old as Maria. How old is each at present?

   Maria  
   Emily

---

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5. Joyce is four times as old as Martha. In nine years’ time, Joyce will be three times as old as Martha. Find their present ages.

6. Marvin is 68 years old and John is 44 years old. How many years ago was Marvin’s age twice John’s age?

7. Two brothers are 19 and 38 years old. In how many years will the sum of their ages be 87 years?

8. Janice is 10 years old and Peter is 52 years old. In how many years will Peter be four times as old as Janice?

9. Malinda is 27 years younger than Lorraine. In eight years’ time, Lorraine will be four times Malinda’s age. How old is each one now?

10. If you decrease Tonya's current age by 30%, you will find her age three years ago. How old is Tonya now?

11. Kurt is 12 years younger than Mark. When Mark's age is increased by 20% and Kurt's age is increased by 25%, the sum of their ages is 83. Find Kurt's age.
Topic 2.5.3  Rate, Time, and Distance Tasks

California Standards: 15.0

1. Use the speed, distance, and time formula to find:
   a. the speed if distance is 420 miles and time is 7 hours.
   b. the distance if speed is 45 miles per hour and time is 2 hours.
   c. the time if distance is 880 miles and speed is 55 miles per hour.

2. A passenger ferry travels between two cities. The ferry travels at an average speed of 35 mph and the journey one way takes 30 minutes. What distance does the ferry cover on a one-way trip?

3. a. A race car driver takes 3 hours to complete a 345 mile race. Find the driver’s average speed during the race.
   b. During his qualifying race, the same driver averaged 96 mph. The qualifying race was 120 miles. What was the driver’s time for the qualifying race?

4. Rob wins a 100 m race with a time of 15 s. What was Rob’s average speed (in m/s)?

Example

A high school basketball team traveled to a game in a bus at a speed of 30 miles per hour. Half an hour after the team set off, the pep band left the school, traveling in the same direction at 35 miles per hour. How long did it take the band bus to catch up with the team bus?

Solution

Say the band bus caught the team bus after x hours. The team bus would then have been traveling for (x + 0.5) hours.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Time (hours)</th>
<th>Speed (mph)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band Bus</td>
<td>x</td>
<td>35</td>
<td>35x</td>
</tr>
<tr>
<td>Team Bus</td>
<td>(x + 0.5)</td>
<td>30</td>
<td>30(x + 0.5)</td>
</tr>
</tbody>
</table>

Distance = speed × time

Equation: $35x = 30(x + 0.5)$

Solve the equation for $x$

$35x = 30x + 15$

$5x = 15$

$x = 3$

It took the band bus 3 hours to catch up to the team bus. Answer the question in English
5. Tour buses A and B both cover the same route. Tour Bus B takes 20 minutes longer than Tour Bus A to complete the route. How long does it take Tour Bus A to complete the route if it travels at 60 mph and Tour Bus B travels at 50 mph?

6. A race car starts out 15 miles behind a sports car. How long will it take the race car to overtake the sports car if the race car travels at 95 mph and the sports car travels at 75 mph?

7. A delivery truck leaves a town traveling at 35 miles per hour. An hour later an emergency vehicle sets off after the truck from the same place. It takes the same route as the truck but travels at 55 mph. How long will it take the emergency vehicle to catch up with the truck?

8. A van leaves the Springfield museum grounds, traveling at a speed of 30 mph. A taxicab leaves the museum grounds 30 minutes later and drives the same route. If the taxicab travels at 45 mph, and the van maintains its constant speed, how long will the taxicab take to reach the van?

9. Tom and Katherine live in different towns which are linked by a bike trail. They agree to meet for a bike ride, and both set off from home at the same time. Tom rides north at a speed of 14 mph and Katherine rides south at 10 mph. The bike trail between their homes is 42 miles long. How long will it be before they meet?

10. Domingo and Bethany drove along the same highway, in opposite directions. Domingo left at 1 p.m., traveling westbound at 55 mph. Bethany left at the same time, traveling eastbound at 60 mph. Domingo and Bethany were 575 miles apart when they started driving. At what time did they pass each other?
Investment Tasks

California Standards: 15.0

1. Work out the return on investment for each person, in the first year of their investment.
   a. Hector invests $2500 at an annual interest rate of 9%.
   b. Dinah invests $1750 at an annual interest rate of 12%.
   c. Morris invests $1400 at an annual interest rate of 4.5%.

2. a. Debbie invested money in an account which pays 5% annual interest. After one year she had earned $102 interest. How much money did she invest?
   b. At the end of one year, Malcolm earned $161 on an investment with an annual interest rate of 7%. How much money had Malcolm invested at the start of the year?
   c. At the end of one year, Eric’s investment at an annual interest rate of 7.25% earned $65.25. How much money did Eric invest?

3. Work out the annual interest rate for each person’s account.
   a. Jon earned $180 interest in one year, on an initial investment of $3000.
   b. Bill earned $168 interest in one year, on an initial investment of $5600.
   c. Lin earned $258.40 in one year, on an initial investment of $3800.

Example

Juan invested a total of $12,000 for one year. He invested $7000 in stocks, and put the remainder in a savings account. If the stocks paid 11% annually and the savings account paid 6%, how much interest did he make over the year?

Solution

\[ I = p_1 r_1 + p_2 r_2 \]

where:

\[ p_1 = 7000, \ r_1 = 0.11 \ (= 11\%) \]
\[ p_2 = (12,000 - 7000) = 5000, \ r_2 = 0.06 \ (= 6\%) \]

\[ I = (7000 \times 0.11) + (5000 \times 0.06) = 770 + 300 = 1070 \]

Juan made $1070 interest over the year.
4. Lucy invested $28,000 in two different savings accounts. She put $15,500 in an account with an annual interest rate of 5%. She invested the remaining money in an account earning 8% annual interest. How much interest did she earn in one year, in total?

5. Dylan opened two bank accounts. He deposited $5700 in an account earning an annual interest rate of 5.5%, and $3900 in a bank account earning an annual interest rate of 6.5%. How much interest would he earn in the first year?

6. Jorge invested a total of $5000. He invested some of his money at 4% and the rest at 7%. If the total return at the end of the year was $320, how much was invested at each rate?

7. Mrs. Martinez has $10,000 to invest. One bank offers her a return of 7.5% on the whole sum. Another bank offers her a return of 8% on part of the sum, and 6% on the rest of the sum. How much of the $10,000 would need to be invested at 8% at the second bank, for the overall return to be the same as at the first bank?

8. A banker invests $20,000 for a client. He invests a certain amount at an annual rate of interest of 7%, and the remainder at a rate of 9%. If the total return on the money after one year is $1670, how much was invested at each rate?

9. Paulina had some money to invest, and decided to split the total between two savings schemes, one paying 11% annual interest and one paying 7%. She invested twice as much in the 11% account as in the 7% account. In one year she earned $6090 interest in total. How much did she invest in total?

10. Jason borrowed money from two different banks. One bank charged an annual interest rate of 6.5% and the other charged 8% per year. He borrowed $1500 more from the bank charging the lower interest rate. After having the loans for one year, Jason owed $67.50 more interest on the loan with the lower interest rate. Work out the amounts he borrowed from each bank.
**Example**

A 198 gram bottle of shaving gel is 15% water, by mass. How many grams of water are contained in the bottle of shaving gel?

### Solution

\[
\text{Percent of substance} = \frac{\text{mass of substance}}{\text{total mass}}
\]

Convert the percent to a decimal first

\[
0.15 = \frac{x}{198} \quad \text{Let} \quad x = \text{the mass of water}
\]

\[
x = 0.15 \times 198
\]

\[
x = 29.7 \text{ g}
\]

There are **29.7 g of water** in the bottle of shaving gel.

---

1. A 3.6 quart detergent bottle contains 0.72 quarts of a brightening agent. What is the percent of brightening agent in the bottle?

2. An 80-fluid ounce bottle of soap contains 1.8% of an antibacterial substance. How many fluid ounces of antibacterial substance does the bottle of soap contain?

3. The label on a tube of children’s toothpaste says that the toothpaste contains 4% flavoring, which is 0.52 grams of flavoring per tube. What mass of toothpaste is in one tube?

4. A punch mix contains two parts soda and one part pre-sweetened drink mix. What percent of the punch mix is soda?

5. A cleaning mixture for wipes contains 2.5 cups water and 0.5 cups cleaning solution. What percent of the cleaning mixture is water?
7. A 15 ounce trail mix bag contains 15% pretzels. The 25 ounce trail mix bag contains 10% pretzels. If you combine the two bags of trail mix then what is the percent of pretzels?

8. A 225 gram bag of a dried berry mix contains 12% strawberries. A 400 gram bag of a dried berry mix contains 5% strawberries. If you combine the two bags of dried berry mix then what is the percent that is not strawberries?

9. Two bubble bath bottles are mixed together. The 4 fluid ounce bottle contains 5% fragrance and the 12 fluid ounce bottle contains 15% fragrance. What is the percent of fragrance when the two bubble bath bottles are mixed?

10. A 21 ounce cereal box contains 30% red pieces, a 12 ounce cereal box contains 20% red pieces, and a 30 ounce cereal box contains 35% red pieces. What is the percent of red pieces if all three boxes of cereal are combined?

11. 1.5 ounces of 5% benzyl alcohol is combined with 1.5 ounces of 10% benzyl alcohol and 1.5 ounces of water. What is the percent of benzyl alcohol in the mixture?

Example

A 128 fl. oz. orange juice bottle with 10% calcium is combined with a 64 fl. oz. orange juice bottle with 5% calcium. What is the percent of calcium in the combined orange juice?

Solution

Find the volume of calcium in the 10% orange juice: $128 \times 0.1 = 12.8$ fl. oz.

Find the volume of calcium in the 5% orange juice: $64 \times 0.05 = 3.2$ fl. oz.

Percent of calcium in combined orange juice = volume of calcium ÷ volume of orange juice

$= (12.8 + 3.2) \div (128 + 64)$

$= 16 \div 192$

$= 0.0833$

$= 8.3\%$ (to 1 d.p.)

There is **8.3% calcium** in the combined orange juice.

6. 500 ml of drink A containing 10% sugar was combined with 250 ml of drink B containing 1% sugar. What percent of the 750 ml combined drink is sugar?

7. A 15 ounce trail mix bag contains 15% pretzels. The 25 ounce trail mix bag contains 10% pretzels. If you combine the two bags of trail mix then what is the percent of pretzels?

8. A 225 gram bag of a dried berry mix contains 12% strawberries. A 400 gram bag of a dried berry mix contains 5% strawberries. If you combine the two bags of dried berry mix then what is the percent that is not strawberries?

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11. 1.5 ounces of 5% benzyl alcohol is combined with 1.5 ounces of 10% benzyl alcohol and 1.5 ounces of water. What is the percent of benzyl alcohol in the mixture?
More Mixture Tasks

California Standards: 15.0

Example

A mixture containing 10% citric acid is to be mixed with 5 liters of a mixture made of 25% citric acid. A new mixture which is 15% citric acid is needed. How many liters of the 10% mixture must be used?

Solution

Let \( x \) = volume of 10% mixture \( \Rightarrow \) amount of acid = 10% of \( x \) liters = 0.10(\( x \))
5 liters = volume of 25% mixture \( \Rightarrow \) amount of acid = 25% of 5 liters = 0.25(5)
\( x + 5 \) = volume of new 15% mixture \( \Rightarrow \) amount of acid = 15% of \( (x + 5) \) liters = 0.15(\( x + 5 \))

**Sum of amounts of acid in initial mixtures = amount of acid in new mixture**

\[
\begin{align*}
0.10(x) + 0.25(5) &= 0.15(x + 5) \\
0.10x + 1.25 &= 0.15x + 0.75 \\
0.5 &= 0.05x \\
x &= 10
\end{align*}
\]

10 liters of 10% citric acid solution must be mixed with the 5 liters of 25% acid.

1. A chemist wants to dilute a 30% phosphoric acid solution to a 15% solution. He needs 10 liters of the 15% solution. How many liters of the 30% solution and water must the chemist use?

2. A student wants to dilute 55 liters of a 20% salt solution to a 12% salt solution. How many liters of distilled water does the student need to add to the 20% salt solution to obtain the 12% salt solution?

3. An alloy containing 30% gold is mixed with a 60% gold alloy to get 300 kilograms of an alloy that is 50% gold. How many kilograms of each alloy were used?

4. Mike has a 5% salt solution and a 20% salt solution. How many milliliters of each of these solutions will he need to mix together in order to get 450 ml of a 10% salt solution?
5. Susan is mixing up some paint for her kitchen. She starts with 500 ml of an orange paint that is 30% yellow. How much of an orange paint that is 80% yellow must she add to get an orange paint that is 50% yellow as she would like?

6. 4 liters of 2% milk contains 8% cholesterol. A container of skim milk contains 2% cholesterol. How many liters of skim milk need to be added to the 4 liters of 2% milk to create a mixture with 4% cholesterol?

7. Barbara has a bucket that holds 18 fl. oz. She mixes 14.4 fl. oz. of water and 3.6 fl. oz. of ammonia to make a cleaning solution. She then decides to replace a portion of the cleaning solution to create a cleaning solution with 25% ammonia. How much cleaning solution should be replaced by ammonia?

8. A manufacturing company makes 500 ml bottles of flavored water. They sell a bottle with 30% flavoring and another bottle with 10% flavoring. The company then decides to combine the two bottles of water to create another bottle with 18% flavoring. How much of the bottled water with 10% flavoring needs to be replaced by the 30% solution to produce the 18% flavoring?

9. Roberta mixed together 20 ounces of tea that cost $3.25 per ounce with an unknown amount of tea that cost $4.50 per ounce. She sold all the mixture at $4.00 per ounce. How many ounces of $4.50 tea did she add if she just broke even?

10. Mrs. Schmeiderer is making a cheese tray containing Cheddar and Swiss cheese that will cost $5.50 per pound. The Cheddar cheese costs $5.20 per pound and the Swiss cheese costs $6.00 per pound. If Mrs. Schmeiderer has 2.5 pounds of Cheddar cheese then how many pounds of Swiss cheese can she add and still break even?
Work Rate

California Standards: 15.0

Example

Laverne can rake the leaves in the yard in 5 hours. Tyrone can rake the leaves in the same yard in 3 hours. How long would it take them to rake the yard together?

Solution:

Laverne’s work rate = \( \frac{1}{5} \)

Tyrone’s work rate = \( \frac{1}{3} \)

\[
\frac{1}{5} + \frac{1}{3} = \frac{3}{15} + \frac{5}{15} = \frac{8}{15}
\]

\[
1 \div \frac{8}{15} = \frac{15}{8} \text{ hours to rake the yard.}
\]

1. Assume (for each case) that the workers, machines or pipes do not affect each other — so their work rates can be added together.

a. Machine A can drain a well in 3 hours and Machine B can drain the same well in 4 hours. How long will it take the machines to drain the well when working together?

b. Henry can vacuum a house in 20 minutes. His uncle Guadalupe can vacuum the same house in 12 minutes. How long would it take the two of them to vacuum the house if they worked together?

c. Demolition Machine 1 can destroy a house in 60 minutes whereas Machine 2 can destroy the same house in 40 minutes. How long would the two machines take to destroy the house together?

d. Adiella can paint her bedroom in 80 minutes, whereas her mother can paint the room in 50 minutes. How long would the two take to paint the room together?
e. Joshua and Vanessa can finish a project in 5 hours. If Joshua can finish the project in 7 hours, how long will it take Vanessa to finish the project alone?

You have to assume that the combined work rate is the sum of the individual work rates.

f. Mom can fold laundry in 30 minutes. Her daughter can fold the same load of laundry in 60 minutes. How long would it take them together to fold the laundry?

g. Together pipes A and B can fill a tank in 4 hours. If Pipe A can fill the tank in 7 hours alone, how long would it take Pipe B to fill the tank alone?

h. Pipe A can fill a tank in 14 hours. Pipe B can empty the same tank in 10 hours. Miscommunication between Emilio and Stephen causes Emilio to release water into Pipe A at the same time as Stephen opens Pipe B to empty the tank. With both pipes open to the tank, how long does it take to empty the tank?

i. Janice can wax the family car in 2 hours. Her younger brother, Lewis, can wax the family car in 2.5 hours. How long would it take them to wax the car together?

j. The gymnasium is being decorated for the Homecoming Dance. Jenny can decorate the gym in 8 hours. Reuben can decorate the gym in 12 hours and Jon can decorate the gym in 8 hours. How long will it take the three of them working together to decorate the gymnasium for the Homecoming Dance?
Mary and Jose work at a car wash. Mary can wash a car in 9 minutes and Jose can wash the same car in 15 minutes. How long would it take the two to wash a car together if they did not affect each other’s work rates?

Solution

Let \( x \) = time for the two to wash the car together

\[
\frac{1}{9} = \text{Mary's work rate} \quad \frac{1}{15} = \text{Jose's work rate} \quad \frac{1}{x} = \text{combined work rate}
\]

Equation:

\[
\frac{1}{9} + \frac{1}{15} = \frac{1}{x}
\]

Multiply by \( 45x \) — the LCM of 9, 15 and \( x \)

\[
45x \left( \frac{1}{9} + \frac{1}{15} \right) = \frac{45x}{x}
\]

\[
5x + 3x = 45
\]

\[
x = \frac{5}{8}
\]

It would take Mary and Jose \( \frac{5}{8} \) minutes to wash a car together.

1. Create and solve an equation for each task in exercises a. – i. Assume (for each case) that the workers, machines or pipes do not affect each other — so their work rates can be added together.

   a. Miguel can paint a mural on a school wall in 7 days. Alfonso can paint an identical mural in 4 days. How long would it take the two boys to paint the mural together?

   b. Pipe A can drain a water tank in 30 hours. Pipe B can fill the same tank in 5 hours. Assuming that the rate of flow is constant in each pipe and that the tank is initially empty, how long would it take to fill the tank if both pipes were open?
c. Lorraine can clean a house and yard in 2 hours, and Andrew can clean them in 3 hours. How long would it take them to clean the house and yard if they worked together?

d. How long will it take both James and Sophie to mow some grass which James can mow in 6 hours and Sophie in 10 hours?

e. A technician and his assistant can repair a bus in 9 hours. The technician works 4 times as fast as his assistant. How long would it take the assistant to repair the bus alone?

f. Simone can change the engine oil in 60 minutes. With Raphael’s help, Simone can change the engine oil in 24 minutes. How long would it take Raphael to change the engine oil by himself?

g. Jackie can do a job in 9 days, Catrina in 12 days, and Mike in 15 days. How long would it take the three of them to finish the job?

h. A tank can be filled by Pipe A in 9 hours and by Pipe B in 15 hours. It can be drained, from full, by Pipe C in 30 hours. How long would it take to fill the tank from empty with all three pipes open?

i. A bucket can be filled in 10 minutes. However, there is a small hole in the bottom of the bucket so the bucket fills in 15 minutes. How long does it take the hole to empty the bucket (from full)?
Absolute Value Equations

California Standards: 3.0

Example

Solve: \(|8x - 3x| = 35\)

Solution

Combine like terms.

\[ 5x = 35 \text{ or } 5x = -35 \quad \text{Write out two equations} \]

\[ x = 7 \text{ or } x = -7 \]

1. Find the distance of each of the following letters from zero.

![Distance scale with letters W, X, Y, Z]

Distance is always positive.

2. Simplify:
   a. \(|-22| \]
   b. \(|-m| \]
   c. \(|-5| \]
   d. \(|-12| \]

3. Solve:
   a. \(|x| = 15 \]
   b. \(|3x| = 9 \]
   c. \(|-x| = 45 \]
   d. \(|x| = \sqrt{100} \]

4. Explain why \(|x - 9| = 22\) has two solutions whereas \(x - 9 = 22\) has only one solution.

5. Find the solution: a number has a distance of 17 from zero.

----

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6. Solve:
   a. $|x + 25| = 26$  
   b. $|x - 6.3| = 6.3$

7. Solve:
   a. $|8x| = 56$  
   b. $|-6x| = 54$  
   c. $|5.2x| = 20.8$  
   d. $|\frac{2}{3}x| = 42$

8. Solve:
   a. $\left|\frac{x}{3}\right| = 8$  
   b. $\left|\frac{x}{2.5}\right| = 50$

9. Why does $|-4x| = 12$ have a solution but $-|4x| = 12$ does not have a solution?

10. Solve:
    a. $|8x - 3x| = 35$  
    b. $|-9x - 2x| = 88$  
    c. $|2x - 7x + 3x| = 26$

11. Solve:
    a. $|8x + 3| = 27$  
    b. $|5(2x - 6)| = 90$  
    c. $|3(4x - 1) + 8(x - 6)| = 49$

12. Find the solution: the sum of half of a number and four times a number is 9 units from zero.

13. Find the solution: the sum of two consecutive even integers is a distance of 34 units from zero.
More on Absolute Value

California Standards: 3.0

Example

Solve \(|5x - 9| = 11\).

Solution

An absolute value equation \(|ax + b| = c\) can be rewritten as two separate equations: \(ax + b = c\) and \(ax + b = -c\).

\[
\begin{align*}
5x - 9 &= 11 \\
5x - 9 + 9 &= 11 + 9 \\
x &= 4
\end{align*}
\]

or

\[
\begin{align*}
5x - 9 &= -11 \\
5x - 9 + 9 &= -11 + 9 \\
x &= -\frac{2}{5}
\end{align*}
\]

1. Solve:

   a. \(|3x + 5| = 14\)

   b. \(|\frac{4}{7}x| = 4\)

   c. \(|2x - 13| = 5\)

   d. \(|4x + 3| + 1 = 10\)

   e. \(|2x - 7| - 3 = 8\)

   f. \(|3 - 3x| = 15\)

   g. \(2|3x + 5| - 8 = 6\)

   h. \(5|3x - 6| - 3 = 12\)

2. Given \(-2|3 - x| = -10\), show that \(x = -2\) or \(x = 8\).

3. Given \(\frac{|2x + 6|}{3} = 10\), show that \(x = 12\) or \(x = -18\).
4. Given that \( \frac{2}{3} |1 - x| = 6 \), find the possible values of \( x \).

5. Given \(-7|2x + 5| + 9 = -12\), find all the possible values of \( x \).

---

**Example**

Solve \( |5x - 11| = |3x + 9| \).

**Solution**

To solve equations of the form \( |ax| = |bx| \), express the absolute value equation as two separate equations: \( ax = +(bx) \) and \( ax = -(bx) \).

\[
|5x - 11| = |3x + 9|
\]

\[
5x - 11 = +(3x + 9) \quad \text{or} \quad 5x - 11 = -(3x + 9)
\]

\[
\begin{align*}
5x - 11 &= 3x + 9 \\
5x - 3x &= 9 + 11 \\
2x &= 20 \\
x &= 10
\end{align*}
\]

\[
\begin{align*}
5x - 11 &= -3x - 9 \\
5x + 3x &= -9 + 11 \\
8x &= 2 \\
x &= \frac{1}{4}
\end{align*}
\]

6. Solve:

a. \( |2x - 1| = |x + 1| \)

b. \( |3x + 1| = |-8| \)

c. \( |2x + 8| = |x - 8| \)

d. \( |4x - 9| = |2x + 5| \)

e. \( 2|x + 5| = |x + 8| \)

f. \( 3|3x - 1| = 6|x + 3| \)

g. \( 7|5x - 7| = 21|x + 5| \) \( \text{Simplify, if possible, before solving} \)

h. \( \frac{2x - 9}{5} = \frac{3x + 1}{2} \)
Inequalities

1. Match the graph with the corresponding inequality and interval notation.
   a. ⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️bie ➕⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️bie ➕
      A. \(-3 \leq x\)  1. \([3, \infty)\)
   b. ⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️bie ➕⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️bij ➖
      B. \(x \geq 3\)  2. \((3, \infty)\)
   c. ⬇️⬇️⩽ biopsy ➖⬇️ biopsy ➖⬇️ biopsy ➖⬇️ biopsy ➖
      C. \(x > 3\)  3. \([-3, \infty)\)
   d. ⬇️⬇️⩽ biopsy ➖⬇️ biopsy ➖⬇️ biopsy ➖⬇️ biopsy ➖
      D. \(x < -3\)  4. \((-\infty, 3)\)
   e. ⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️bie ➕⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️bij ➖
      E. \(3 > x\)  5. \((-\infty, -3)\)

2. Graph each inequality and rewrite each inequality using interval notation.
   a. \(w > 0\) ⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️bij ➖
      b. \(4 \geq g\) ⬇️⬇️⬇️⬇️⬇️⬇️⬇️⬇️bij ➖
      c. \(f \geq -4\) ⬇️⬇️⬇️⬇️bij ➖

3. Why does the inequality \(x > 8\) not contain any brackets when written using interval notation?

Example

Graph the inequality \(w < -5\) on the number line, and write the inequality using interval notation.

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Solution

In interval notation, \(w < -5\) is \((-\infty, -5)\)
4. Write an inequality to match each graph.

a. 

b. 

c. 

d. 

5. Write each inequality using interval notation.
   a. \( x < -12 \)
   b. \( x \leq -9 \)
   c. \( 20 < x \)
   d. \( x \geq 7 \)

6. Write each interval as an inequality.
   a. \( (-\infty, 36] \)
   b. \( (1, \infty) \)
   c. \( [-13, \infty) \)
   d. \( (-\infty, 12) \)

7. A gymnast needs a score of at least 8.6 on the balance beam to regain first place. Write an inequality that represents the score the gymnast needs to attain to regain first place.

8. The play area at the shopping mall requests that only children under 40 inches use this area. Write an inequality that represents the height restriction on the play area at the shopping mall.

9. A trash can holds no more than 16 gallons. Write an inequality to represent the capacity of the trash can.

10. Tricia's cell phone plan costs $39.50 with 60 free minutes each month. Any minute over 60 minutes is an additional cost. Write an inequality to represent Tricia's possible cell phone bill each month.
Addition and Subtraction Properties of Inequalities

California Standards: 4.0, 5.0

Example

Determine the values of $x$ for which $2x - 2 > 3x$. Write the solution set in interval notation and graph it on a number line.

Solution

\[2x - 2 > 3x\]

\[-2 > 3x - 2x\]

\[-2 > x\]

or, $x < -2$

-2 is not part of the solution, so it is marked by an open circle.

1. Match the graph with the corresponding inequality.

a. \[\begin{array}{cc}
\text{a. } & -10 -8 -6 -4 -2 0 2 4 6 8 10 \\
\end{array}\]

b. \[\begin{array}{cc}
\text{b. } & -10 -8 -6 -4 -2 0 2 4 6 8 10 \\
\end{array}\]

c. \[\begin{array}{cc}
\text{c. } & -10 -8 -6 -4 -2 0 2 4 6 8 10 \\
\end{array}\]

d. \[\begin{array}{cc}
\text{d. } & -10 -8 -6 -4 -2 0 2 4 6 8 10 \\
\end{array}\]

e. \[\begin{array}{cc}
\text{e. } & -10 -8 -6 -4 -2 0 2 4 6 8 10 \\
\end{array}\]

A. $m + 6 \leq -4$
B. $m - 6 \leq 4$
C. $m + (-4) > 6$
D. $m + 4 > -6$
E. $m + 4 > 4$

2. In each exercise, determine the solution set for each inequality, write it in interval notation and graph it on the number line provided.

a. $x + 5 > 7$

b. $x - 3 < 1$

c. $8 + x < 3$

d. $x - 7 \geq 12$
3. Solve and graph each inequality.
   a. \( k - 3 < 2k \)
   
   b. \( 14m \geq 6 + 13m \)
   
   c. \( 9r > 8r - 1 \)

4. Solve each inequality and write each solution using interval notation.
   a. \( 8x - 7(x + 3) > -20 \)
   
   b. \( 3(x - 6) < 2(x - 5) \)
   
   c. \( -9(x - 3) + 7 \leq -(10x - 27) \)
   
   d. \( 3(4 - 2x) \geq -7(x - 1) + 5 \)

5. Find the maximum integer value of \( x \) which satisfies each inequality.
   a. \( x - 9 \leq -34 \)
   
   b. \( 10x < 3(3x - 1) \)

6. Determine the minimum or maximum integer value of \( x \) which satisfies each inequality. State whether the integer value is a minimum or a maximum.
   a. \( -3(x - 11) < -2x \)
   
   b. \( 18x - 8(2x + 6) \leq x - 8 \)

7. Why is 11 not the maximum integer value of \( x \) for the inequality \( 2(2x - 7) < 3(x - 1) \)?

8. The sum of two consecutive even integers is greater than 18. What is the minimum value for the first even integer?

9. Reginald's quota in the shoe department is $550 per month. This month he has $370 towards his quota. Assume in both parts of the question that his sales are spread evenly among his working days.
   a. If Reginald is working ten more days this month, then what is the minimum amount of shoe sales required each day to meet his monthly quota of $550?
   
   b. If Reginald is working six more days this month, then what is the minimum amount of shoe sales required each day to meet his monthly quota of $550?
**Example**

Solve the inequality \(-\frac{x}{5} > -6\)

**Solution**

\[-\frac{x}{5} > -6\]
\[-x > -6 \times 5\]
\[-x > -30\]
\[x < 30\]

There are two ways to make \(x\) positive from the step \(-x > -30\):

1) multiply through by \(-1\) and flip the inequality sign

2) subtract \(-x\) and \(-30\) from both sides of the inequality

---

1. Solve each inequality.

   a. \(\frac{x}{6} < -4\)
   b. \(\frac{2}{3}x \geq 8\)
   c. \(10 < \frac{1}{5}x\)

   d. \(\frac{x}{9} > -\frac{1}{27}\)
   e. \(\frac{4x}{3} \geq 0\)
   f. \(\frac{x}{(2 + 3)} \leq \frac{3}{10}\)

2. Solve each inequality.

   a. \(4k < 12\)
   b. \(36 \geq 72m\)
   c. \(14 < 14x\)

   d. \(2(3x) \leq -72\)
   e. \(x + 3x > 8 - 16\)
   f. \(2(3 + x) > (-4)^2\)

3. Solve each inequality.

   a. \(-x < 3\)
   b. \(\frac{-x}{2} > 4\)
   c. \(-2x < 12\)

   d. \(-12x \leq 12\)
   e. \(0 \leq \frac{-5x}{2}\)
   f. \(-18x \leq 3(10 - x)\)

Remember to reverse the inequality sign if you are multiplying or dividing by a negative number.
4. Solve each inequality.
   a. $\frac{x}{3} \leq -6$  
   b. $\frac{3}{4}x > -12$
   c. $20 < -4x$
   d. $-10 < 20x$
   e. $-18y \geq -54$
   f. $-5 < -\frac{1}{3}x$
   g. $\frac{3}{4} \leq -4(x - 1)$
   h. $3(5 + x) > -2x$

5. The inequality, $-4x < 16$, is solved and graphed below. Use the graph to justify why the inequality was not solved correctly.

   $-4x < 16$
   \[
   \frac{-4x}{4} < \frac{16}{4} \\
   \Rightarrow x < -4
   \]

Example

Solve the inequality $\frac{x}{-3} - 6 > -8$

Solution

\[
\frac{x}{-3} - 6 > -8 \\
\Rightarrow \frac{x}{-3} > -2 \\
x < -2 \times (-3) \\
x < 6
\]

6. Determine, graph, and write in interval notation the solution set for $2x - 3 > 5$.

7. Solve each inequality.
   a. $3 + 2(x + 2) \leq 3$
   b. $-(3x - 2) > 8$
   c. $-3 > \frac{3}{2} - x$
   d. $-5 - \frac{x}{2} \geq 4$
   e. $-3(1 - x) \geq \frac{3}{12}$
   f. $\frac{3}{2} \geq \frac{3}{4} + x$
   g. $9 - (x + 3) < x$
   h. $-36 + 2^x \leq -4x$

8. Hector has $45 to spend on dry cleaning per month. This month he has spent $24. One long sleeve shirt costs $1.75 to dry clean. What is the maximum number of long sleeve shirts Hector can have dry cleaned for the remainder of the month?
**Example**

Solve $5(2x - 1) - 2(x - 3) \leq -3(3 - 2x)$, justifying each step. Write the solution in interval notation and graph it on a number line.

**Solution**

$5(2x - 1) - 2(x - 3) \leq -3(3 - 2x)$

$10x - 5 - 2x + 6 \leq -9 + 6x$ \hspace{1cm} \text{Distributive property of multiplication over addition}

$10x - 2x - 5 + 6 \leq -9 + 6x$ \hspace{1cm} \text{Commutative property of addition}

$8x + 1 \leq -9 + 6x$ \hspace{1cm} \text{Combining like terms}

$8x - 6x + 1 \leq -9 + 6x - 6x$ \hspace{1cm} \text{Subtraction property of inequalities}

$2x + 1 \leq -9$ \hspace{1cm} \text{Combining like terms}

$2x + 1 - 1 \leq -9 - 1$ \hspace{1cm} \text{Subtraction property of inequalities}

$2x \leq -10$ \hspace{1cm} \text{Combining like terms}

$\frac{2x}{2} \leq -\frac{10}{2}$ \hspace{1cm} \text{Division property of inequalities}

$x \leq -5$ \hspace{1cm} \text{Canceling}

Interval notation: $(-\infty, -5]$

Graph: 

---

1. Solve $4x + 3 > 15$. Graph the solution set and write it in interval notation.

---

2. In each exercise, solve the inequality.

   a. $3x + 2 < 8$

   b. $2x - 3 \leq -1$

   c. $-5 - x \leq -5$

   d. $7(x - 2) + 6 > 6$

   e. $9 - 3(2 + x) \geq 12$

   f. $5x - 2(x + 3) < 0$
3. Solve $2(7 - 3x) < 16$, justifying each step of your solution.

4. Solve each inequality in exercises a. – n.

   a. $3(4 - 3x) > -24$
   
   b. $2(3x - 2) - 8x > 6$
   
   c. $-7 - 3(x + 2) \leq 10x$
   
   d. $5x - 3 \leq -3(x - 1) + 10$
   
   e. $\frac{x + 1}{3} + \frac{1}{2} \leq \frac{4}{3}x$
   
   f. $\frac{x - 3}{6} - \frac{7}{15} \leq \frac{3x - 1}{10}$
   
   g. $11 - 3(2 - x) \leq -2x - 5$
   
   h. $5(1 - x) - 3(2x - 7) \leq -6(x - 1)$
   
   i. $2(2x + 1) - 3(1 - 2x) \geq 5(x - 2)$
   
   j. $2(x - 3) + 3(3 - x) \geq 3(3x - 5)$
   
   k. $\frac{x}{2} - \frac{5}{6}x - 5 < -\frac{7}{9}x$
   
   l. $\frac{1}{6}x - 5 \leq \frac{3}{4}x + \frac{1}{4}$
   
   m. $\frac{1}{2}(x - 3) - \frac{1}{3}(2x + 1) \leq \frac{1}{6}(3x - 3)$
   
   n. $\frac{4x}{6} + \frac{x}{3} - \frac{3}{2} \geq \frac{1}{2} + \frac{x}{2}$

5. Solve $\frac{6(2.2 - x) - 4(x - 1)}{9} \leq \frac{-7(2x + 1.8)}{3}$. Graph the solution set and write it in interval notation.
Applications of Inequalities

California Standards: 4.0, 5.0

Example

The sum of the numbers \( k \) and 8 is at least 19. Find all possible values of \( k \).

Solution

\[
\begin{align*}
k + 8 &\geq 19 \\
k &\geq 11
\end{align*}
\]

Translate the question into an inequality. The sum of \( k \) and 8 is at least 19, so a \( \geq \) inequality sign is used.

1. Five less than the number \( 2t \) is at most 15. Find all possible values of \( t \).

2. Find the three smallest consecutive even integers whose sum is greater than 126.

3. Find the three largest consecutive odd integers whose sum is less than 159.

4. The sum of four consecutive even integers is at least 244. Find the least possible value of the first integer.

5. The sum of four consecutive odd integers is 368 or less. Find the largest possible value of the fourth integer.

6. A rectangle has a length of \( (2w + 7) \) feet and a width of \( (w + 5) \) feet. Find the possible values of \( w \) for the perimeter of the rectangle to be at least 72 feet.

Remember to give the value of the fourth integer, not the first.
7. The width of a rectangle is 8 feet. If the length is \((2x + 5)\) feet, find the maximum integer value of \(x\) for the area to be at most 152 square feet.

8. Find the area of a rectangle whose length is the least integer value of \(l\) that satisfies \(2l - 15 > 5\), and whose width is the largest integer value of \(x\) that satisfies \(3x - 7 < 17\).

9. A man weighing 340 pounds is admitted into a hospital for a special diet to lose no more than 6 pounds per week. What is the shortest time in weeks it could take the man to reduce his weight to 243 pounds?

10. The senior class of a High School plans to sponsor a holiday dance. Band A offers to play for \$350 plus 40\% of the ticket sales, and Band B offers to play for the flat rate of \$830. The dance organizers expect 400 of the seniors to attend. What is the most the organizers can charge each person and still expect Band A's offer to be the cheapest?

11. Car Rental Company A rents cars for \$15 per day plus 8 cents per mile driven. Car Rental Company B rents cars for \$9 per day plus 14 cents per mile driven. For what daily mileage is Car Rental Company A a better deal than Car Rental Company B?

12. Telephone Company A offers its customers a rate of \$40 per month for 500 minutes of anytime airtime plus 17 cents per minute for additional airtime. Telephone Company B offers its customers a rate of \$20 per month for 500 minutes of airtime plus 50 cents per minute for additional airtime. For how many additional minutes is Telephone Company A's plan a better offer than the plan offered by Telephone Company B?
Compound Inequalities

California Standards: 4.0, 5.0

1. Express each conjunction as a single mathematical statement.
   a. $5 \leq 3a - 4$ and $3a - 4 < 17$
   b. $12b - 1 < -2$ and $12b - 1 > -3$
   c. $5 < 6 - 3c$ and $21 \geq 6 - 3c$
   d. $-3 \geq 2.5d - 1.1$ and $2.5d - 1.1 \geq -12.3$

Example

Solve $-4 \leq 2(x - 5) \leq 6$, justifying each step. Write the solution in interval notation and graph it.

Solution

$-4 \leq 2(x - 5) \leq 6$

$-4 \leq 2x - 10 \leq 6$  
Distributive property of multiplication over addition

$-4 + 10 \leq 2x - 10 + 10 \leq 6 + 10$  
Addition property of inequalities

$6 \leq 2x \leq 16$  
Combining like terms

$\frac{6}{2} \leq \frac{2x}{2} \leq \frac{16}{2}$  
Division property of inequalities

$3 \leq x \leq 8$  
Canceling

Interval notation: $[3, 8]$

Example

Solve the compound inequality $\frac{x + 2}{3} > \frac{1}{2}$ or $\frac{x + 2}{3} < -\frac{1}{2}$.  
The solution is the set of $x$-values that satisfy either one inequality or the other.

Solution

$6 \left( \frac{x + 2}{3} \right) > 6 \left( \frac{1}{2} \right)$ or $6 \left( \frac{x + 2}{3} \right) < 6 \left( -\frac{1}{2} \right)$  
Multiply by 6: the LCM of the denominators

$2(x + 2) > 3$ or $2(x + 2) < -3$

$2x + 4 > 3$ or $2x + 4 < -3$

$2x > -1$ or $2x < -7$

$x > -\frac{1}{2}$ or $x < -\frac{7}{2}$

Interval notation: $\left( -\infty, -\frac{7}{2} \right) \cup \left( -\frac{1}{2}, \infty \right)$

There are no common points between the two inequalities.
2. In Exercises a.–l., solve the conjunction and write the solution in interval notation.
   a. \(-7 < 3x + 5 < 14\) .................................................................
   b. \(-5 \leq 2x - 1 \leq 7\) .................................................................
   c. \(-7 \leq 11 - 3x \leq 41\) .................................................................
   d. \(-18 \leq 3(x - 3) \leq 3\) .................................................................
   e. \(-16 \leq 11 - 3(1 - x) \leq 17\) .................................................................
   f. \(-5 \leq \frac{7 - 3x}{5} \leq 7\) .................................................................
   g. \(-9 < \frac{3 - 2x}{2} < 5\) .................................................................
   h. \(-\frac{3}{5} \leq \frac{5x - 7}{15} \leq \frac{11}{3}\) .................................................................
   i. \(\frac{1}{5} < \frac{2x - 1}{5} < \frac{3}{4}\) .................................................................
   j. \(-9 \leq \frac{2(x - 3)}{3}\) and \(\frac{2(x - 3)}{3} \leq 5\) .................................................................
   k. \(-2 \leq \frac{2a - 6}{4} \leq 2\) .................................................................
   l. \(\frac{3}{10} < \frac{2 - x}{5} + \frac{x - 3}{6} \leq \frac{8}{15}\) .................................................................

3. In Exercises a.–i., solve the disjunction and write the solution in interval notation.
   a. \(2y + 5 < 5y - 1\) or \(5y - 1 < 3y - 7\) .................................................................
   b. \(3(x + 2) \geq 1\) or \(3(x + 2) \leq -1\) .................................................................
   c. \(5x + 3 > 13\) or \(5x + 3 < -13\) .................................................................
   d. \(5x - 7 > 8\) or \(4x + 5 < -7\) .................................................................
   e. \(7(x + 2) \geq 30\) or \(7(x + 2) \leq 6\) .................................................................
   f. \(3(x - 4) < -5\) or \(3(x - 4) > 5\) .................................................................
   g. \(\frac{x}{3} < \frac{x}{2}\) or \(\frac{x}{3} > \frac{x}{2}\) .................................................................
   h. \(\frac{4x - 4}{2} > \frac{2}{3}\) or \(\frac{4x - 4}{2} < -\frac{26}{3}\) .................................................................
   i. \(\frac{1}{3}(1 - 5y) \geq 1\) or \(\frac{1}{3}(1 - 5y) \leq -\frac{16}{3}\) .................................................................
**Topic 3.4.1**  
**Absolute Value Inequalities**

**California Standards:** 3.0

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**Example**

Solve $|2x - 5| \leq 11$. Graph the solution and write it in interval notation.

**Solution**

$-11 \leq 2x - 5 \leq 11$  \hspace{1cm} Write the absolute value inequality as a compound inequality  
$-6 \leq 2x \leq 16$ \hspace{1cm} Add 5 (addition property of inequalities)  
$-3 \leq x \leq 8$ \hspace{1cm} Divide by 2 to get $x$ on its own (division property of inequalities)  

Interval notation: $[-3, 8]$

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1. For each inequality, write the equivalent compound inequality, graph the solution set, and say whether each one is a conjunction or disjunction.

   a. $|m| < 2$
   
   b. $|x| \geq 7$

2. Solve each inequality. Write the solution using interval notation.

   a. $|8j| \leq 24$
   
   b. $9 < \frac{m}{3}$
   
   c. $\frac{c}{11} \geq 5$

3. In Exercises a. – i., solve each inequality. Graph the solution set of every third problem.

   a. $|x - 4| < 5$
   
   b. $|x + 3| > 5$
   
   c. $|2x - 5| < 3$
   
   d. $|3x - 2| < 7$
   
   e. $|5x - 1| > 9$
   
   f. $|2x - 3| > 7$
   
   g. $|3x - 4| \leq 5$
   
   h. $|4x - 1| \leq 6$
   
   i. $|2x - 1| \leq 4$
4. Solve each inequality. Write the answer using interval notation.
   a. $|3x + 2| < 4$  
   b. $|5x - 3| \geq 2$
   c. $\frac{|4e - 6|}{6} \leq 5$  
   d. $\frac{|7x + 5|}{2} > 1$
   e. $\frac{|x - 9|}{5} < 12$  
   f. $|6x - 2| \geq x + 33$

5. A wedding coordinator estimates that the Ramirez's wedding will cost approximately $18,000. The wedding coordinator has to stay within $1,500 of the estimated amount.
   a. Write an absolute value inequality to show the cost boundaries of the Ramirez’s wedding.
   b. Use this inequality to find the maximum and minimum cost of the Ramirez's wedding.

6. A factory produces plastic pipe. They receive an order with a requirement for each pipe to be 1.5 m in length. Quality Assurance decides that the pipe length should differ by no more than 6% of the specified 1.5 m.
   a. Write an absolute value inequality with which Quality Assurance can specify the pipe length.
   b. Use this inequality to find the pipe lengths which would pass the Quality Assurance requirements.

7. A pan of water is liquid at a temperature of 122°F. If its temperature is increased by more than 90°F it becomes a gas, or if it drops by more than 90°F it becomes a solid. A scientific experiment requires water to not be in a liquid state. Use these figures to write an absolute value inequality to specify the range of temperatures which fulfil these criteria. Solve it to find the range of temperatures at which water is not in its liquid form.

---

**Example**

Write $-1 < x < 3$ as a single inequality that involves an absolute value.

**Solution**

$x$ is greater than $-1$ but less than $3$. The midpoint of this interval is $1$.
There is a difference of $2$ between each end of the interval and its midpoint.
$x$ minus the midpoint is always less than the difference
between the midpoint and one end of the interval, so: $|x - 1| < 2$

8. Write each of the following as a single inequality involving an absolute value.
   a. $4 < x < 9$  
   b. $-3 < x < 5$
   c. $-3 < x < 7$  
   d. $x \leq -21$ or $x \geq 19$
The Coordinate Plane

California Standards: 6.0

1. Fill in the blanks.
   a. The origin is the point of intersection of the _____________ and _____________ at zero.
   b. The vertical number line is called the _____________ on the coordinate plane.
   c. 4 and –6 in the ordered pair, (4, –6) are called ________________.
   d. A plane extends indefinitely in _____________ directions.

Example

Plot and label the points on the coordinate plane.
K(1, 5), L(1, –4), M(–2, –1), N(–2, 2)

Name the figure formed by connecting the points in alphabetical order.

Solution
The figure is a trapezoid.

2. Plot and label the points on the coordinate plane.
   A (4, 0)       B (–5, –5)
   C (–3, 5)      D (2, 7)
   E (4, 8)       F (0, –3)

3. Identify the coordinates of each point.
   J _____________  K _____________
   L _____________  M _____________
   N _____________  P _____________
4. Are the points (2, 5) and (5, 2) the same? Explain.

5. a. Identify the x-coordinate for point W. 
   b. Identify the y-coordinate for point R. 
   c. Identify the x-coordinate for point T. 
   d. What is the sum of the x and y-coordinate for the point U? 
   e. What is the product of the x and y-coordinate for the point V?

6. Plot the following points on the coordinate plane.
   A (–3, 3) B (–6, –2) C (5, –2) D (2, 3)
   Connect the points sequentially and name the resulting geometric shape.

   Geometric shape: 

7. Plot and label the points on the coordinate plane.
   Q (5, 1) R (2, –7) S (8, –7) W(–7, –2) X(1, 6) Y (3, 4) Z (–5, –4)
   a. Name the type of triangle formed by connecting points Q, R, and S in alphabetical order. Find its area.
      Name 
      Area 
   b. Name the figure WXYZ formed by connecting the points in alphabetical order.
**Topic 4.1.2**

**Quadrants of the Plane**

**California Standards: 6.0**

1. Identify the quadrant or axis for each point.

F ................. G .................

H ................. J .................

K ................. L .................

M .................

---

**Example**

Given: $r > 0, t > 0, u < 0$, state the quadrant or axis where each point is located.

A $(r, t)$  B $(r, u)$  C $(t, -6)$  D $(u, 0)$

**Solution**

A – Quadrant I  B – Quadrant IV  C – Quadrant IV  D – x-axis

---

2. In exercises a. – d., state the quadrant or axis where each point is located and justify your answer.

a. $(-2, 3)$

b. $(3, 0)$

c. $(-15,000, -11,788)$

d. $(5, -12)$
3.  a. Explain how to identify a point that lies on the $y$-axis without graphing.

b. Explain how to identify a point that lies in quadrant III without graphing.

4. State the quadrant or axis where the point $\left( -\frac{1}{10}, -\frac{1}{17} \right)$ is located.

5. In exercises a. – f., state the quadrant or axis where each point is located and justify your answer.
   a. $(0, k)$, $k > 0$
   b. $(m, n)$ such that $m > 0$ and $n > 0$
   c. $(a, k)$ such that $a > 0$ and $k < 0$
   d. $(x, 0)$ such that $x < 0$
   e. $(k, 0)$ such that $k \neq 0$
   f. $(0, 0)$

6. Complete the following sentences.
   a. Point $(x, y)$ lies in quadrant IV. The $y$-coordinate has a \textbf{negative} sign.
   b. Point $(x, y)$ lies on the $y$-axis. The $y$-coordinate has a \textbf{positive} sign.
   c. Point $(x, y)$ lies in quadrant III. The $x$-coordinate has a \textbf{negative} sign.
   d. Point $(x, y)$ lies in quadrant I. The $x$-coordinate has a \textbf{positive} sign.
1. Fill in the blanks.

   a. A line extends indefinitely in ___________ directions.

   b. A plane extends indefinitely in ___________ directions.

   c. A line and plane have ___________ thickness. Both have an infinite number of points.

2. Draw the line defined by the pair of points. Label the line with the corresponding letter.

   A: (0, 6) and (–2, 1)
   B: (–8, –1) and (1, –6)
   C: (–1, –3) and (5, 0)
   D: (–3, 8) and (4, –2)
3. Identify two additional points on each line.

   a. Given: \((-4, 2), (0, 0)\) and \((4, -2)\)

   b. Given: \((-1, -1)\) and \((1, 5)\)

4. Draw the graph for each situation. Label each line.

   a. The set of all points such that \((x, -x)\).

   b. The set of all points such that \((x, 3x)\).
If \( x \in \{ -3, -\frac{3}{4}, 1, 6 \} \), find the set of points defined by \((x, 3x - 4)\).

**Solution**

**Substitute each value of** \( x \) **into** \((x, 3x - 4)\) **to find the coordinates**

- If \( x = -2 \) then \( 3(-2) - 4 = -10 \).
  \((-2, -10)\)
- If \( x = 0 \) then \( 3(0) - 4 = -4 \).
  \((0, -4)\)
- If \( x = 5 \) then \( 3(5) - 4 = 11 \).
  \((5, 11)\)

5. **a.** If \( x \in \{ -3, -\frac{3}{4}, 1, 6 \} \), find the set of points defined by \((x, -2x)\).

6. **a.** Graph the following sets of points on the coordinate plane.

   - Set M of points defined by \((x, 2x + 4)\).
   - Set N of points defined by \((x, -\frac{1}{2}x - 6)\).

   **b.** What point do the two lines share?
7. If $x$ is any integer between 1 and 5, not including those values, find the set of all points defined by $(x, 4x)$.

8. If $x$ is any odd integer between 2 and 10, find the set of all points defined by $(x, x - 2)$.

9. If $x$ is any even integer between $-5$ and 3, find the set of all points defined by $(x, -3x + 2)$.

10. If $x$ is any prime number between 1 and 10, find the set of all points defined by $(x, -x)$.

11. If $x$ is any multiple of 3 larger than 9, but less than 21, find the set of all points defined by $(x, x + 5)$.

12. If $y$ is any odd integer between $-3$ and 4, not including those values, find the set of all points defined by $(3y - 1, y)$.

13. Given: $(0, 0), (1, 2), (2, 4), (3, 6), (4, 8)$.
   Write an ordered pair such that the $y$-coordinate is in terms of $x$ for the set of points listed above.

14. Write an ordered pair such that the $y$-coordinate is in terms of $x$ for the line shown on the coordinate plane.
**Horizontal and Vertical Lines**

**California Standards: 6.0**

### Example

Draw the following lines on the coordinate plane. Label each line.

\[ x = 3 \quad y = -8 \]

**Solution** (see coordinate plane)

1. **Plot and label the lines in exercises a. – d.**
   
   **a.** \( x = 2 \) and \( y = 4 \)
   
   **b.** \( x = -3 \) and \( y = -2 \)
   
   **c.** \( x = -5 \) and \( y = 2 \)
   
   **d.** \( x = 7 \) and \( y = -3 \)
2. **a.** Draw each line on the coordinate plane. Label each line.

\[ y = 3 \quad x = -3 \quad y = 0 \quad x = 7 \]

**b.** A portion of the four lines form a rectangle. State the coordinates for the endpoints that form the rectangle.

3. **a.** Draw each line on the coordinate plane. Label each line.

\[ y = 6 \quad x = 4 \quad y = -5 \quad x = 7 \]

**b.** A portion of the four lines form a rectangle. Calculate the area of the rectangle.

4. State three ordered pairs on each of the following lines.

**a.** \( y = 12 \)

**b.** \( x = -33 \)

5. Justify the statement as true or false.

All equations in the form \( x = \) any real number are parallel to the \( y \)-axis.

6. Write the equation for each line.

**a.**

**b.**

**c.**
Topic 4.2.1

Points on a Line

California Standards: 6.0, 7.0

1. Determine whether each equation is linear or nonlinear. Justify your answer for each nonlinear equation.
   a. \( y = -15 \)
   b. \( 4x = y \)
   c. \( y^2 = x + 2 \)
   d. \( 3xy = 12 \)
   e. \( 4x - 6y = 12 \)

Example

Determine whether \((-1, -3)\) lies on the line whose equation is \(4y + 3x = -15\).

Solution

\[
4y + 3x = -15 \\
4(-3) + 3(-1) = -15 \quad \text{Substitute the } x \text{ and } y \text{ values} \\
-12 - 3 = -15 \\
-15 = -15 \quad \text{The statement is true}
\]

So, the point \((-1, -3)\) lies on the graph of \(4y + 3x = -15\).

2. a. Show that \((-4, 1)\) is a point on the graph of \(y = \frac{3}{2}x + 7\).
 b. Show that \((5, -6)\) is a point on the graph of \(x + 5y = -25\).

3. In exercises a.– c., determine whether or not the given point lies on the graph of the equation.
   a. \((2, 1): y + 2x = 5\)
   b. \((-3, -4): 5y - 2x = -14\)
   c. \((-1, 3): 2x - 3y = 11\)
   d. \((3, -4): 6y - 5x = -39\)
   e. \((2, 5): 2x - 5y = 10\)
4. a. Show that \( \left( \frac{1}{2}, 1 \right) \) is a point on the graph of \( y = 6x - 2 \).

b. Show that \( \left( \frac{1}{5}, 4 \right) \) is a point on the graph of \( y = 5x + 3 \).

c. Show that \( \left( \frac{1}{4}, \frac{3}{4} \right) \) is a point on the graph of \( x = 1 - y \).

5. Determine whether each point lies on the graph of \( 18x - 12y = 48 \).
   a. \((-4, -16)\)
   b. \((-10, -19)\)
   c. \((6, 5)\)
   d. \((22, 37)\)

6. Determine whether each point lies on the graph of \( 9x + 3y = 21 \).
   a. \(\left( \frac{11}{2}, 2 \right)\)
   b. \((2.4, 0.2)\)
   c. \((-8, 31)\)
   d. \((-4.3, 19.9)\)

7. Determine whether the point \((2, 5)\) lies on the line \(2y - 7x = -4\).
   Justify your answer.

8. Determine which line(s) the point \((33, 45)\) lies on: \(x = 33, x = 45, y = 33,\) or \(y = 45\).
   Justify your answer.

9. Determine whether the point \((-4, -6)\) lies on the lines \(x - y = 2\) and \(x + 4y = -28\).
Graph the equation $2x + y = 5$.

**Solution**

Solve for $y$: $2x + y = 5$

$y = -2x + 5$

**Rearrange to the form $y = Px + Q$.**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>$y = -2(-1) + 5 = 7$</td>
<td>(-1, 7)</td>
</tr>
<tr>
<td>3</td>
<td>$y = -2(3) + 5 = -1$</td>
<td>(3, -1)</td>
</tr>
</tbody>
</table>

**Choose two values of $x$ and substitute them into your equation.**

Plot these two points and draw a straight line between them.

Plot a third point to check that the line is correct.

Check $(0, 5)$: Does $2(0) + 5 = 5$? Yes.
2. Match each line on the graph with its corresponding equation.
   a. \(3x - 4y = 28\)  
   b. \(4x + 3y = 3\)  
   c. \(x - 4y = -20\)  
   d. \(4x + y = -5\)  

3. Graph and label each of the following lines.
   Line A: \(4y = -8x + 4\) 
   If the coefficient of \(x\) is a fraction then choose \(x\)-values that will simplify the fraction to a whole number.
   Line B: \(-6y = 3x\)  
   Line C: \(-y = 4x - 3\)  

4. Graph and label each of the following lines.
   Line P: \(x + 3y = -9\)  
   Line Q: \(-2x + y = 0\)  
   Line R: \(3x - 9y = 63\)  

5. Graph and label each of the following lines.
   Line J: \(2x - 5y = 20\)  
   Line K: \(5x + 2y = 6\)  
   Line M: \(x - 5y = -25\)  
   Line N: \(5x + y = -3\)
Using Line Equations

California Standards: 7.0

Example

(3p, 8) is a point on the line 4x + y = 14.


b. Find the coordinates of the point.

Solution

a. 4x + y = 14
   4(3p) + 8 = 14
   12p = 6
   p = \frac{1}{2}

b. 3p = 3 \times \frac{1}{2} = \frac{3}{2} \quad \text{Substitute in your value of } p

So the coordinates of the point are \( \left( \frac{3}{2}, 8 \right) \).

1. If \((-k, 5)\) is a solution to the equation \(-2x - 3y = -14\), find \(k\).

2. For each of the exercises a. – i., find the unknown value and the coordinates of the point.

   a. \((-4, 3f)\) is a point on the line \(8x - y = -5\).

   b. \((r + 7, 1)\) is a point on the line \(4x - y = 3\).

   c. \((5c, 0)\) is a point on the line \(8x - y = 0\).

   d. \(\left(\frac{2}{5}k, -2\right)\) is a point on the line \(5x + y = 14\).

   e. \((0.6e, 3.8)\) is a point on the line \(4x + 5y = 11.8\).

   f. \(\left(\frac{1}{5}h, -\frac{2}{3}h\right)\) is a point on the line \(15x - 18y = 45\).

   g. The point \((-4, -2k)\) lies on the graph of \(x + 3y = 14\).

   h. The point \(\left(\frac{1}{2}n, \frac{5n}{3}\right)\) lies on the graph of \(8x - y = -17\).

   i. The point \((7, 3w - 2)\) lies on the graph of \(3x + 2y = 53\).
Example

(3, –2) lies on the graph of \(-2kx + 5y = 26\). Find the value of \(k\).

Solution

\[-2kx + 5y = 26\]

\[-2k(3) + 5(-2) = 26\]  Substitute in the \(x\)- and \(y\)-values to give you an equation in terms of \(k\)

\[-6k - 10 = 26\]  Solve to find \(k\)

\[-6k = 36\]

\[k = -6\]

3. In exercises a. – f., solve for \(k\) if the given point lies on the line whose equation is given.

a. \((2, –2):\) \(kx + 2y = 12\)  
   b. \((3, 4):\) \(4x – 3ky = -9\)

   c. \((-3, –2):\) \(-4x – ky = -15\)  
   d. \(\left(\frac{1}{2}, \frac{-4}{5}\right):\) \(-2kx + 10y = -1\)

   e. \(\left(\frac{1}{4}, 15\right):\) \(-4kx – ky = -32\)  
   f. \(\left(\frac{1}{3}, \frac{-11}{2}\right):\) \(4x – 2ky = 5\)

4. The point \((-4, 9)\) lies on the line \(2kx – 3ky = 35\).

   a. Find the value of \(k\).

   b. Use the value of \(k\) to rewrite the equation.

   c. Find the coordinates of the point with an \(x\)-coordinate of 4.

5. The point \((3, 3)\) lies on the line \(4rx + 6ry = -15\).

   a. Find the value of \(r\).

   b. Use the value of \(r\) to rewrite the equation.

   c. Find the coordinates of the point with a \(y\)-coordinate of \(-3\).

6. The point \((-1, 5)\) lies on the line \(3kx + ky = -4\). Find the value of \(k\).

   Use the value of \(k\) to rewrite the equation.
The \( x \)- and \( y \)-intercepts

California Standards:  6.0

1. Identify the \( x \)- and \( y \)-intercept, if it exists, for each line.

<table>
<thead>
<tr>
<th></th>
<th>( x )-intercept</th>
<th>( y )-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line A</td>
<td>( ............. )</td>
<td>( ............. )</td>
</tr>
<tr>
<td>Line B</td>
<td>( ............. )</td>
<td>( ............. )</td>
</tr>
<tr>
<td>Line C</td>
<td>( ............. )</td>
<td>( ............. )</td>
</tr>
<tr>
<td>Line D</td>
<td>( ............. )</td>
<td>( ............. )</td>
</tr>
</tbody>
</table>

Example

Find the \( x \)- and \( y \)-intercepts of the line \( 4x + 3y = 16 \).

**Solution**

**Let \( y = 0 \), then solve for \( x \):**

\[
4x + 3y = 16 \\
4x + 3(0) = 16 \\
4x = 16 \\
x = 4 \\
(4, 0)
\]

**Let \( x = 0 \), then solve for \( y \):**

\[
4x + 3y = 16 \\
4(0) + 3y = 16 \\
3y = 16 \\
y = \frac{16}{3} \\
\left(0, \frac{16}{3}\right)
\]

The graph crosses the \( x \)-axis when \( y = 0 \), so to find the \( x \)-intercept, substitute \( y = 0 \) into the equation and solve.

2. In exercises a. – e., compute the \( x \)- and \( y \)-intercepts of the graph of the given equation.
   
   a. \( x - 3y = 15 \)
   
   b. \( 2x + 5y = 20 \)
   
   c. \( -3x + 5y = -15 \)
   
   d. \( 2x - 3y = 9 \)
   
   e. \( 8x + 5y = 80 \)
3. Find the \( x \)- and \( y \)-intercepts of the line \( 3y + 7x = 42 \).

4. Find the \( x \)- and \( y \)-intercepts for each line.
   a. \( 2.4x - 0.4y = 12 \)
   b. \( \frac{1}{4}x + \frac{2}{3}y = 24 \)
   c. \( 0.5x - 2.5y = -25 \)

5. The point \((a, 0)\) is the \( x \)-intercept of \( 2y - 3x = 15 \). Find the value of \( a \).

6. a. \((-6h, 0)\) is the \( x \)-intercept of the line \( 3x + y = 18 \). Find the value of \( h \).
   b. \( \left(0, \frac{3}{4}e\right)\) is the \( y \)-intercept of the line \( 9x - 20y = -30 \). Find the value of \( e \).
   c. \((0, 2w - 3)\) is the \( y \)-intercept of the line \( 2x + 5y = 25 \). Find the value of \( w \).

7. a. Write the equation of a line that does not have an \( x \)-intercept.
   b. Write the equation of a line that only has an \( x \)-intercept.

8. a. The point \((2, -2)\) lies on the line \( 2kx - 9ky = 44 \). Find the \( x \)-intercept of the line.
   b. The point \(\left(\frac{1}{2}, 5\right)\) lies on the line \( 6kx + 2ky = -26 \). Find the \( y \)-intercept of the line.
   c. The point \((-5, -7)\) lies on the line \(-rx + 3ry = 48 \). Find the \( y \)-intercept of the line.
Graphing Lines

California Standards: 6.0, 7.0

1. Use the given intercepts to graph each line. Label each line.
   a. \((-4, 0)\) and \((0, 6)\)
   b. \((9, 0)\) and \((0, -9)\)
   c. \((5, 0)\) and \((0, 3.5)\)

2. Use the given intercepts to graph each line. Label each line.
   a. \(\left(\frac{1}{2}, 0\right)\) and \(\left(0, \frac{19}{2}\right)\)
   b. \((3.5, 0)\) and \((0, -8.5)\)
   c. \((7, 0)\) and \(\left(0, \frac{14}{5}\right)\)

Example

Graph the equation \(3x + 8y = -24\) using \(x\)- and \(y\)-intercepts.

Solution

\[
\begin{align*}
\text{\(y\)-intercept:} & & \text{\(x\)-intercept:} \\
3(0) + 8y = -24 & & 3x + 8(0) = -24 \\
8y = -24 & & 3x = -24 \\
y = -3 & & x = -8
\end{align*}
\]

Therefore \((0, -3)\) is the \(y\)-intercept and \((-8, 0)\) is the \(x\)-intercept.
3. In exercises a. – l., graph each line using the intercepts. Label each line.
   a. $x + y = -6$
   b. $-x - y = 1$
   c. $x - y = 8$
   d. $7x - 3y = -21$
   e. $4x + 8y = 32$
   f. $-x - 2y = 6$
   g. $2x + 4y = -6$
   h. $-5x - 4y = 10$
   i. $5x - 3y = 15$
   j. $y = 7x + 8$
   k. $y = -x + 6$
   l. $y = \frac{2}{3}x + 2$

4. a. Find the intercepts for the lines $10x + 5y = 25$ and $2x + y = 5$.  
   b. Are the two equations equivalent?  
   c. Explain whether it is necessary to graph the two lines to determine if the equations are equivalent.

5. Determine the direction of the following lines.
   a. The x-intercept has a positive x-coordinate and the y-intercept has a negative y-coordinate.
   b. The x-intercept has a negative x-coordinate and the y-intercept has a negative y-coordinate.

6. Why can the line $3x - y = 0$ not be graphed using intercepts only?
Slope of a Line

California Standards: 7.0

1. State whether each line has a positive or negative slope.
   - Line A
   - Line B
   - Line C
   - Line D

2. Find the slope of each line.
   - Line W
   - Line X
   - Line Y
   - Line Z

Example

The slope $m$ of the straight line containing any two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Find the slope of the line through $(-4, 7)$ and $(5, 1)$.

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{5 - (-4)}$$

$$m = \frac{-6}{9}$$

$$m = -\frac{2}{3}$$ Simplify

3. In exercises a. – n., find the slope of the straight line through each pair of points.
   a. $(5, -8)$ and $(-3, 4)$
   b. $(-4, -5)$ and $(3, 5)$
   c. $(-5, -7)$ and $(3, 12)$
   d. $(4, 6)$ and $(-3, 7)$
5. a. Draw a line that has an undefined slope.
b. Use the slope formula to confirm the line has undefined slope.

c. Write the equation of the line with undefined slope.

d. State a conclusion about all lines with undefined slope.
The equation of the line through \((x_1, y_1)\) with slope \(m\) is given by the formula \(y - y_1 = m(x - x_1)\).

Find the equation of the line through \((-2, -7)\) that has a slope of \(-\frac{2}{9}\).

**Solution**

\[
y - y_1 = m(x - x_1)
y - (-7) = -\frac{2}{9}(x - (-2))\]

Substitute the \(x\)- and \(y\)-values into the formula

\[
y + 7 = -\frac{2}{9}(x + 2)
\]

\[
\frac{2}{9}x + y = -\frac{67}{9}
\]

Rearrange the equation into the form \(Ax + By = C\)

1. In exercises a. – j., write the equation of the line through the given point and with the given slope. Give your answer in the form \(Ax + By = C\)

a. \((4, -3), m = 2\)

b. \((0, 7), m = -5\)

c. \((5, 6), m = 0\)

d. \((-3, -4), m = 4\)

e. \(\left(\frac{1}{2}, 5\right), m = -3\)

f. \((4, 9), m = \frac{1}{4}\)

g. \((-3, 2), m = -\frac{3}{7}\)

h. \((5, -2), m = \frac{1}{2}\)

i. \((-3, -4), m = -\frac{2}{3}\)

j. \((-1, 7), m = \frac{3}{8}\)
Example
Find the equation of the straight line through \((-4, 5)\) and \((7, -3)\).
Give your answer in the form \(Ax + By = C\).

Solution
First find the slope of the line, using the “rise over run” formula

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Then find the equation using the point-slope formula

\[
y - y_1 = m(x - x_1)
\]

\[
\therefore y - 5 = -\frac{8}{11}(x - (-4))
\]

\[
y - 5 = -\frac{8}{11}(x + 4)
\]

\[
11y - 55 = -8x - 32
\]

\[
8x + 11y = 23
\]

2. In exercises a. – i., write the equation of the straight line passing through the given pair of points. Give your answer in the form \(Ax + By = C\).

a. \((8, 4)\) and \((12, -8)\)

b. \((1, -5)\) and \((-7, -13)\)

c. \((-9, -11)\) and \((-4, -1)\)

d. \((4, -2)\) and \((-3, 5)\)

e. \((-3, -1)\) and \((5, -7)\)

f. \((-4, -5)\) and \((7, 8)\)

g. \((2, 3)\) and \((-8, 9)\)

h. \(\left(\frac{2}{3}, -\frac{2}{5}\right)\) and \(\left(-\frac{1}{3}, \frac{1}{5}\right)\)

i. \((9, 10)\) and \((-3, -7)\)

3. Points \(A(-5, 1)\), \(B(1, 5)\) and \(C(5, -1)\) are graphed.

a. Plot point \(D\) to form square \(ABCD\).

b. Write the equation of the line containing points \(A\) and \(D\).

c. Write the equation of the line containing points \(C\) and \(D\).
California Standards: 8.0

1. Determine whether the lines whose equations are \(6y - 5x = -27\) and \(6y = 5x + 11\) are parallel or not. Justify your answer.

---

Example

Determine whether the straight line through \((3, 7)\) and \((-5, -17)\) is parallel to the straight line through \((2, 11)\) and \((-4, -7)\).

**Solution**

**Two lines are parallel if their slopes \(m_1\) and \(m_2\) are equal.**

Line through \((3, 7)\) and \((-5, -17)\).

\[
m_1 = \frac{-17 - 7}{-5 - 3} = \frac{-24}{-8} = 3
\]

Line through \((2, 11)\) and \((-4, -7)\).

\[
m_2 = \frac{-7 - 11}{-4 - 2} = \frac{-18}{-6} = 3
\]

Since \(m_1 = 3\) and \(m_2 = 3\), the lines are parallel.

---

2. In exercises a. – d., determine whether the lines passing through each pair of points are parallel or not.

a. \((0, 7)\) and \(\left(\frac{1}{2}, 9\right)\), \((0, -3)\) and \((2, 5)\)

b. \((7, 0)\) and \((2, -3)\), \((5, -2)\) and \((0, -5)\)

c. \((-3, -4)\) and \((15, 2)\), \((4, -5)\) and \((3, -2)\)

d. \((3, -5)\) and \((-5, -9)\), \((-3, -1)\) and \((3, 2)\)

---

3. If the straight line through \((-2, 5)\) and \((k, 7)\) is parallel to \(2x - 7y = 14\), what is \(k\)? Explain or justify your answer.

---

4. Determine whether the straight line through \((-8, -1)\) and \((3, 11)\) is parallel to the line \(11y - 12x = 1\).
5. Determine whether the straight line through \((5p, 4p)\) and \((-3p, 2p)\) is parallel to the line \(8y - 2x = -1\).

Example

Find the equation of the line through \((5, -2)\) that is parallel to the line through \((7, 4)\) and \((3, -4)\).

Solution

The lines have the same slope, so:

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-4)}{7 - 3} = \frac{8}{4} = 2
\]

First, find the slope of the line through \((7, 4)\) and \((3, -4)\).

Substitute \((7, 4)\) and \((3, -4)\) into the slope formula to find the slope

\[
m = \frac{y - y_1}{x - x_1} = \frac{y - (-2)}{x - 5} = 2(x - 5)
\]

Substitute \((5, -2)\) and 2 into the point-slope formula

\[
y + 2 = 2x - 10
\]

\[
y - 2x = -12
\]

You know that the line going through \((5, -2)\) also has a slope of 2. Use the point-slope formula to find the equation of the line.

6. In exercises a. – e., find the equation of the line that satisfies the given conditions:

a. Contains the point \((-1, -2)\) and is parallel to the line through \((5, 1)\) and \((0, -8)\).

b. Passes through \((-5, -3)\) and is parallel to a line with slope \(\frac{2}{7}\).

c. Parallel to a line through \((-4, 6)\) and \((5, -8)\) and contains the point \((-7, -3)\).

d. Contains the point \((2, 1)\) and is parallel to a line through \((-3, -5)\) and \((-7, 5)\).

e. Parallel to a line through \((4, -3)\) and \((3, 3)\) and contains the point \((-2, -2)\).

7. Write the equation of a line through \((-5, 7)\) that is parallel to the line whose equation is \(3y - 2x = 5\).

8. The straight line through \((3a, 2a - 1)\) and \((-5, -4)\) is parallel to the line whose equation is \(2y + 5x = -3\). Find the value of \(a\).
Perpendicular Lines

California Standards: 8.0

1. Find the negative reciprocal for each of the following.
   a. 5
   b. \(-\frac{4}{5}\)
   c. \(\frac{t}{3z}\)
   d. \(\frac{1}{3}\)
   e. \(-4\)

Example

Determine whether the line through \((-2, 2)\) and \((3, -1)\) is perpendicular to the line through \((-3, -2)\) and \((0, 3)\).

Solution

Two lines in a plane are perpendicular if their slopes, \(m_1\) and \(m_2\), are negative reciprocals of each other. That is, if \(m_1 \times m_2 = -1\), the lines are perpendicular.

\[
m_1 = \frac{-1 - 2}{3 - (-2)} = \frac{-3}{5} \quad \text{and} \quad m_2 = \frac{3 - (-2)}{0 - (-3)} = \frac{5}{3}
\]

Since \(m_1 \times m_2 = \frac{-3}{5} \times \frac{5}{3} = -1\), the lines must be perpendicular to each other.

2. Determine whether a pair of lines with these slopes would be perpendicular, parallel, or neither.
   a. \(4\) and \(\frac{1}{4}\)
   b. \(\frac{2}{3}\) and \(-\frac{2}{3}\)
   c. \(\frac{3}{6}\) and \(\frac{1}{2}\)
   d. \(-2\) and \(\frac{1}{2}\)

3. a. Prove that lines \(k\) and \(l\) are perpendicular.
    
   b. Draw in line \(m\) perpendicular to line \(k\) through point \((3, -8)\).
    
   c. State a conclusion about lines \(l\) and \(m\).
4. Show that the line through the points (–7, 3) and (–3, 9) is perpendicular to the line through points (2, –3) and (8, –7).

Example

Find the equation of the line that passes through point (–2, 9) and is perpendicular to the line through (–3, –4) and (5, 7).

Solution

Let \( m_1 = \frac{7 - (-4)}{5 - (-3)} = \frac{11}{8} \)

So, \( m_2 = -\frac{8}{11} \) \[\text{Find the negative reciprocal of } m_1\]

\( y - y_1 = m(x - x_1) \)

\( y - 9 = -\frac{8}{11}(x - (-2)) \) \[\text{Substitute into the point-slope formula}\]

\( 11(y - 9) = -8(x + 2) \)

\( 11y - 99 = -8x - 16 \)

\( 11y + 8x = 83 \)

5. Determine whether the two lines are perpendicular to each other.

a. Line A through points (–3, –8) and (2, 12), and line B through points (–2, –4) and (–10, –6).

b. Line C through points (7, 1) and (–2, 4), and line D through points (–7, 2) and (–9, –4).

c. Line E through points (8, 3) and (6, –1), and line F through points (1, 9) and (5, 7).

6. Determine the equations of the following lines:

a. Line P through point (–5, 9) and perpendicular to a line with slope \( \frac{9}{10} \).

b. Line Q through point (–5, 2) and perpendicular to a line with slope 0.

7. Determine the equations of the following lines.

a. Line W through point (–1, 6) which is perpendicular to a line through points (1, 4) and (7, –2).

b. Line Y through point (3, –4) which is perpendicular to a line through points (9, 4) and (0, –8).
Topic 4.4.3  The Slope-Intercept Form of a Line

California Standards: 8.0

1. Determine whether each equation is in slope-intercept form.
   a. $3x + 6y = -11$  
   b. $(y - 4) = \frac{2}{3} (x + 6)$  
   c. $y = -\frac{1}{4}x + 15$  
   d. $y = 5x$  
   e. $5 + 6x = 8y$  
   f. $y = -\frac{2}{3}x + \frac{7}{8}$

2. Find the slope and $y$-intercept for each equation.
   a. $y = 5x - 9$  
   b. $y = -\frac{3}{7}x$  
   c. $y = \frac{1}{6}x + 2$  
   d. $y = x - 1$

Example

Graph $y = 4x - 5$ using the slope and $y$-intercept.

Solution

$y = mx + b$
Slope $= m = 4$
For a slope of 4, go up 4 units for every 1 across

$y$-coordinate of the $y$-intercept $= b = -5$
$y$-intercept $= (0, -5)$

3. Graph each line using the slope and $y$-intercept. Label each line.
   a. $y = 3x$
   b. $y = \frac{2}{5}x - 6$
   c. $y = -x + 7$
   d. $y = -\frac{1}{2}x + 3$
4. Determine the slope and $y$-intercept for each equation.
   a. $3x + 7y = -14$
   b. $8x - y = 7$
   c. $x + y = -17$
   d. $y = 8$
   e. $-4x + 3y = 12$

5. Determine the slope and $y$-intercept for each equation.
   a. $y = -4(x - 6)$
   b. $y - 6 = -\frac{2}{3}(x + 4)$
   c. $9 - y = 2(x - 5)$
   d. $y = \frac{3}{4} \left(x + \frac{2}{3}\right)$

6. Graph and label each equation using the slope and $y$-intercept.
   a. $5x + 15y = 45$
   b. $x + y = -2$
   c. $4x + 8y = 0$
   d. $2x - 10y = 10$

7. Write each equation in slope-intercept form.
   a. A line that passes through points $(0, -12)$ and $(5, 7)$.
   b. A line that passes through points $(0, -4)$ and $(-4, -4)$.
   c. A line that passes through points $(2, 3)$ and $(5, -6)$.
More About Slopes

California Standards: 8.0

1. Determine whether the following pairs of lines are parallel, perpendicular, collinear, or none of these.

   a. \( y = 4x + 8 \) and \( y = -4x + 8 \)
   
   b. \( y = \frac{1}{5}x - 15 \) and \( y = -5x + 18 \)
   
   c. \( y = 4x - 8 \) and \( 2y = 8x - 16 \)
   
   d. \( y = \frac{6}{7}x \) and \( y = \frac{6}{7}x - 12 \)
   
   e. \( y = \frac{8}{9}x - 1 \) and \( y = -\frac{9}{8}x + 8 \)
   
   f. \( x = 3 \) and \( y = -12 \)
   
   g. \( y = 2 \) and \( y = 0 \)
   
   h. \( 3y + 2x = 11 \) and \( 2y - 3x = 16 \)
   
   i. \( y + 2x = 10 \) and \( 3y - x = 4 \)
   
   j. \( 5y - x = -4 \) and \( 5y = x + 13 \)
   
   k. \( 8x - 12y = 46 \) and \( 4x - 6y = 23 \)
   
   l. \( x + 10y = -40 \) and \( 10x - y = 15 \)
   
   m. \( 5x - 8y = 16 \) and \( 10x - 16y = -32 \)

2. Determine whether the straight line through \((4, 2)\) and \((-2, -4)\) and the straight line through \((-2, 2)\) and \((3, -3)\) are parallel, perpendicular, collinear, or none of these.

---

**Example**

Find the equation of the line that is parallel to \(4x - y = 14\) and passes through the point \((7, 13)\).

Write the equation in slope-intercept form.

**Solution**

Rearrange the equation for \(y\) to identify the slope

\[
4x - y = 14 \\
-y = -4x + 14 \\
y = 4x - 14 \\
m = 4
\]

Write the equation of the other line

\[
(y - 13) = 4(x - 7) \\
y - 13 = 4x - 28 \\
y = 4x - 15
\]
3. Write each equation in slope-intercept form.
   a. Find the equation of the line through the point \((-4, 1)\), parallel to a line with a slope of \(-3\).

   b. Find the equation of the line through the point \((0, 2)\), parallel to a line with a slope of \(\frac{1}{9}\).

   c. Find the equation of the line through the point \((-6, 2)\), perpendicular to a line with a slope of \(-5\).

4. Write each equation in slope-intercept form.
   a. Find the equation of a line that is parallel to \(y = -\frac{2}{3}x\) and passes through the point \((-6, 10)\).

   b. Find the equation of a line that is parallel to \(y = 11x - 2\) and passes through the point \((7, -8)\).

   c. Find the equation of the line perpendicular to \(y = -\frac{1}{6}x + 8\) and passing through the point \((2, -1)\).

   d. Find the equation of the line perpendicular to \(y = x - 10\) and passing through the point \((-6, -9)\).

5. Write each equation in the form \(Ax + By = C\).
   a. Find the equation of a line through \((-6, 10)\), parallel to the line through points \((3, 2)\) and \((7, -4)\).

   b. Find the equation of a line through \((-3, -13)\), parallel to the line through points \((6, 2)\) and \((8, -8)\).

   c. Find the equation of the line perpendicular to \(9x + 27y = 135\) and passing through the point \((-3, -3)\).

   d. Find the equation of the line perpendicular to \(-3x + 2y = -16\) and passing through the point \((5, -10)\).

6. Show that the points \(R(-4, -2)\), \(S(0, -8)\) and \(T(-10, -6)\) are vertices of a right triangle. Identify the vertex of the right angle.
1. Use the graph to determine if the points given are in the solution set.
   a. (0, 0)  
   b. (4, –4)  
   c. (–2, –9)  
   d. (5, 7)  

2. State whether the points given are solutions of $x + y > 3$.
   a. (2, 1)  
   b. (3, 2)  
   c. (–1, –1)  
   d. (2, –5)  

3. State whether the points given are solutions of $6x – 5y < 15$.
   a. (0, 0)  
   b. (0, –3)  
   c. (–1, –9)  
   d. (2, 5)  

4. State whether the points given are solutions of $7x – 4y > 19$.
   a. (2, –1)  
   b. (4.7, 2)  
   c. (–3, –7.1)  
   d. (6.5, 3.6)  

California Standards: 6.0
5. State the inequality which defines the shaded region.

6. State the inequality which defines the shaded region.

7. State the inequality which defines the shaded region.
Example

Sketch the region defined by $3x + 4y < -8$.

Solution

3x + 4y = -8

$4y = -3x - 8$

$y = \frac{-3}{4}x - 2$

Equation of border line

Rearrange into the form $y = mx + b$

Pick two values of $x$ and work out corresponding values of $y$.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>-5</td>
</tr>
</tbody>
</table>

Plot the points calculated and connect with a dotted line.

Pick a point on one side of the line and enter into the original inequality.

$(1, 1)$:

$3x + 4y < -8$

$3 \times 1 + 4 \times 1 < -8$

$7 < -8$

This statement is false, so the point $(1, 1)$ is not a solution of the inequality.

Shade the area of the graph that does not include $(1, 1)$.

8. Sketch the region defined by $y > -x - 1$. 

---

CGP Education Algebra I — Homework Book

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9. Sketch the region defined by \( y < 2x - 3 \).

10. Sketch the region defined by \( x + y > 0 \).

11. Sketch the region defined by \( 6x - 4y < -24 \).
1. Explain the difference between a solid line and a dashed line with respect to the solution set.

2. Use the graph to determine if the points given are in the solution set.
   a. (0, 0) ..........................................................
   b. (–1, –8) ..........................................................
   c. (5, 8) ..........................................................
   d. (5, –2) ..........................................................

3. State whether the points given are solutions of \( x + 4y \geq 2 \).
   a. (0, 0) ..........................................................
   b. (–6, 2) ..........................................................
   c. (5, 9) ..........................................................
   d. (–3, –8) ..........................................................

4. State the inequality which defines the shaded region.

   ..........................................................

5. Which of the following has (8, 10) in its solution set? Justify your answers.
   a. \( 9x - 6y = 12 \) ..........................................................
   b. \( 9x - 6y < 12 \) ..........................................................
   c. \( 9x - 6y \leq 12 \) ..........................................................
Example

Sketch the region defined by \( 3 - x \geq 3y \).

Solution

\[
3 - x = 3y \\
3y = 3 - x
\]

Equation of border line

\[
y = 1 - \frac{1}{3}x
\]

Rearrange to the form \( y = mx + b \)

\[
m = -\frac{1}{3} \quad \text{and} \quad b = 1
\]

Draw the border line using the values of \( m \) and \( b \) calculated.

\[
(0, 0): \\
3 - 0 \geq 3 \times 0 \\
3 \geq 0
\]

Pick a point on one side of the line and enter into the original inequality.

This statement is true, so the point \((0, 0)\) is a solution of the inequality.

Shade the area of the graph that includes \((0, 0)\).

6. Sketch the regions defined by the following inequalities.

a. \( y \geq 0 \)

b. \( y \leq -5x + 2 \)

c. \( 4x - 8y \leq -24 \)

d. \( 3x + y \geq -5 \)
### Example

Sketch the region defined by \( x < 6 \), \( 5y - 3x \leq 12 \), and \( 5y + 3x > 18 \).

#### Solution

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Equation of borderline</th>
<th>Type of line</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; 6 )</td>
<td>( x = 6 )</td>
<td>Dashed</td>
</tr>
<tr>
<td>( 5y - 3x \leq 12 )</td>
<td>( y = \frac{3x + 12}{5} )</td>
<td>Solid</td>
</tr>
<tr>
<td>( 5y + 3x &gt; 18 )</td>
<td>( y = \frac{18 - 3x}{5} )</td>
<td>Dashed</td>
</tr>
</tbody>
</table>

Work out the equations of all three border lines in the form \( y = mx + b \).

Choose a point that does not lie on any of the lines, e.g. \((4, 3)\).

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Value at point ((4, 3))</th>
<th>Satisfied?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; 6 )</td>
<td>( 4 &lt; 6 )</td>
<td>Yes</td>
</tr>
<tr>
<td>( 5y - 3x \leq 12 )</td>
<td>( \frac{5 \times 3 - 3 \times 4}{3} \leq 12 )</td>
<td>Yes</td>
</tr>
<tr>
<td>( 5y + 3x &gt; 18 )</td>
<td>( \frac{5 \times 3 + 3 \times 4}{27} &gt; 18 )</td>
<td>Yes</td>
</tr>
</tbody>
</table>

All three inequalities are satisfied by the point \((4, 3)\), so it is part of the solution set.

Shade the region of the graph containing the point \((4, 3)\).
1. Sketch the regions defined by each of the following sets of inequalities.
   a. \( x \geq -2, \ y \leq 3, \ x \leq 3, \) and \( y \geq -2 \)
   b. \( 2y + x \leq -2, \) and \( 4y - x \geq -10 \)
   
   [Graphs showing the regions defined by the inequalities]

   c. \( y \leq -x, \) and \( 2y - 3x \leq 5 \)
   d. \( y - x < 1, \) and \( y + x < 7 \)

   [Graphs showing the regions defined by the inequalities]

   e. \( 4y + 3x < 10, \) and \( 2y - 3x < -10 \)
   f. \( y \geq 3, \ 3y - 4x \leq 24, \) and \( 2y + 5x \leq 6 \)

   [Graphs showing the regions defined by the inequalities]

   \textit{Border types:} \(< \) or \( > \) — dashed lines
   \( \leq \) or \( \geq \) — solid lines
2. Sketch the region defined by the inequalities $2y + x < 6$ and $2y - x > -2$.

3. Sketch the regions defined by each of the following sets of inequalities.
   
   a. $y > -\frac{3}{4}x + \frac{5}{2}$, and $y > x + 2$
   
   b. $x \geq 0$, $y < -\frac{3}{7}x + 3$, and $y > \frac{3}{7}x - 3$
   
   c. $y \leq -\frac{2}{5}x + \frac{18}{5}$, and $y \geq \frac{3}{5}x + \frac{12}{5}$
   
   d. $y \leq \frac{3}{5}x + 2$, $y > -\frac{2}{5}x + \frac{18}{5}$, and $y \geq \frac{5}{3}x - 4$
4. Sketch the figure defined by \( x \geq 2, x \leq 5, y \leq 3, \) and \( y \geq -3. \)

Find its area in square units. 

5. What type of triangle is defined by \( y \leq x, y + x \leq 6, \) and \( 11y - 5x \geq -30? \)

Justify your answer.
The Graphing Method

California Standards: 9.0

Example

Solve the following system of equations graphically.

\[\begin{align*}
3y + 2x &= -5 \\
y - x &= -5
\end{align*}\]

Solution

Rewrite the equations in the form \(y = mx + b\).

Find two points on each line. Plot the lines. The point where they intersect is the solution.

Draw a table to help work out which points to plot.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = -\frac{2}{3}x - \frac{5}{3})</th>
<th>(y = x - 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

The lines intersect at the point \((2, -3)\), so the solution is \(x = 2, y = -3\).

1. Use the graphs provided to identify the solution to each of the following systems of equations.
   a. \(3x - y = 5, x + y = -5\)
   b. \(-x + y = 4, 4x - y = 11\)
   c. \(x - y = -7, 2x + 3y = 6\)
   d. \(3x - 5y = -5, 3x + 2y = -19\)
Use the graphing method to solve these systems of linear equations:

2. \(y = -3x + 4, \ y - x = 4\)

3. \(x + 2y = 7, \ -2x + 3y = 5\)

4. \(x + 2y = 3, \ y - 2x = 9\)

5. \(y = 4, \ x = -3\)

6. \(x - 5y = 30, \ x - y - 2 = 0\)
The Substitution Method

California Standards: 9.0

Example

Solve the following system of equations by the substitution method.

\[ x = 3y - 4 \]
\[ 3x - 2y = 9 \]

**Solution**

\[ x = 3y - 4 \]
\[ 3x - 2y = 9 \]
\[ 3(3y - 4) - 2y = 9 \]
\[ 9y - 12 - 2y = 9 \]
\[ 7y - 12 = 9 \]
\[ 7y = 21 \]
\[ y = 3 \]
\[ \Rightarrow x = 3y - 4 = 3(3) - 4 \]
\[ x = 5 \]

Solution is \( x = 5, y = 3 \).

1. Solve the following systems of equations by substitution.

   a. \( y = x - 5 \) and \( 2y + x = 2 \)
   
   b. \( y = 2x + 1 \) and \( 3y - x = 13 \)

   c. \( y = \frac{2}{3} x \) and \( 6 - 9y = 12x \)
   
   d. \( y = 4x - 1 \) and \( 2x - 3y = 8 \)

   e. \( y + x = 10 \) and \( 2y = 3x \)
   
   f. \( y = x - 3 \) and \( 3x - y = 23 \)

   g. \( x = y + 4 \) and \( 6y - 2x = -12 \)
   
   h. \( y - x = 3 \) and \( 3x = 9 + y \)
2. Solve the following systems of linear equations by substitution.

   a. \(x + y = 9\) and \(2x + 3y = 22\)
   
   b. \(x - y = 7\) and \(3x + y = 9\)

   c. \(x - 2y = 3\) and \(4x + 3y = 34\)

   d. \(3y - 2x = 16\) and \(4x - y = 8\)

   e. \(x = -6\) and \(3x - 7y = 10\)

   f. \(x - 2y - 2 = 0\) and \(2x - y + 2 = 0\)

   g. \(x + y = -1\) and \(x - 3y = -9\)

   h. \(2x - 3y = 9\) and \(5x + 3y = 12\)

   i. \(3x + y = 14\) and \(2x - 5y = -2\)

   j. \(5y - x = 22\) and \(3y + 4x = 27\)

   k. \(2x + 3y = 27\) and \(3x + 2y = 23\)

   l. \(7x - 4y = 24\) and \(3y = 2x - 5\)

   m. \(6x = 4y - 2\) and \(3y - 4x - 2 = 0\)

   n. \(2x + 2y = -2\) and \(3x - y + 7 = -8\)

3. Solve this system of three linear equations by substitution:
   \[2x + y = 11, \quad 3y - 2z = 1 \quad \text{and} \quad 3z = 33 - 4x.\]

4. Solve this system of three linear equations by substitution:
   \[4x - 14 = 3y, \quad x + y + z = 7 \quad \text{and} \quad 2z + 6y = 2.\]
### Example

Solve the system of equations $2x - y = 4$ and $x + 2y = 2$ using:

**a.** the graphing method,

**b.** the substitution method.

#### Solution

**a.** Find two points to plot for each equation. Then use your points to draw the lines.

<table>
<thead>
<tr>
<th>$2x - y = 4$</th>
<th>$x + 2y = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

The lines intersect at the point $(2, 0)$, so the solution is $x = 2$ and $y = 0$.

**b.** $2x - y = 4$ and $x + 2y = 2$

Rearrange the first equation to find $y$

Substitute for $y$ in the second equation

Rearrange and solve for $x$

Substitute the value of $x$ to find $y$

$y = 2x - 4$
$x + 2(2x - 4) = 2$
$x + 4x - 8 = 2$
$5x = 10$
$x = 2$
$y = 2x - 4 = (2 \times 2) - 4 = 4 - 4$
$y = 0$

The solution is $x = 2$ and $y = 0$.

1. The system of equations below has been solved using the graphing method. Check the solution by using the substitution method. Give the correct solution. The solution of $2x + y = 5$ and $x - 3y = -15$ is $x = 5, y = 0$. 

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2. The system of equations below has been solved using the substitution method. Check the solution by graphing the equations. Give the point of intersection and the correct solution. The solution of \( x - y = 1 \) and \( 2x + 3y = 12 \) is \( x = 2 \) and \( y = 3 \).

3. Given the system of equations \( 5x + 6y = 0 \) and \( x - 2y = 16 \):
   a. Solve using the **graphing** method.
      Show all your work.
   b. Solve using the **substitution** method.
      Show all your work.

4. Given the system of equations \( x - 2y = -8 \) and \( x + 2y = -8 \):
   a. Solve using the **graphing** method.
      Show all your work.
   b. Solve using the **substitution** method.
      Show all your work.

5. Solve the system of equations \( x - 4y = -20 \) and \( 2x + y = -4 \) using the graphing method.

Solve the following systems of equations using the substitution method.

6. \( x = 5 \) and \( 2x + 5y = -5 \)

7. \( 0.5x - 3y = 4.5 \) and \( x + y = 5.5 \)
Inconsistent Systems

California Standards:  9.0

Example

Solve the following system of equations by graphing: \( x - 3y = -9 \) and \( x - 3y = -21 \).
Say whether the system is independent or inconsistent.

Solution

\[
\begin{array}{cc|cc}
   x - 3y &= -9 & x - 3y &= -21 \\
   x & y & x & y \\
   0 & 3 & 0 & 7 \\
   3 & 4 & 3 & 8 \\
\end{array}
\]

The lines are parallel, so there’s no point of intersection.
So the system of equations has no solutions — the system is inconsistent.

1. Solve each system of equations by graphing.
   a. \( y = 2 \) and \( x = -5 \)
   b. \( x = 7 \) and \( x = 1 \)

2. Solve the following systems of equations by graphing, and say whether each system is independent or inconsistent.
   a. \( 6x + y = -5 \) and \( y = -6x + 4 \)
   b. \( x - y = -3 \) and \( x + y = -3 \)
   c. Explain how you could determine whether each of the above systems is inconsistent without solving it.
3. Solve these systems of equations using the substitution method, and say whether each system is independent or inconsistent.
   a. \( y = x - 6 \) and \( x - y = 1 \)
   b. \( y = -3x - 4 \) and \( 4x - y = -10 \)

4. Without solving, determine whether each of the following systems of equations is independent or inconsistent. Justify your answers.
   a. \( y = 8x - 6 \) and \( y = 8x + 3 \)
   b. \( y = 2x + 1 \) and \( y = -2x + 1 \)
   c. \( 4x - y = -3 \) and \( 4x - y = 2 \)

5. Solve these systems of equations using the substitution method, and say whether each system is independent or inconsistent.
   a. \( 2x - y = -1 \) and \( x + 2y = 2 \)
   b. \( 3x - 4y = 0 \) and \( 3x - 4y = -12 \)

6. One of the two equations in an inconsistent system of equations is \( 3x - 9y = -18 \).
   a. Determine the slope of the second equation in this system. 
   b. Give a possible value for the \( y \)-intercept of the second equation.

7. \( 3x - y = 5 \) and \( y = 3x + 2 \) are a system of equations. The equation \( 3x - y = 5 \) is graphed below.

   a. Choose any point on the line above. Does this point satisfy the second equation \( (y = 3x + 2) \)?
   b. Plot \( y = 3x + 2 \) on the graph above.
   c. If both graphs were extended forever, would a solution ever be found? Justify your answer.
1. Complete the following sentences.
   a. A dependent system of equations has ______________________________ solutions.
   b. The lines of equations in a dependent system ______________________________.

2. Determine whether the following systems of equations graphed are dependent.
   a. ______________________________
   b. ______________________________
   c. ______________________________
   d. ______________________________

3. Solve these systems of equations by graphing, and say whether each system is dependent.
   a. $3x - y = 3$ and $y = 3x$
   b. $x + 2y = -14$ and $6y = -3x - 42$
   c. Explain how you could determine whether each system is dependent without solving it.
Use the substitution method to determine if the system of equations is dependent.

\[ y = 2x + 6 \]
\[ 6x - 3y = -18 \]

**Solution**

\[ 6x - 3(2x + 6) = -18 \]  \hspace{1cm} **Substitute the 2x + 6 for y in the second equation**
\[ 6x - 6x - 18 = -18 \]  \hspace{1cm} **Simplify**
\[ -18 = -18 \]  \hspace{1cm} **This is a true statement — the equation holds for any value of x**

This system of equations is dependent.

4. Use the substitution method to determine whether each system is dependent.
   a. \( 5x - y = 0 \) and \( 15x = 3y \)
   b. \( y = 2x - 12 \) and \( 2x - y = -1 \)

5. Use the substitution method to determine whether each system is dependent.
   a. \( 3x + 2y = -6 \) and \( x + 2y = -10 \)
   b. \( x + 4y = 12 \) and \( -20y = 5x - 60 \)

6. Without solving, determine whether the following systems of equations are dependent. Justify your answers.
   a. \( 2x - y = -3 \) and \( x = y - 6 \)
   b. \( 2x + 3y = -9 \) and \( -2x = 3y + 9 \)
   c. \( y + 4 = -3x \) and \( 3x - y = -4 \)
   d. \( 5 = 7x - 2y \) and \( 7x = 28x - (6y + 15) \)

7. One of the two equations in a dependent system of equations is \( 5x - y = 7 \).
   a. Determine the slope of the second equation in this system.
   b. Determine the y-intercept of the second equation in this system.
Systems of Equations — Further Examples

California Standards: 9.0

Example

Solve the following system of equations by graphing: \(5x + y = 6\) and \(x - 2y = 10\). Determine whether the system of equations is independent, dependent, or inconsistent.

Solution

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>-4</td>
</tr>
</tbody>
</table>

The lines intersect at \((2, -4)\), so the solution is \(x = 2\) and \(y = -4\). The system of equations has one solution, so it is independent.

1. Solve the systems of equations in each exercise by graphing. Say whether each system is independent, dependent, or inconsistent.
   
a. \(y + 2x = 4\) and \(2y + 4x = 8\)

b. \(y - 5 = 2x\) and \(-2x + y = -5\)

c. \(x - 3y = 3\) and \(x - y = -3\)

d. \(2y = 5x - 3\) and \(3 - 3x = 2x - 2y\)
2. Solve the following systems of equations by substitution. Say whether each system is independent, dependent, or inconsistent.
   a. $x = -2y - 3$ and $4y - 3x = -11$
   b. $y = -2x + 5$ and $2y + 4x = 15$
   c. $y = 4x - 1$ and $3y - 12x = -15$
   d. $x = \frac{1}{2}y - 3$ and $2x - y = -6$
   e. $y + 3x = -5$ and $2y = -6x - 10$
   f. $x - 2y = 5$ and $2x + 3y = -18$
   g. $-4y = 2x + 7$ and $7 - 2y = 3x + 2y$
   h. $3y - 4 = 6 - 2x$ and $5y - x = 6 - 2x$

You may find it helpful to label the equations (1) and (2) so you can keep track of which is which.

3. Solve the following systems of equations by either substitution or graphing. Determine whether each system is independent, dependent, or inconsistent.
   a. $y - 2x = 15$ and $y = 2x + 2$
   b. $3y - 2x = -4$ and $6y = 3x - 6$

4. Without solving, determine whether each of the following systems of equations is independent, dependent or inconsistent. State the number of solutions for each system.
   a. $x - y = 9$ and $-2y = -2x + 18$
   b. $y = 4$ and $y = 12$
   c. $5x - y = -2$ and $5x - y = 4$
   d. $x - y = -10$ and $2y = x - 14$
**Topic 5.2.1** Systems of Equations — Elimination

California Standards:  9.0

### Example

Solve the following systems of equations by elimination to find $x$ and $y$.

**a.** $3y – 2x = 21 \quad \text{and} \quad 2y – x = 13 \quad \text{\(\text{\(\text{\(1\)}}\)}} + 2$ \quad \text{\(\text{\(\text{\(2\)}}\)}}$

**Solution**

**a.** $4y – 2x = 26 \quad \text{Multiply equation \(\text{\(\text{\(2\)}}\)}}$ by $2$

$-3y – 2x = 21 \quad \text{Subtract equation \(\text{\(\text{\(1\)}}\)}}$ from the result

$\begin{align*}
\frac{y}{2} &= 5 \\
\Rightarrow y &= 5 \\
\Rightarrow 2(5) – x &= 13 \quad \text{Find } x \text{ by substituting 5 for } y \text{ in equation \(\text{\(\text{\(2\)}}\)}} \\
\Rightarrow 10 – x &= 13 \\
\Rightarrow x &= -3
\end{align*}$

The solution is $x = -3, y = 5$.

**b.** $7y + x = -16 \quad \text{and} \quad 5y – 2x = -6 \quad \text{\(\text{\(\text{\(1\)}}\)}}$

**Solution**

$14y + 2x = -32 \quad \text{Multiply equation \(\text{\(\text{\(1\)}}\)}}$ by $2$

$+ 5y – 2x = -6 \quad \text{Add equation \(\text{\(\text{\(2\)}}\)}}$ to the result

$\begin{align*}
19y &= -38 \\
\Rightarrow y &= -2 \\
\Rightarrow 7(-2) + x &= -16 \quad \text{Find } x \text{ by substituting } -2 \text{ for } y \text{ in equation \(\text{\(\text{\(1\)}}\)}} \\
\Rightarrow x &= -2
\end{align*}$

The solution is $x = -2, y = -2$.

### 1. Solve the following systems of equations using the elimination method.

**a.** $2y + x = 3$ and $3y – x = 7$

**b.** $3y – 2x = -21$ and $7y + 2x = -29$

**c.** $3y – 2x = 8$ and $4y + 2x = 13$

**d.** $9y – 2x = -6$ and $7y + 2x = -26$

**e.** $5y – 2x = -6$ and $3y – 2x = 2$

**f.** $7y + 5x = -46$ and $7y – 6x = 9$

Check your answers by substituting them into the other equation.
Try to keep the algebra as simple as possible — it doesn’t matter which equation is subtracted from which.

Some of these questions require both equations to be multiplied before elimination.

If it helps, rearrange the equation.

It’s sometimes easier to multiply out the fractions before rearranging.

Some of these questions require both equations to be multiplied before elimination.

Try to keep the algebra as simple as possible — it doesn’t matter which equation is subtracted from which.
Malinda bought three notebooks and five art sets for a total of $16.50.
Anna bought two notebooks and three art sets for a total of $10.25.
How much was each notebook and each art set?

**Solution**
Let \( n \) = price per notebook.
Let \( a \) = price per art set.

\[
\begin{align*}
3n + 5a &= 16.50 \quad \text{(1)} \\
2n + 3a &= 10.25 \quad \text{(2)}
\end{align*}
\]

\[
\begin{align*}
2(3n + 5a) &= 2(16.50) \Rightarrow 6n + 10a &= 33.00 \quad \text{(3)} \\
3(2n + 3a) &= 3(10.25) \Rightarrow 6n + 9a &= 30.75 \quad \text{(4)}
\end{align*}
\]

Subtract equation (4) from equation (3) to eliminate \( n \)

\[
\begin{align*}
a &= 2.25 \\
3n + 5a &= 16.50 \Rightarrow 3n + 5(2.25) &= 16.50 \\
\Rightarrow 3n + 11.25 &= 16.50 \\
\Rightarrow n &= 1.75
\end{align*}
\]

Each art set cost $2.25 and each notebook cost $1.75.

---

1. For each exercise, create a system of equations and solve it by elimination to answer the question.

   a. Two books and five music CDs cost $110.50. Four books and three music CDs cost $98.50. If all the books are the same price, and all the CDs are the same price, how much does each book and each CD cost?

   b. Two cups and three spoons cost $5.55. If Felicity bought ten cups and twelve spoons for $25.20, how much does each spoon and each cup cost?

   c. Cindy bought three T-shirts and six pairs of socks for $23.10. Celia bought seven T-shirts and ten pairs of socks for a total of $45.50. How much is each T-shirt and each pair of socks?
d. A teacher ordered eight electric pencil sharpeners and five dictionaries for $555. Another teacher ordered three of the electric pencil sharpeners and two dictionaries for $215. How much is each pencil sharpener and each dictionary?

---

e. Two books and three executive pens cost $26. If three books and five executive pens cost $42.50, how much does each book and each pen cost?

---

2. a. Carl is 7 years older than Angela. The sum of five times Carl’s age and four times Angela’s age is 197. How old is each now?

---

b. The combined age of the Weismillers’ two automobiles is 12 years. Three years ago, Mrs. Weismiller’s automobile was double the age of Mr. Weismiller’s automobile. How old are Mr. and Mrs. Weismiller’s automobiles now?

---

c. The sum of Ramón and Yvette’s ages is 69 years. Eight years ago, Ramón was two more than twice Yvette’s age. How old are Ramón and Yvette now?

---

d. The sum of Fernando’s and Martin’s ages is 56 years. Five years from now, Martin will be nine more than half of Fernando’s age. How old are Fernando and Martin now?

---

3. Find the values of $x$ and $y$ in the rectangle below.

---

4. The isosceles trapezoid below has a perimeter of 122 units. Determine the values of $x$ and $y$. 

---
Topic 5.3.2

Systems of Equations — Integer Problems

California Standards: 9.0, 15.0

Example

a. The sum of two integers is 98. The second number is one less than twice the first number. Find the two numbers.
b. The sum of the digits of a two-digit number is ten. The tens digit is four less than the ones digit. Find the number.

Solution

a. Let \( x \) = the first number.
   Let \( y \) = the second number.
   \[ x + y = 98 \]
   \[ y = 2x - 1 \]
   \[ x + 2x - 1 = 98 \]
   \[ 3x = 99 \]
   \[ x = 33 \]
   \[ y = 2 \times 33 - 1 \]
   \[ y = 65 \]
   The two numbers are 33 and 65.

b. Let \( x \) = the tens digit of the number.
   Let \( y \) = the ones digit of the number.
   \[ x + y = 10 \]
   \[ y - 4 + y = 10 \]
   \[ 2y = 14 \]
   \[ y = 7 \]
   \[ x = 7 - 4 \]
   \[ x = 3 \]
   The number is 37.

1. For each exercise, create a system of equations and solve it by substitution to answer the question.
   a. The sum of two integers is 116. The second number is three times the first number. Find the numbers.

   b. The sum of two integers is 64. When the smaller number is subtracted from the larger number, the difference is 26. Find the two numbers.

   c. Three times a smaller number plus four times a larger number is 233. Find the two numbers if twice the smaller number minus the larger number is 16.

2. The sum of two numbers is –6. The smaller number is 24 less than the larger number. Find the two numbers.
3. The difference between two numbers is 9.
   Twice the larger number subtracted from the smaller number is 7. Find the two numbers.

4. The difference between two numbers is 16. If the smaller number is one-third of the larger number then find the two numbers.

5. The difference between two numbers is 52.
   The sum of one-fourth the larger number and twice the smaller number is 85. Find the two numbers.

6. The sum of the digits of a two-digit number is 10.
   The sum of twice the tens digit and three times the ones digit is 23. Find the number.

7. The ones digit of a two-digit number is 4 less than the tens digit.
   The sum of three times the tens digit and four times the ones digit is 47. Find the number.

8. The sum of the digits of a two-digit number is 13.
   The ones digit is one more than twice the tens digit. Find the number.

9. The tens digit of a two-digit number is five less than the ones digit.
   The sum of four times the tens digit and one-third the ones digit is 19. Find the number.

10. The ones digit of a two-digit number is one less than the tens digit.
    One half the sum of the ones digit and double the tens digit is 13. Find the number.

11. The ones digit of a two-digit number is four more than the tens digit. If the ones digit and the tens digit are interchanged, the value of the new number is 36 more than the original number. Find the possible original number(s).
Systems of Equations
— Percent Mix Problems

California Standards: 9.0, 15.0

Example

A pharmacist has a 15% acid solution and a 25% acid solution. A prescription calls for 50 milliliters of 21% acid solution. How much of each solution should the pharmacist mix to obtain the 21% acid prescription?

Solution

Let \( x \) = milliliters of 15% acid solution
Let \( y \) = milliliters of 25% acid solution

\[ x + y = 50 \] \hspace{1cm} \text{(Total volume of 15% solution + total volume of 25% solution = 50 ml)}

\[ 0.15x + 0.25y = 0.21(50) \] \hspace{1cm} \text{(15% times } x \text{) plus (25% times } y \text{) = 21% times 50 ml)}

\[ x = 50 - y \]

\[ 0.15(50 - y) + 0.25y = 10.5 \]
\[ 7.5 - 0.15y + 0.25y = 10.5 \]
\[ 0.10y = 3 \]
\[ y = 30 \Rightarrow x = 50 - 30 = 20 \]

The pharmacist should mix \( 20 \text{ ml of 15\% solution} \) with \( 30 \text{ ml of 25\% solution} \).

1. Anne has two solutions that contain 20% citric acid and 45% citric acid. How many liters of each solution must she mix to produce 20 liters of a 30% citric acid solution?

2. A manufacturing company of syrup produces light syrup with 5% molasses and regular syrup with 10% molasses. How many ounces of each syrup must be mixed to obtain a 25-ounce syrup containing 8% molasses?

3. A gardener has two sorts of plant food containing 4% nitrogen and 10% nitrogen. How many pounds of each plant food are needed to create 12.5 pounds of 8% nitrogen plant food?
4. Two makes of fabric softener contain 14% scented ingredients and 24% scented ingredients. A 50 ounce bottle of fabric softener with 20% scented ingredients is created from the two makes. How many ounces of each make of fabric softener are needed to create the 50 ounce bottle?

5. A record label produced 40 CDs in two years, 25% of which were hip-hop. In the first year 10% of the CDs were hip-hop and in the second year 30% of the CDs were hip-hop. Find the number of hip-hop CDs the company produced in each year.

6. Two different face creams cost $0.40 per ounce and $0.90 per ounce to produce. A chemist creates a 12-ounce bottle of face cream costing $0.60 per ounce. How many ounces of each face cream were used to create the 12-ounce bottle?

7. Sugar on assembly line A costs $0.20 per pound and sugar on assembly line B costs $0.45 per pound. How many pounds of sugar from each assembly line needs to be mixed to produce 50 pounds of sugar costing $0.30 per pound?

8. Sandy invested her savings of $4500 in two different accounts one year ago. One account had an interest rate of 5% per year and the other had a rate of 7% per year. The total return from the investments was $280. How much money was invested at each rate?

9. Omar invested $2600 in two different companies one year ago. The first company gave a return of 3% that year and the other gave a return of 4% that year. The total return from the investments was $90. How much money was invested in each company?

10. Lilly’s piggy bank contains only quarters and dimes and totals $8.75. If the total number of coins is 53 then how many quarters are in Lilly’s bank?
Topic 5.3.4 Systems of Equations — Rate Problems

California Standards: 9.0, 15.0

Example

A boat takes 5 hours to travel 180 miles upriver and 4 hours to travel 180 miles back downriver. Determine the speed of the boat in still water and the speed of the water current.

Solution

Let \( x \) = the boat's speed in still water
Let \( y \) = water current speed

Upriver:

\[
\text{Speed of boat} = x - y
\]

\[
180 = (x - y) \times 5
\]

\[
36 = x - y \quad (1)
\]

Downriver:

\[
\text{Speed of boat} = x + y
\]

\[
180 = (x + y) \times 4
\]

\[
45 = x + y \quad (2)
\]

The system of equations to solve is \( 36 = x - y \) \( (1) \) and \( 45 = x + y \) \( (2) \)

\[
\begin{align*}
36 &= x - y \\
+ 45 &= x + y \\
81 &= 2x \\
\Rightarrow x &= 40.5 \\
36 &= x - y \\
\Rightarrow 36 &= 40.5 - y \\
\Rightarrow y &= 4.5
\end{align*}
\]

The speed of the boat is 40.5 mph and the speed of the water current is 4.5 mph.

1. A boat can travel 222 miles in 6 hours upriver.
   It takes the same boat 4 hours to travel downriver the same 222 miles.
   Determine the speed of the boat in still water and the speed of the water current.

2. An airplane takes off from airfield A and flies, directly into the wind, to airfield Z.
   The flight takes 60 minutes and is a distance of 80 miles. If the return flight to airfield A
   takes 48 minutes, find the speed of the airplane in no wind and the wind speed.
   (Assume the wind speed and direction are constant throughout.)
3. An airplane travels 400 miles in 2 hours with the wind. It takes the same airplane 3.2 hours to travel the same 400 miles against the wind. Determine the airplane’s speed in still air and the wind speed.

4. A crew of eight can row five kilometers downstream in 20 minutes. The same crew of eight can row the same river section upstream in one hour. Determine the boat’s speed in still water and the speed of the water current.

5. A wave runner can travel 90 miles in 2 hours with the waves and in 3 hours against the waves. Determine the speed of the wave runner in still water.

6. Joanne and Karen run the 5000 feet from their house to the park. Karen sets off first, but runs 2.5 feet per minute slower than Joanne. The girls both reach the park at the same time. If Karen takes 15 minutes to run to the park, how long does Joanne take?

7. Drew and Kirk are meeting at the mall at 12:30. Drew catches the 12:02 bus for the 7 mile journey to the mall. Kirk walks the 0.6 miles from his house to the mall and travels 10 times slower than Drew. What time should Kirk have set off to arrive at the mall on time?

8. It is 74.4 miles from Dwane and Patti’s house to Martha’s house. Patti leaves home at 10:00 and drives toward Martha’s house. Dwane and Martha set off half an hour later from their own houses and drive toward the other’s house. The average speed of the three friends is 54 mph and they all meet at 11:06. How fast does each drive?
Polynomials

California Standards: 10.0

Example

Simplify the following polynomial and state whether the answer is a monomial, binomial, or trinomial.

\[ 4x + 2x^3 - 5 + 3x^2 - 2x + 3 \]

Solution

Combine all like terms to simplify:

\[ 5x^2 + 2x - 2 \]
\[ 2x^3 + 3x^2 = 5x^2, \quad 4x - 2x = 2x, \quad -5 + 3 = -2 \]
The polynomial has three terms and so is a trinomial.

1. State whether each of the following is a monomial, binomial, trinomial, or none of the three.

   a. \( 4x + 7 \)
   b. \( 7x^4 + 3x^3 - 2x^2 + 3x - 1 \)
   c. \( 5 \)
   d. \( 5x^5 - 3x^2 + 7x \)
   e. \( 0 \)
   f. \( 3x^4 + 2x^2 - 6 \)
   g. \( 7x^7 - 3x^3 \)
   h. \( 234 \)

2. When simplifying \( 4x + 3 + 2x \) the result is \( 6x + 3 \). Where did the 6 in the simplified form come from?

3. When simplifying \( 3x^3 - 5x + 7 - 5x^2 - 4 \) the result is \( -2x^2 - 5x + 3 \). Where did the 3 in the simplified form come from?

4. Simplify the following polynomials.

   a. \( 7x + 5x \)
   b. \( 3x + 7 - 5x \)
   c. \( 2x^2 + 5x + 3x^2 \)
   d. \( 5x^3 + 3x^2 - 2x \)
   e. \( 12x^4 - x^2 + 2x^4 + x^2 - x \)
   f. \( 9x + \frac{2}{7}x^2 - 3 + \frac{4}{7}x^2 \)
   g. \( x^5 + \frac{3}{10} - \frac{5}{8}x^5 \)
   h. \( x^{11} + \frac{1}{4}x^2 - \frac{6}{8}x^2 + 2 \)
5. Simplify the following polynomials.
   a. \( \frac{1}{2}x^2 + \frac{1}{2}x^2 - 5x + 1 \) .................................................................
   b. \( 7 + \frac{2}{3}x^4 + \frac{4}{3}x^4 - \frac{7}{3}x^4 + x^3 \) .................................................................
   c. \( \frac{2}{5}x + 4 - 2x^3 + \frac{1}{5}x - 7 \) .................................................................
   d. \( \frac{2}{3}x + \frac{3}{4}y - \frac{5}{6}x + \frac{5}{8}y \) .................................................................
   e. \( \frac{1}{2}x^2 - \frac{3}{5}x + \frac{3}{4}x^2 + 4 - \frac{7}{10}x \) .................................................................
   f. \( 3 + 5x^2 + \frac{1}{4}x - \frac{2}{6}x - \frac{1}{8}x^2 \) .................................................................

6. State the degree of each of the following polynomials.
   a. \( 4x^5 + 3x^3 - x \) .................................................................
   b. \( 7y^3 - 5y^4 + 2y^2 - 3y + 5 \) .................................................................
   c. \( w^6 - 5 + 3w^8 + 2w^5 - w^4 \) .................................................................

7. Simplify each of the following polynomials and state the degree of your answer.
   a. \( 5x^4 + 3x^2 - 6x^3 + 4x + 5 - 5x^4 + 4x^3 - 3x^2 \) .................................................................
   b. \( 7y^2 + 3y + 5 - 3y^3 - 7y^2 - 3y + 3 + 3y^3 \) .................................................................
   c. \( 2x^2 - 3 + 3x^2 - 5x + 4 + (-5x^2) + 3x - 7 \) .................................................................

8. Simplify \( 3y^3 + 2y - 5 - 2y^3 + 3y^2 + 8 - y^3 - 3y^2 - 2y + 4 \) and state the degree of your answer.
   .................................................................

9. What type of polynomial is formed by adding a second degree binomial to a fourth degree monomial? State the degree of this polynomial.
   .................................................................

10. Solve the equation \( 3x^2 - 4x - 5 - 3x^2 + 5x - 7 = 20 \).
    .................................................................

11. Solve the equation \( 4x^2 + 3x - 5 - 3x^2 - 3x + 5 = 36 \).
    .................................................................
**Adding Polynomials**

**California Standards:** 2.0, 10.0

1. Find the opposite of the following polynomials:
   a. $4w + 8$
   b. $3x^2 - 5x + 7$
   c. $-4y^2 + 3y - 7$
   d. $-4p - 3 + 2p^2$
   e. $\frac{1}{2}x^2 - 4y^2 + 3z^2$
   f. $w^4 - \frac{2}{3}w^2 + 6w - (-2)$

2. John is asked to write the opposite of $3x + 5$. He writes $-3x + 5$. Explain what John has done wrong.

---

**Example**

Find the sum of $8x^3 - 7x^2 + 5x - 3$, $-5x^3 + 3x^2 - 3x - 2$, and $-x^3 + 2x^2 - x + 1$ by either the vertical method or by the distributive property method.

**Solution**

**Vertical method:**

\[
\begin{array}{ccc}
8x^3 - 7x^2 + 5x - 3 \\
-5x^3 + 3x^2 - 3x - 2 \\
-x^3 + 2x^2 - x + 1 \\
\hline
2x^3 - 2x^2 + x - 4
\end{array}
\]

Line up like terms

Combine like terms

**Distributive property method:**

\[
(8x^3 - 7x^2 + 5x - 3) + (-5x^3 + 3x^2 - 3x - 2) + (-x^3 + 2x^2 - x + 1)
\]

\[
= 8x^3 - 7x^2 + 5x - 3 - 5x^3 + 3x^2 - 3x - 2 - x^3 + 2x^2 - x + 1
\]

\[
= (8x^3 - 5x^3 - x^3) + (-7x^2 + 3x^2 + 2x^2) + (5x - 3x - x) + (-3 - 2 + 1)
\]

\[
= 2x^3 - 2x^2 + x - 4
\]

Collect together like terms

Simplify

3. Simplify the expressions in exercises a. – d.
   a. $(-3x^2 - 2x + 1) + (5x^2 + 3x - 2)$
   b. $(3k^3 - 7k^2 + 5) + (-3k^3 + 5k^2 - 3)$
   c. $(4t^3 - 2t + 3) + (2t^3 - 9t^2 - 3t + 2)$
   d. $(-3y^3 - y^3 + 2y^2 - 8y) + (-y^4 + y^3 - 6y^2 + 7y - 2)$
4. Find the sum of the following polynomials: \(3x^3 + 5x - 2x^2, 4x - x^3 + 3, 4x^2 - 6\).

5. Find the sum of the following polynomials: \(6 + 3y^4 - 2y^2 + 2y, 2y^2 + 3 - 3y^4 - 5y, 3y - 9\).

6. Find the sum of the following polynomials. State the degree of your answer.
   a. \(3y^2 + 4 + 2y, 6 - 2y^2 + 5y^3 - 6y, 5y - 4y^2 - 4, 3y^2 - 6\)
   b. \(4x + 7 + 3x^3, 2x^2 + 7x^3 - 7x, -4x^2 + 3x^3 - 2x + 3x^2, 4x^2 - 5x^3 + 5x - 6\)
   c. \(3w^3 - 2w^2 + 5w + 6\)
      \(+ 4w^2 - 3w - 8\)
      \(-5w^3 - 2w\)
      \(+ 3w^2 + 3\)
   d. \(z^3 - 2z^2 - 2z\)
      \(+ z^2 - 8\)
      \(-2z^3 + z\)
      \(-6z^3 + 5z^2 + z - 9\)
   e. \(x^{22} - 3x^5 + 6x + 2, \frac{1}{2} x^2 - 3x^{22} + 12x - 16, 4x^5 - x^2 + 53, 2 - 5x^{22}\)
   f. \(0.3x^{15} + 0.2x^3 + 0.6x^3, 2.1x^8 - 1.7x^4 - x - 12, 0.5x^{15} - 0.9x^3 - 2.8, 0.2x - 0.4x^2 - 14\)

7. Find the perimeter of the triangle in its simplest form. State the degree of your answer.

8. If a fourth degree polynomial is added to a third degree polynomial, what is the degree of the answer?

9. Multiply each of the following polynomials by the given number.
   a. \(6x^2 + 5x - 4\) by 5
   b. \(6y^5 - 4y^3 + 3y^2 + 2\) by \(-2\)
   c. \(-4y^4 + 3y^2 - 2\) by 6
   d. \(3x^2 + 2x + 4\) by 3
   e. \(4w^3 - 3w^2 - 1\) by \(-3\)
   f. \(-5y^3 + 2y^2 - 6y + 4\) by \(-2\)

10. Multiply each of the following polynomials by \(3t\).
    a. \(2t^2 + 5t + 1\)
    b. \(3t^2 - t^4 + t^2 - 7\)
Subtracting Polynomials

Example

a. Subtract $-5x^2 + 6x - 2$ from $-7x^2 + 5x - 3$.
b. Subtract $-5x^2 - 7x - 8$ from $-x^2 - 10x - 12$.

Solution

a. $( -7x^2 + 5x - 3 ) - ( -5x^2 + 6x - 2 )$
   $= -7x^2 + 5x - 3 + 5x^2 - 6x + 2$
   $= ( -7x^2 + 5x^2 ) + ( 5x - 6x ) + ( -3 + 2 )$
   $= -2x^2 - x - 1$
   \[ \text{Collect together like terms} \]
   \[ \text{Simplify} \]

b. $( -x^2 - 10x - 12 ) - ( -5x^2 - 7x - 8 )$
   $= -x^2 - 10x - 12 + 5x^2 + 7x + 8$
   $= ( -x^2 + 5x^2 ) + ( -10x + 7x ) + ( -12 + 8 )$
   \[ \text{Collect together like terms} \]
   \[ \text{Simplify} \]

1. Subtract:
   a. $x^2 + 2$ from $x^2 + 5$
   b. $x^2 + 3$ from $x^2 - 12$
   c. $2x + 5$ from $x^2 - 8x + 2$
   d. $x^2 + 2$ from $x^3 + 3x - 1$
   e. $x^2 + x$ from $4x^3 - 2x + 14$
   f. $3x^3 - x$ from $-6x^3 + 5x - 4$

2. In her homework, Jenna writes $(3x + 4) - (2x - 3) = (3x + 4) + (-2x + 3)$. How has Jenna changed the calculation?

3. Subtract $-3h^2 - 4h - 5$ from $-2h^2 - 3h - 7$. 
4. Subtract each of the following:
   a. $5x^2 + 3x - 4$ from $3x^2 - 5x + 4$
   b. $-3y^2 - 5y + 6$ from $-4y^3 + 2y^2 + 3y - 3$

5. Simplify the expressions in exercises a. – h.
   a. $(4x^2 + 3x - 5) - (2x^2 + 5x - 3)$
   b. $(7x^2 - 9x + 5) - (6x^2 - 7x + 4)$
   c. $(2y^2 - 5y + 7) - (5y^2 + y - 1)$
   d. $(-5y^2 + 3y + 8) - (2y^2 - y - 6)$
   e. $(2x^3 - 9x + 14x - 3) - (12x^4 - 12x^3 + 15x)$
   f. $(3y^4 - 5y^3 + 6) - (5y^3 + 3y^2 - 2y + 4)$
   g. $(0.2x^4 - 0.4x^3 + 0.7) - (0.8x^4 + 0.1x^3 - 2x + 1.6)$
   h. $(0.8x^5 - 1.4x^4 + 2.3x^2) - (-1.7x^6 + 1.2x^4 - 2.8x^2 + 1.9)$

6. Subtract $-m^3 + 2m^2 - m - 1$ from $m^3 - m^2 + 5m + 3$.

7. John left his house and walked northward $(3x^2 + 4x - 7)$ feet.
   He then turned around and walked southward $(6x + 2)$ feet.
   How far from home was he at the end of his walk?

8. Mr. Jones bought $(5y^3 + 2y - 3)$ tons of hay and fed $(4y^2 - 3y + 2)$ tons to his cattle.
   How much hay did he have left after feeding his cattle?

9. When a fourth degree polynomial is subtracted from a third degree polynomial,
   what degree of polynomial will be produced?

10. Subtract the polynomials and simplify the resulting expression.
    a. $(2x^2 - 5x + 2) - (4x^3 - 9x^2 + 2x) - (-x^2 + 4x - 6)$
    b. $(-5y^4 + y^3 - 6y) - (7y^3 + y + 4) - (6y^4 + 4y^3 - 6y + 9)$
    c. $(-0.5z^6 + z^5 - 6z^2) - (4z^4 + 0.8z^2 + 1.8) - (2z^3 + 4z^2 - 6z^3)$
Adding and Subtracting Polynomials

California Standards: 2.0, 10.0

1. Simplify each of the following:
   a. \((4x + 3) - (2x - 5)\)
   b. \((4y - 7) + (-2y + 5)\)
   c. \((3z - 4) - (-2z + 5)\)

2. Simplify:
   a. \(4x^2 + 3x - 5 + (3x - 2x^2 - 3)\)
   b. \(5x^2 - 7x + 2 - (3x^2 - 5x - 1)\)
   c. \(5x + 3x^2 + 4 - (6 + 5x + x^3)\)

3. Simplify:
   a. \(3k(k - 4) - 2k(k - 3)\)
   b. \(2(3x + 7) + 3(4x - 5) - 4(-2x + 3)\)
   c. \(3(2x^2 + 3x - 5) - (-4)(2x + 7)\)

4. Simplify \(-2(-x^2 - 3x + 5) + (x^2 - x - 7)\)

Example

The surface area of sphere A = \((2x^3 + 3x^2 - 4x - 5)\) sq in.
The surface area of sphere B = \((3x^3 - 3x^2 - 6x + 4)\) sq in.
a. Find the sum of the areas.
b. Sphere B is larger than sphere A. Find the difference between the surface areas of the two spheres.

Solution
a. \((2x^3 + 3x^2 - 4x - 5) + (3x^3 - 3x^2 - 6x + 4)\)
   \[= 2x^3 + 3x^2 - 4x - 5 + 3x^3 - 3x^2 - 6x + 4\]
   \[= 5x^3 - 10x - 1\]
   Collect together like terms

b. \((3x^3 - 3x^2 - 6x + 4) - (2x^3 + 3x^2 - 4x - 5)\)
   \[= 3x^3 - 3x^2 - 6x + 4 - 2x^3 - 3x^2 + 4x + 5\]
   \[= 3x^3 - 2x^3 - 3x^2 - 3x^2 - 6x + 4x + 4 + 5\]
   \[= x^3 - 6x^2 - 2x + 9\]
   Sphere B is larger than sphere A, so subtract A from B

Like addition, collect together like terms
5. Find the difference between the circumferences of the larger circle and the smaller circle.

\[ C = 2\pi r \]

6. Find the difference between the areas of triangle A and triangle B.

\[ \text{Area} = \frac{1}{2} bh \]

7. Simplify \( 5[(m - 2b) - (2m + 3b) - 1] - [3(m + b) - 2(m - 3b) - 2] \).

8. Simplify \( \frac{5(x - 1)}{6} - \frac{2(3x - 4)}{4} + \frac{2x}{3} \).

9. Jason writes down two polynomials, \( A = (3x^2 + 2x + 4) \) and \( B = (5x^2 - 2x + 3) \). Subtract the difference between the two polynomials \( (A - B) \) from the sum of the two polynomials.

10. The perimeter of the rectangle shown below is 104 inches.

\[ 2(x - 1) \text{ in.} \]
\[ 4(x + 3) \text{ in.} \]

a. Find the value of \( x \).

b. Find the area of the rectangle.
Rule of Exponents

Example

Find the product $6x^3y^4 \times 9x^2y^5$ and the quotient $6x^3y^4 \div 9x^2y^5$.

**Solution**

**Product**

Put all the same variables together and add the powers.

$(6x^3y^4)(9x^2y^5) = (6)(9)(x^3)(x^2)(y^4)(y^5) = 54x^{3+2}y^{4+5} = 54x^5y^9$

**Quotient**

Separate out into terms that only have one variable and subtract the powers.

$\frac{6x^3y^4}{9x^2y^5} = \frac{2}{3} \cdot \frac{x^3}{x^2} \cdot \frac{y^4}{y^5} = \frac{2x}{3y}$

1. Simplify the following expressions:
   a. $(3x^3y^2z^3)(4x^2y^3z^4)$
   b. $(5a^2b^3c)(3a^3b^2c^2)$
   c. $(2r^3s^2)(3r^4t^6)$
   d. $(5x^2y^3z^4)(3x^3y^2)$
   e. $(-3x^3k)(-x^2k^4)$
   f. $(-2a^3b^4c^2)(-4a^3b^2)$

2. Simplify the following expressions:
   a. $\frac{3x^3y^4}{x^2y^2}$
   b. $\frac{12a^5b^4c^4}{4a^3bc^2}$
   c. $\frac{5s^4t^5}{15r^3s^5t^3}$
   d. $\frac{6x^4y^5z^6}{3x^3y^3z^3}$

3. Expand and simplify each expression:
   a. $(6r^6s^3t^2)^2$
   b. $(-2a^2xb^3)^3$
   c. $(4x^2y^3z^4)^3$
   d. $(-3a^2b^6c^3)^4$
4. Simplify each of the following:
   a. \((-4ab^2)(3a^2b^3)\)  
   b. \(3a^3b^4(4ab^2 + 3a^2b - 2a^2b^3)\)  
   c. \(2a(3a^2 + 4) + 3a^2(3a + 5) - 4(5a + 6)\)

5. Simplify \(\left(\frac{x^2}{3}\right)^3\)

6. Karen simplified the expression \(\left(\frac{2x^3}{3y^4}\right)^{-1}\) to get the answer \(\frac{3y^4}{2x^3}\). Explain how she reached this answer.

7. Simplify the following expressions.
   a. \((4x^2y^{-3}z^4)^0\)  
   b. \((3a^2b^4)^{-2}\)  
   c. \(\left(\frac{4x^3y^2}{12x^6y^3}\right)^{-1}\)  
   d. \(\left(\frac{3x^2y^3z^4}{9x^5yz^2}\right)^{-3}\)  
   e. \(\left(\frac{7x^4yz}{21x^5y^9z^3}\right)^{-2}\)  
   f. \(\left(\frac{20x^6y^3w^{23}}{6u^9v^3w^{14}}\right)^2\)  
   g. \((-2a^2b^3c^4)^{-4}\)  
   h. \((-4a^2a^3b^5c^6)^{-3}\)  
   i. \(\left(\frac{3r^2s^{-1}t^{11}}{r^{-3}s^{-2}}\right)^3\)  
   j. \(w\left(\frac{-16u^{15}v^{-2}w^7}{u^3v^3}\right)^{-1}\)

8. Find the value of \(z\) for each of the following statements.
   a. \((w^6x^7y^4)^2 = w^{24}x^{14}y^8\)  
   b. \((p^8q^2r^4)^2 = p^{16}q^4\)  
   c. \((r^4s^{-1}t^2)^2 = r^{-8}s^{-2}t^4\)

9. If \((x^2y)^3 = x^6y^{12}\), what is the value of \(r\)?
### Example

Simplify \((3x - 5)(2x + 3)\).

**Solution**

Multiply out the parentheses:  

\[
\begin{align*}
(3x - 5)(2x + 3) & = 6x^2 + 9x - 10x - 15 \\
& = 6x^2 - x - 15
\end{align*}
\]

#### Distributive method:

\[
\begin{align*}
(3x - 5)(2x + 3) & = 3x(2x + 3) - 5(2x + 3) \\
& = 6x^2 + 9x - 15x - 15 \\
& = 6x^2 - 6x - 15
\end{align*}
\]

#### Stacking method:

\[
\begin{align*}
\begin{array}{c}
3x - 5 \\
\times & \phantom{2x + 3} \\
\hline
9x - 15 & \phantom{2x + 3} \\
+ 6x^2 - 10x & \phantom{2x + 3} \\
\hline
6x^2 - x - 15 & \phantom{2x + 3}
\end{array}
\end{align*}
\]

1. Use the stacking method to multiply the following polynomials.
   
   a. \(2x(5x + 2)\) 
   
   b. \(6x(8x + 2)\)
   
   c. \((4x + 6)(x + 3)\) 
   
   d. \((2x + 2)(2x - 2)\)

2. Expand and simplify each expression using the distributive method:
   
   a. \((5x + 3)(2x - 7)\)
   
   b. \((5x - k)(2x + k)\)
   
   c. \((3x - 1)(2x - 5)\)
   
   d. \((8x - 3)(2x - 6)\)
   
   e. \((5a - 6)(-2a + 3)\)
   
   f. \((0.5a + 2)(4a - 6)\)
   
   g. \((3a^2 - 2)(4a - 5)\)
   
   h. \((2x^2 - 4x)(9x - 2)\)
3. Expand and simplify each product below using the distributive method.
   a. \((4kt + 3)(2k - t)\)  
   b. \((ab + c)(bd + ac)\)  
   c. \((5kp - 3k)(2k - 7p)\)  
   d. \((6xy - y)(0.5y + x)\)  

4. Rico says “If two binomials are multiplied, the answer will always be a trinomial.” Is he right? Give an example to justify your answer.  

5. Expand and simplify each product below using the distributive method.
   a. \((2m + 5)(m^2 - 5m - 1)\)  
   b. \((2y^2 - 3y + 1)(3y + 2)\)  
   c. \((-5x^2 + 2x - 3)(7x + 1)\)  
   d. \((0.25x - 6)(4x^2 + 12x + 8)\)  
   e. \((-3t^2 + t - 4)(-t^2 + 5t + 2)\)  
   f. \((x^3 + x - 1)(x^2 + 2x - 1)\)  

6. Expand and simplify the product \((x^2 + 3x + 5)(2x^2 - 3x - 4)\) using the stacking method. State the degree of your answer.  

7. What degree of polynomial will be produced by multiplying a third degree polynomial and a fourth degree polynomial?  

8. Simplify \((y - 3)^2\).  

   Multiply the first two binomials, ignoring the third. Once simplified, multiply in the third binomial.  

9. Expand and simplify \((2y - 1)^2(3y + 2)^2\).
Example

Find the area, A, of this rectangle:

\[ (4x + 5) \text{ inches} \times (3x - 2) \text{ inches} \]

Solution

Since \( A = \text{length} \times \text{width} \),

\[ A = (4x + 5)(3x - 2) \]

\[ = 4x(3x - 2) + 5(3x - 2) \]

\[ = 12x^2 - 8x + 15x - 10 \]

\[ = (12x^2 + 7x - 10) \text{ in}^2 \]

---

1. Find the area of this rectangle:

\[ (3x + 7) \text{ cm} \times (2x + 3) \text{ cm} \]

2. Find the area of this triangle:

\[ \frac{1}{2} \times (4x - 2) \text{ inches} \times (2x + 5) \text{ inches} \]

3. Find the circumference and area of a circle whose radius is \((2x - 1)\) cm.

4. Find the area of this square:

\[ (3x^2 + 2x - 4) \text{ cm} \times (3x^2 + 2x - 4) \text{ cm} \]
5. Penelope goes to the zoo to see the penguins. She walks round the penguin enclosure on a walkway that is 5.5 feet wide. If the enclosure is three times as long as it is wide, find the combined area of the enclosure and walkway.

6. Frank has a rectangular garden which measures \((2x + 3) \text{ m} \times 5 \text{ m}\). He knows the area is 55 sq m. Find \(x\).

7. A cube has an edge \((e)\) of length \((2x - 3) \text{ inches}\). Find its volume \((V)\) in terms of \(x\).

8. The shipping container shown has a square bottom with sides of length \(b = (4k - 3) \text{ inches}\). Find the volume \((V)\) of the container if its height \(h = (3k + 1) \text{ inches}\).

9. Find the volume of this box.

10. Use the prism shown on the right to answer the following questions.
   a. Find the volume of this shape.

   b. If the height of the shape is reduced by 50%, what is the new volume of the shape?
Division by Monomials

1. Simplify each of the following quotients:
   a. \((6x^3y^2z^5) \div (2xy^2z^2)\) 
   b. \((12a^3b^2c^3d^4) \div (3a^2b^2d)\) 
   c. \((8r^4s^{-3}t^6) \div (4r^3s^2t^4)\) 
   d. \((32x^3y^2z^2) \div (4x^3y^2z)\) 
   e. \((25a^{12}b^4c^8) \div (5a^6bc^8)\) 
   f. \((24x^3y^2z^4) \div (6x^3y^2z^9)\) 

2. When \(\frac{x^5}{x^3}\) is simplified, the result is \(x^2\). Explain where this simplification came from.

3. When \(\frac{x^3}{x^3}\) is simplified the result is 1. Explain where this simplification came from.

Example

James draws a rectangle with a width of \(x\), and an area \(A = (x^2 + 3x)\) in\(^2\). Find the length of the rectangle.

Solution

Since \(A = \text{length} \times \text{width}\),

\[
\text{length} = \frac{A}{\text{width}} = \frac{x^2 + 3x}{x} = x + 3(x^{-1}) \quad \text{Subtract the exponents} \\
= x + 3(1) \quad x^0 = 1 \\
= (x + 3) \text{ in.}
\]
4. Simplify each of these quotients:
   a. \[ \frac{10x^3 + 15x^2}{5x^2} \] 
   b. \[ \frac{10a^4b^5 - 6a^6b^3 + 9a^2b^3}{3a^3b^3} \] 
   c. \[ \frac{12s^{14}t^{27} - 24s^{18}t^{19}}{6s^6t^{12}} \] 
   d. \[ \frac{12u^2v^8 - 16u^6v^3 + 4u^4v^4}{4u^8v^2} \]

5. When \( \frac{x^2}{x} \) is simplified, the result is \( x \). Explain where this simplification came from.

6. Roger has a rectangle whose area is \( 3x^2 + 9x \) and is \( 3x \) in length. What is the width of the rectangle in terms of \( x \)?

7. Jane has a triangle with an area of \( 8x^3 + 6x^2 + 10x \) and a height of \( 4x \). What is the length of the base in terms of \( x \)?

8. Divide \( 5a^4b^3c - 5a^3b^2c^2 + 10a^2b^3c \) by \( 5a^2b^3c \).

9. Divide \( 9r^6s^4t^6 + 6r^4s^7t^3 - 12r^2s^4t^6 \) by \( 3r^2s^4t^3 \).

10. Fran has a playhouse that has a parallelogram shaped floor with area \( 14x^3 + 21x^4 + 7x^3 \), which is \( 7x^2 \) wide. Find the length of the floor.

11. Wanda wishes to carpet her rectangular floor. She knows the floor has a total area of \( (25x^4 + 15x^3) \) sq ft and that one side of the floor is \( 5x^2 \) ft, where \( x = 2 \).
   
   a. What are the dimensions (the length and the width) of the room?
   
   b. How much must Wanda spend to cover her floor if the carpet costs $3.00 per sq ft?
Polynomials and Negative Powers

California Standards: 2.0, 10.0

1. Determine the multiplicative inverse of each of the following:
   a. \( rs^2 \)  
   b. \( 2a + 5b \)  
   c. \( \frac{1}{xy} \)  
   d. \( \frac{1}{3a^2e^3} \)

Example

Simplify \((2 - x)^{-2}(2 - x)\).

Solution

\[
\begin{align*}
\text{Express both brackets as fractions} & \quad \text{—} \\
(2 - x)^{-2} & \text{is the same as} \quad \frac{1}{(2 - x)^2} \\
\text{Cancel out top and bottom} & \quad \frac{1}{2 - x}
\end{align*}
\]

2. Write each of the following in simplest form without using negative exponents.
   a. \( r^2s^3 \)  
   b. \( (3x^2y^3)^{-2} \)  
   c. \( (4x^3y^4)(2xy^{-3})^{-3} \)  
   d. \( 5x^2y^2(5x^3y^3)^{-2} \)  
   e. \( (a^2b^3)^{-4}(4a^4b^2)^3 \)  
   f. \( 9^{-2}(12x^2y^{-4})(4xy)^{-1} \)  
   g. \( 12x^4(6x^2y^3)(3xy)^{-2} \)
3. What is meant by the multiplicative inverse?

4. Expand and simplify each of the following:
   a. $(x - 1)^{-2}(x - 1)$
   b. $(x + y)^{3}(x + y)^{-1}$
   c. $(a + b)^{-3}(a + b)$
   d. $(3x - y)^{-2}(3x - y)^{3}$
   e. $(-5x - y)^{-3}(-5x - y)^{2}$
   f. $(6a + 2b)^{-1}(6a - 2b)^{2}$
   g. $(7y^2x - 3xy)^{-23}(7y^2x - 3xy)^{24}$

5. Simplify the following expressions:
   a. $6rs(2r^2s^3)^{-1}(3r)^{-1}$
   b. $5x^2y(5xy)^{-1}(3x)^{-1}$
   c. $16u^2v^3(u^2v)^{-1}(4uv)^{-2}$
   d. $7xy(2xy)^{-4}(xy)^{3}(4xy)^2$
   e. $(2r^2s^4)(4r^3s^2t)^{-1}(2rst)^{-3}(rst)^2$
   f. $(6x^2y^2z^3)^2(2x^2y^3z^4)^{-1}(4x^3y^2z^5)(6x^1y^2z^4)^{-1}$
   g. $(2fg^2)^3(fg^{-2}h^3)^{-1}(3f^2g^4h^2)^{-2}(6f^2gh)^2$
   h. $(2u^3v^6w^4)(2u^4v^6w^{-2})^{-2}(2u^2v^2w)^{-3}(4u^1v^4w^{-3})^2$

6. Franklin draws a rectangle with an area of $(2xy^{-1}z^3)^3$ in$^2$. The length of the rectangle is $(4x^{-2}yz)^2$ in.
   Find the width of the rectangle.
Topic 6.3.3  Division by Polynomials — Factoring

California Standards:  10.0

1. Simplify each of the following expressions.
   
   a. \( \frac{(x+5)(x+7)}{x+5} \)

   b. \( \frac{x+2}{(x+2)(x-2)} \)

   c. \( \frac{(x+3)(2x-5)(3x+1)}{(x+3)(3x+1)} \)

   d. \( \frac{5x+3}{2x-5} \)

   e. \( \frac{6x+2}{3x+9}(3x+9)(6x+2)(x-5) \)

   f. \( \frac{3x-y}{(2x-y)(4x-2y)} \)

2. Greg writes down the simplification \( \frac{x+3}{x+6} = \frac{x+1}{x+2} \) in his homework. What has he done wrong?

Example

Simplify \((x^3 + 3x^2) ÷ (4x + 12)\).

Solution

\[
\frac{x^3 + 3x^2}{4x + 12} = \frac{x^2(x+3)}{4(x+3)}
\]

Factor the numerator and denominator

Cancel \(x + 3\) from top and bottom

3. Simplify each expression.

   a. \( \frac{4x + 4}{x+1} \)

   b. \( \frac{8x+16}{x+2} \)

   c. \( \frac{2x^2 + 4x}{2x} \)

   d. \( \frac{6x^2 - 9x}{2x - 3} \)

   e. \( \frac{x^2 + x^3}{1 + x} \)

   f. \( \frac{2x + x^2}{xy + 2y} \)
4. Simplify each of the following expressions.
   a. \( \frac{(x+1)(x-3)}{x+4} \div \frac{x+1}{x+4} \)
   b. \( \frac{x^2(2x+5)(4-x)}{3x+2} \div \frac{x(2x+5)}{3x+2} \)

5. Simplify each of the following expressions.
   a. \( \frac{2x^3(3x-6)(5-x)}{7-x^2} \div \frac{x^2(3x-6)}{7-x^2} \)
   b. \( \frac{2x^4(-3x+1)(-2x-3)}{2(-2x-1)} \div \frac{x^4(-2x-3)}{-2x-1} \)

6. Jan has a rectangular garden whose area is \( 4x - 4 \) sq ft and length is \( (x - 1) \) ft.

   Find the width of her garden.

7. The volume of a box is \( 3x^3 + 6x^2 \), one side measures \( x \) and another side measures \( x + 2 \).
   Find the size of the third side.

8. Find the ratio of the area to perimeter of the rectangle below.
Example

Find the quotient: \((x - 3) \div 3x^2 - 7x - 6\)

Solution

3x multiplied by \((x - 3)\) gives you \((3x^2 - 9x)\)
Subtract \(3x^2 - 9x\) from \(3x^2 - 7x - 6\)
Subtract \(2x - 6\) from \(2x - 6\)

1. Calculate: \(x + 1 \div 2x^2 - x - 3\).

2. Divide using the long division method.
   a. \(x - 1 \div x^2 + 2x - 3\)
   b. \(x + 2 \div 2x^2 + 7x + 6\)
   c. \(x + 1 \div 3x^2 + 8x + 5\)

3. Calculate: \(2x - 5 \div 2x^2 - 7x + 5\).

4. Calculate the following using the long division method.
   a. \(2x - 1 \div 4x^2 + 4x - 3\)
   b. \(3x + 1 \div 6x^2 - x - 1\)
   c. \(3x + 4 \div 9x^2 + 9x - 4\)
5. Calculate the following using the long division method:
   a. $2x + 5 \overline{6x^2 + 19x + 12}$
   b. $6x + 4 \overline{12x^2 - 22x - 16}$
   c. $2x - 3 \overline{14x^2 - 23x + 3}$
   d. $4x - 2 \overline{16x^2 - 16x - 6}$

6. Calculate: $2x - 1 \overline{12x^3 - 4x^2 - 5x + 2}$

7. Simplify each quotient by dividing using the long division method.
   a. $\frac{4x^2 + 4x - 3}{2x - 1}$
   b. $\frac{2x^3 + 9x^2 + 14x + 8}{x + 2}$
   c. $\frac{6x^3 - 23x^2 + 27x - 15}{3x - 7}$
   d. $\frac{24x^3 - 4x^2 + 2x + 7}{6x + 2}$

8. Find the remaining factors of the following polynomials.
   a. $(x^3 + 4x^2 + x - 6)$, given that $(x - 1)$ is a factor.
   b. $(x^3 + 2x^2 - 5x - 6)$, given that $(x + 3)$ is a factor.

9. Calculate the height of a parallelogram which is $(3x - 2)$ inches long and has an area of $(6x^2 + 11x - 10)$ sq in.

10. A trapezoid has bases $(x + 1)$ and $(2x - 3)$ and an area of $\frac{1}{2} (3x^2 - 8x + 4)$. What is the height of the trapezoid?
    (Area of a trapezoid = $\frac{1}{2} \times $ sum of bases $\times$ height)
A circular cylinder has a radius of \((x + 1)\) in. and a volume of \(\pi(2x^3 + 7x^2 + 8x + 3)\) cu in. What is the height of the cylinder?

**Solution**

\[
V = \pi r^2 h,
\]

\[
\pi \left( 2x^3 + 7x^2 + 8x + 3 \right) = \pi \left( x + 1 \right)^2 h
\]

Divide by \(\pi\) on both sides

\[
2x^3 + 7x^2 + 8x + 3 = (x^2 + 2x + 1)h
\]

Expand the terms in parentheses

\[
h = 2x^3 + 7x^2 + 8x + 3 ÷ \left( x^2 + 2x + 1 \right)
\]

\[
\begin{array}{c|ccccc}
& 2x + 3 \\
\hline
x^2 + 2x + 1 & 2x^3 + 7x^2 + 8x + 3 \\
\hline
& - (2x^3 + 4x^2 + 2x) \\
& \hline
& 3x^2 + 6x + 3 \\
& - (3x^2 + 6x + 3) \\
& \hline
& 0
\end{array}
\]

\(\Rightarrow\) Use the long division method

So the height of the cylinder must be \((2x + 3)\) in.

---

1. The length of a rectangle is \((x + 7)\) inches. Find the width of the rectangle if its area is \((x^2 + 10x + 21)\) square inches.

---

2. Find the length of the rectangle below in terms of \(y\), if the area is \((15y^2 - 4y - 3)\) square inches.

\[
(3y + 1)\text{ in}
\]

---

3. The base of a prism has an area of \((4x^2 - 5x + 3)\). If the prism volume is given by \((8x^3 - 42x^2 + 46x - 24)\), find its height.
4. The area of a triangle is given by \( A = \frac{1}{2}bh \), where \( b \) is the length of the base and \( h \) is the height of the triangle. If the area of the triangle below is \((2x^3 + x^2 - 8x + 21)\) cm\(^2\), find its height.

\[
\text{Area} = (2x^3 + x^2 - 8x + 21) \text{ cm}^2
\]

5. The area of a triangle is \( \frac{1}{2}(6x^2 + 7x + 2) \) cm.
If the base is \((2x + 1)\) cm, find its height.

6. The surface area of a sphere is given by \( A = 4\pi r^2 \).
If the surface area of a sphere is \(4\pi(4x + 12x + 9)\) sq ft, then what is its radius?

7. A box has volume \((2x^3 - x^2 - 13x - 6)\) cu in.
It has a height = \((x + 2)\) in and a width = \((x - 3)\) in.
Find the length of the box.

8. The volume of a cone is found by \( V = \frac{1}{3}\pi r^2 h \).
If the volume of a cone is \(\frac{1}{3}\pi(4x^3 + 4x^2 - 15x - 18)\) cu in,
and the height is \((x - 2)\) in, what is its radius?
**Example**

Simplify \((x - 2k)^2\).

**Solution**

\[
(x - 2k)(x - 2k) = x^2 - 2kx - 2kx + 4k^2 = x^2 - 4kx + 4k^2
\]

California Standards: 2.0, 10.0, 11.0

1. Simplify the following expressions.
   a. \((k + 1)^2\)
   b. \((3x + 2)^2\)
   c. \((5x^a - 1)^2\)
   d. \((x + \#)^2\)

2. Simplify the following expressions.
   a. \((2 - x)^2\)
   b. \((x - m)^2\)
   c. \((x^3 - 1)^2\)
   d. \((ma - 2)^2\)

3. Roger said \((2x + 3)^2 = 4x^2 + 9\). Explain why Roger is incorrect.

4. Simplify the following expressions:
   a. \((2k - 3)(2k + 3)\)
   b. \((x - 3)(x^2 + 3)\)
5. Find the area of the square in square centimeters:

\[(3x + 4)\text{ cm}\]

6. Find the area of a circle of radius \((2x + 3)\) cm. (Area of a circle = \(\pi r^2\))

7. The area of a triangle is given by \(A = \frac{1}{2}bh\), where \(b\) is the length of the base and \(h\) is the height of the triangle. Find the area of the triangle shown.

8. A square garden plot is surrounded by a walkway. Find the area of the walkway shown.
Topic 6.5.1
Factors of Monomials

California Standards: 11.0

1. List all the factors for each of the following numbers.
   a. 9
   b. 12
   c. 15
   d. 18
   e. 20
   f. 28

2. Write each of the following numbers as a product of primes.
   a. 15
   b. 24
   c. 30
   d. 36
   e. 45
   f. 50

3. Frank listed the prime factorization of 14 as $1 \times 2 \times 7$. Explain what is wrong with this listing.

4. Write each of the following monomials as the product of the smallest possible factors.
   a. 30
   b. $24x^2$
   c. $36x^3y^2$
   d. $44a^2b^2c^2$
   e. $-12$
5. Determine the Greatest Common Factor (GCF) for each of the following sets of numbers.

\begin{align*}
\text{a.} & \quad 6, 9, 15 \\
\text{b.} & \quad 8, 12, 20 \\
\text{c.} & \quad 12, 18, 30 \\
\text{d.} & \quad 15, 25, 30 \\
\text{e.} & \quad 20, 50, 80 \\
\text{f.} & \quad 30, 60, 120
\end{align*}

6. Find the Greatest Common Factor (GCF) for each of the following sets of expressions.

\begin{align*}
\text{a.} & \quad lsv \text{ and } msv \\
\text{b.} & \quad 3x^2 \text{ and } 12x \\
\text{c.} & \quad 6x^4, 12x^3, 24x^5 \\
\text{d.} & \quad 6a^2b^2c, 24a^3c^2b^2, \text{ and } 30c^2d^4b^3 \\
\text{e.} & \quad 15x^3y^5, 24x^2y^8, 30x^2y^6 \\
\text{f.} & \quad 7x^4y^2 \text{ and } 21x^3y \\
\text{g.} & \quad 16a^7b^4c^8, 32a^5b^8c^6, 40a^8b^7c^5 \\
\text{h.} & \quad 6x^7(y + 4)^5, 18x^4(y + 4)^7, 30x^3(y + 4)^3
\end{align*}

7. A rectangle has an area of 247 sq ft, and its dimensions are prime numbers. What are the dimensions of the rectangle?

8. Joe says he needs a garden plot that is 50 sq ft. List the possible dimensions in whole numbers of the garden plot.
1. Find the greatest common factor (GCF) of the terms in each of the following polynomials.
   a. $6y^2 + 9y$
   b. $12x^4 - 18x^2$
   c. $15x^3 + 12x^2 + 6x$
   d. $24x^5y^5 + 32x^4y^2$
   e. $16x^4y^2 - 12x^3y^4 + 6x$
   f. $3(x + 2)(y - 1)^4 + 21(x + 2)^3(y - 1)^2$

Example

Factor $x^2 - 2x$.

Solution

The Greatest Common Factor (GCF) of the two terms is $x$.

So $x\left(\frac{x^2 - 2x}{x}\right)$ Take out the Greatest Common Factor (GCF)

$= x(x - 2)$ Cancel out $x$ on the top and bottom

Factor $-a(x - 1) + (x - 1)^2$.

Solution

The GCF of the terms is $(x - 1)$.

So $(x - 1)\left[\frac{-a(x - 1)}{(x - 1)} + \frac{(x - 1)^2}{(x - 1)}\right]$ Take out the Greatest Common Factor (GCF)

$= (x - 1)[-a + (x - 1)]$ Cancel out $(x - 1)$ on the top and bottom

$= (x - 1)(x - a - 1)$ Rearrange terms

Check your answer by multiplying out your factors — you should get the original expression.

2. Francis said $8x + 12 = 2(4x + 6)$ and it is completely factored. Is she correct? Explain your answer.
3. Factor the following expressions.
   a. $3k^2 - 12k$  
   b. $4 - 10x$  
   c. $4x^3 + 6x^2$  
   d. $3x^3 + 12x$  
   e. $6xy^2 + 4x^2y^3$  
   f. $16x^2y^4 - 20x^3y^3$  
   g. $10x^3y^6 - 15x^4y^5$  
   h. $25x^4y^3 + 5x^5y^2$  
   i. $9x^3y^2 - 17x^4y^4$  
   j. $(y - 1) - 2(y - 1)$  
   k. $(m + 1)^2 + 3(m + 1)$  
   l. $a(r - 2) - 2b(r - 2)$  
   m. $(br - 2b) + (3r - 6)$  
   n. $y(a - 1)^2 - (a - 1)$  
   o. $k(a - 2)^2 + (a - 2)$  
   p. $12x^4(y + 3)^3 + 8x^3(y + 3)^5$

4. Factor and simplify the following expressions.
   a. $k^2 + 2k - (3k + 6)$  
   b. $-3t + 3 - (2t^2 - 2t)$  
   c. $7x^3 + 21x^2 - 28x$  
   d. $(x^2 + 3)(x + 4) + (x^2 + 3)(7x + 9)$  
   e. $(x^2 - 1)(x^2 + 3) + (x^2 + 3)(2x + 7) + 4x(x^2 + 3)$  
   f. $(4 - x^2)(x + 3) - (x^2 + 2)(4 - x^2) + 4x^2(4 - x^2)$

5. a. Show that $(x + 7)$ is a factor of $x^2 - 5x + 4x + 14$.

b. Show that $(x - 3)$ is a factor of $x^2 - 3x + 6x - 18$.

6. Darlene’s yard is a rectangle with an area of $(18x^4 + 24x^3)$.
   Find the dimensions of the yard in the form $ax^2 + b$, where $a$ and $b$ are integers.

7. Ann has a dog run, which is a rectangle whose area is $6x^4y^2 + 9x^2y^5$.
   Find the dimensions of the dog run in the form $ax^2y + by$, where $a$ and $b$ are integers.
**Topic 6.6.1**

**Factoring Quadratics**

**California Standards:** 11.0

**Example**

Factor \( x^2 - 6x - 7 \).

**Solution**

\[
\begin{align*}
&\text{Factors must multiply to give } -7 \text{ and add up to } -6 \\
&\text{Factors of } -7 \text{ are } -7 \text{ and } 1 \\
&\text{They add up to } -6
\end{align*}
\]

\[
\begin{align*}
x^2 - 6x - 7 &= (x - 7)(x + 1)
\end{align*}
\]

1. Determine the value of \( t \) in the factors of the following quadratics.

   a. \( x^2 + x - 6 = (x - t)(x + 3) \)

   b. \( x^2 + 8x + 15 = (x + 3)(x + t) \)

   c. \( x^2 + 2x - 8 = (x + 4)(x - t) \)

   d. \( x^2 + 4x - 5 = (x - t)(x + 5) \)

   e. \( x^2 - 11x + 24 = (x - 8)(x - t) \)

2. Replace \( s \) with the appropriate sign in the factors of the following quadratics.

   a. \( x^2 - 2x - 3 = (x + 1)(x - s 3) \)

   b. \( x^2 - 6x + 8 = (x - s 4)(x - 2) \)

   c. \( x^2 + 3x - 10 = (x - s 2)(x + 5) \)

   d. \( x^2 - 2x - 15 = (x - 5)(x s 3) \)

3. The class was asked to factor \( x^2 - 2x - 15 \). Bill had \((x - 5)(x + 3)\) while Jane had \((x + 3)(x - 5)\). Who was correct? Explain your answer.

4. Factor each quadratic as the product of two binomials.

   a. \( x^2 - 4x + 3 \)

   b. \( x^2 - 7x + 10 \)

   c. \( x^2 + x - 12 \)

   d. \( x^2 - 2x - 35 \)
6. Factor each quadratic as the product of two binomials. If it won’t factor state so.

a. \(x^2 - 9\)  

b. \(x^2 + 9\)  

c. \(x^2 + 3x + 8\)  

d. \(x^2 + 5x - 14\)  

e. \(x^2 - 6x + 8\)  

f. \(x^2 + 2x - 3\)  

g. \(x^2 - 3x - 18\)  

h. \(x^2 + 13x + 22\)  

i. \(x^2 - 7x + 12\)  

j. \(x^2 + 11x + 28\)  

7. The area of the rectangle shown is \((x^2 + 2x - 141)\) square inches. If the length is \((x + 13)\) inches, how wide is the rectangle in terms of \(x\)?

\[A = (x^2 + 2x - 141) \text{ in}^2\]

\((x + 13)\) in.

8. Factor completely each of the 3rd degree polynomials using the information given.

a. \((x + 2)\) is a factor of \(x^3 + 4x^2 + x - 6\)  

b. \((x - 2)\) is a factor of \(x^3 - 4x^2 + x + 6\)  

c. \((x + 3)\) is a factor of \(x^3 + 4x^2 - 9x - 36\)  

d. \((x - 3)\) is a factor of \(x^3 - 9x^2 + 26x - 24\)
Factoring Quadratics – $ax^2 + bx + c$

Example

Factor completely $2x^2 + 10x + 12$.

Solution

$2x^2 + 10x + 12 = 2(x^2 + 5x + 6) = 2(x + 2)(x + 3)$. 2 and 3 are factors of 6 and they add up to 5.

1. Factor the following expressions completely.

   a. $2x^2 + 8x + 6$
   b. $\pi r^2 + 2\pi rh$
   c. $2x^3 - 6xk + 4k$
   d. $4mx^2 + 6mx - 18m$
   e. $3m^2p - 3mp - 126p$
   f. $\frac{1}{2}qx^2 - 4qx + 8q$
   g. $\frac{3}{5}x^2 + \frac{33}{5}x + \frac{84}{5}$

Example

Factor $3x^2 + x - 2$.

Solution

$(3x - 1)(x + 2)$. Factors of 3 (the coefficient of $x^2$) are 1 and 3.

Factors of -2 (the last term of the quadratic) are -2 and 1, or -1 and 2.
The sum of the products of the outer factors and the inner factors must equal +1, the coefficient of x. Try both sets of factors and see which works.

$(3x - 1)(x + 2)$, $-x$ and $+6x$ equal +5x.

$(3x - 2)(x + 1)$, $-2x$ and $+3x$ equal +x. This is correct.

2. Factor $2x^2 + x - 1$. .................................................................
3. Factor the following expressions. Some expressions may not be able to be factored.

a. $2x^2 - 3x + 1$ .................................................. b. $2x^2 + 3x + 1$ ..................................................

c. $2x^2 - 5x + 3$ .................................................. d. $3x^2 - 4x + 1$ ..................................................

e. $5x^2 - 4x - 1$ .................................................. f. $6x^2 - x - 1$ ..................................................

g. $6x^2 + x - 1$ .................................................. h. $4x^2 + 12x + 9$ ..................................................

i. $27x^2 - 18x + 3$ .................................................. j. $4x^2 - 4x + 1$ ..................................................

k. $2x^2 + x + 1$ .................................................. l. $6x^2 - 4x - 10$ ..................................................

m. $6x^2 - 11x + 4$ .................................................. n. $3x^2 - x + 1$ ..................................................

o. $14x^2 - 12x - 2$ .................................................. p. $16x^2 - 8x + 1$ ..................................................

Remember to take out the GCF first where possible.

4. Factor $2 + x - x^2$. ..................................................

5. The area of a rectangle with length $(5x + 1)$ inches is $(15x^2 - 7x - 2)$ square inches. Find its width $(w)$ in terms of $x$. ..................................................

Be careful when there is a minus sign.

6. Factor the following expressions completely.

a. $3abx^2 - 3abx - 60ab$ ..................................................

b. $x^2(x + 2) - 4(x + 2)x - 21(x + 2)$ ..................................................

c. $4x^3 + 14x^2 + 12x$ ..................................................

d. $-4ab^2x + 64ax$ ..................................................

7. The diagram below shows Kaitlin’s garden. The garden measures $(5x + 4)$ by $(x + 19)$. The area of lawn is $3x^2 + 88x + 97$. Find the dimensions of the pool in the form $ax + b$ where $a$ and $b$ are integers.
**Topic 6.7.1 Factoring Quadratics in Two Variables**

California Standards: 11.0

---

**Example**

Factor $x^2 - 2xy - 3y^2$.

**Solution**

$$x^2 - 2xy - 3y^2 = (x - 3y)(x + y)$$

$-3y$ and $y$ are factors of $-3y^2$ and their sum is $-2y$

---

1. Factor the following expressions.

   a. $x^2 - 2xy + y^2$

   b. $x^2 + 3kx + 2k^2$

   c. $x^2 + mx - 6m^2$

   d. $k^2 + 2kr + r^2$

   e. $x^2 + 3xy + 2y^2$

   f. $a^2 - ab - 2b^2$

   g. $r^2 + 5rs + 6s^2$

---

2. The area of a rectangle is $x^2 + 7xy + 12y^2$. What are its dimensions in terms of $x$ and $y$?

---

3. Factor the following expressions.

   a. $2x^2 + 7xy + 3y^2$

   b. $3a^2 - 7ab - 6b^2$

   c. $2u^2 - 5uv - 3v^2$

   d. $2g^2 - 7gh + 6h^2$

   e. $2x^2k - 6xk + 4k$
4. Factor the following expressions.
   a. $8x^2 + 22xy + 15y^2$
   b. $6a^2 + 11ab - 10b^2$
   c. $12u^2 - uv - 20v^2$
   d. $10g^2 - 29gh + 21h^2$
   e. $6x^2 + 31xy + 40y^2$
   f. $5p^2 + 23pq + 12q^2$
   g. $2(3z^2 - yz - 4y^2)$
   h. $2(10m^2 - 7mn - 6n^2)$

5. Factor the following expressions. Look for a common factor first.
   a. $8ax^2 - 2axy - 15ay^2$
   b. $30a^2x + 22abx - 24b^2x$
   c. $30u^2x^2 + 57uvx^2 - 45v^2x^2$
   d. $112a^2 + 332ab + 66b^2$
   e. $52xy - 12x^2 - 16y^2$

6. Assume you factored $a^2 + 3ab - 4b^2 = (a + x)(a + y)$. What can you tell about the signs of $x$ and $y$?

7. The volume of a rectangular box is $50ax^2 + 35axy - 60ay^2$. What are the dimensions of the box in terms of $a$, $x$, and $y$?
### Example

Factor \(2x^3 + 4x^2 - 6x\).

**Solution**

\[
2x^3 + 4x^2 - 6x \\
2x(x^2 + 2x - 3)
\]

2\(x\) is a factor of all three terms, so take it out first.

Now you need to factor the quadratic inside the parentheses.

\[
2x(x + 3)(x - 1)
\]

The factors of \(-3\) (the last term of the quadratic) are \(-3\) and 1, or \(-1\) and 3. The sum of these numbers must equal \(+2\), the coefficient of \(x\).

\[2x(x + 3)(x - 1) \quad \text{+3x and -x equal +2x} \]

You can multiply this out again to give \(2x^3 + 4x^2 - 6x\).

---

1. Factor completely each of the following expressions. Look for a common factor first.

   a. \(x^3 + 6x^2 + 8x\)  
   b. \(x^3 + x^2 - 6x\)  
   c. \(x^3 + 9x^2 + 20x\)  
   d. \(x^3 + x^2 - 12x\)  
   e. \(x^3 - 9x^2 + 20x\)

2. Factor completely each of the following expressions.

   a. \(5x^3 + 10x^2 + 5x\)  
   b. \(3x^3 + 15x^2 + 18x\)  
   c. \(5x^3 + 5x^2 - 60x\)  
   d. \(4x^3 - 8x^2 - 140x\)  
   e. \(6x^3 - 30x^2 + 36x\)  
   f. \(21y^3 - 154y^2 - 112y\)
3. Factor completely each of the following expressions.
   a. $ax^3 - ax^2 - 2ax$
   b. $4am^3 - 6am^2 + 2am$
   c. $3ax^3 + 21ax^2 + 36ax$
   d. $7b^2x^3 + 7b^3x^2 - 140b^2x$
   e. $5c^3x^3 + 5c^4x^2 - 150c^3x$
   f. $cx^3 - 4cx$

4. Factor completely each of the following expressions.
   a. $12a^2b^2x^3 + 42a^2b^2x^2 + 36a^2b^2x$
   b. $24u^4v^2x^3 - 92u^4v^2x^2 + 80u^4v^2x$
   c. $18a^3b^2x^3 + 48a^3b^2x^2 + 32a^3b^2x$

5. Factor completely the following polynomials.
   a. $-2x^3 + 4x^2 + 16x$
   b. $-6x^3 + 21x^2 - 9x$
   c. $-10x^3 + 25x^2 + 15x$
   d. $-3x^3 - 12x^2 - 12x$

6. The product of 3 consecutive even integers is $x^3 + 6x^2 + 8x$. Determine the integers in terms of $x$.

7. The diagram shows a rectangular based prism made by cutting a rectangular box in half along the diagonal. The volume of the prism is $36xy^3 + 15x^2y^2 - 6x^3y$. Calculate the dimensions of the box from which it was made in terms of $x$ and $y$. 

Topic 6.8.1  The Difference of Two Squares

California Standards: 11.0

Example

Factor completely $p^2 - 64$.

Solution

$p^2 - 64 = (p - 8)(p + 8)$  
Remember, $m^2 - c^2 = (m + c)(m - c)$.

1. Factor each of the following expressions completely.
   
   a. $k^2 - 1$
   
   b. $m^2 - 36$
   
   c. $r^2 - t^2$
   
   d. $4 - x^2$
   
   e. $100 - c^2$
   
   f. $r^2 - 81$
   
   g. $x^2 - y^2$
   
   h. $r^2 - 16$
   
   i. $25 - z^2$
   
   j. $4b^2 - c^2$
   
   k. $r^4 - s^2$

2. Factor each of the following expressions completely. Look for a common factor first.
   
   a. $3a^2 - 3b^2$
   
   b. $5a^3 - 45ab^2$
   
   c. $24ax^2 - 54ay^2$
   
   d. $45r^3 - 80rs^2$
   
   e. $100lm^2n - 4f^3n$
   
   f. $-4p^3x + 4pq^2x$
   
   g. $27m^6n^2 - 12mn^4$
3. The volume of the rectangular box in the diagram is $12ax^3 - 27axy^2$. What are the dimensions of the box in terms of variables?

![Image of a rectangular box]

4. A triangle has area $\frac{1}{2}(4x^2 - 25y^2)$. What is the length of the base and the height, in terms of $x$ and $y$?

![Image of a triangle with labels for base and height]

5. Joel factored $x^4 - y^4$ as $(x^2 + y^2)(x^2 - y^2) = (x + y)(x + y)(x - y)$. Is Joel correct? Explain.

6. Factor each of the following expressions completely.
   a. $x^4 - r^4$
   b. $x^4 - 16$
   c. $20x^2 - 45y^2$
   d. $36r^3 - 100rs^2$
   e. $112u^2v - 63v^3$

7. The diagram shows Brett’s birthday cake. It is square and has an area of $9x^4y^2$. Brett cuts four square pieces that each measure $xz^2$ by $xz^2$ from the cake.
   a. Write an expression for the area of cake left after the pieces have been cut.

   ![Image of a birthday cake with labels for dimensions]

   b. Determine the dimensions of the cake left in terms of $x$, $y$ and $z$.
Perfect Square Trinomials

California Standards: 11.0

1. Expand each of the following polynomials.
   a. \((r + s)^2\)
   b. \((r - s)^2\)
   c. \((2a + b)^2\)
   d. \((2a - b)^2\)

Example

Factor completely \(4x^2 + 28x + 49\).

Solution

\[
= (2x + 7)(2x + 7) \quad \text{This is a perfect square trinomial.}
\]

\[
= (2x + 7)^2
\]

2. Factor each of the following expressions completely.
   a. \(x^2 + 12x + 36\)
   b. \(x^2 - 4x + 4\)
   c. \(4x^2 + 12x + 9\)
   d. \(9x^2 - 6x + 1\)
   e. \(9x^2 + 24xy + 16y^2\)
   f. \(4a^2 - 20ab + 25b^2\)
   g. \(16a^2 + 56ab + 49b^2\)
   h. \(9r^2 - 30rs + 25s^2\)

3. Explain why Ann said she knows \(9x^2 + 12xy + 16y^2\) is not a perfect square trinomial.
4. The expression $9x^2 + 12xy + ay^2$ is a perfect square trinomial. Find the number $a$.

5. Factor each of the following expressions completely.
   a. $25x^2 - 70xy + 49y^2$
   b. $4a^4 - 4a^2b^2 + b^4c^2$
   c. $a^2c^2 + 18ab^2cd + 81b^4d^2$
   d. $3a^6 - 24a^3bc + 48b^2c^2$
   e. $16a^2b^2 + 24abx + 9x^2$
   f. $9a^2b^2 + 12abc^2d + 4c^4d^2$
   g. $9x^2y^2 - 102a^2xy + 289a^4$
   h. $36a^2b^2c^2 + 84abcdef + 49d^2e^2f^2$
   i. $9a^2b^4 + 30a^2b^2c + 25a^2b^2c^2$
   j. $f^2k^2 + 88k^2l + 16f^2k^2l$
   k. $9a^4b^2 - 72a^2bxy + 144x^2y^2$
   l. $4a^2b^2 - 12abr^2s^2 + 9r^4s^4$

6. The area of a circle is given by the formula $A = \pi r^2$. What is the radius in terms of $x$ and $y$ of a circle with an area of $25\pi x^2 + 70\pi xy + 49\pi y^2$?

7. The volume of a circular cylinder is given by the formula $V = \pi r^2h$. Determine $r$ and $h$, for a circular cylinder with a volume of $108\pi ax^2 + 252\pi axy + 147\pi ay^2$.

8. Factor $16a^4b^4 - 72a^2b^2x^2y^2 + 81x^4y^4$ completely.
Topic 6.8.3

Factoring by Grouping

California Standards: 11.0

Example

Factor completely \(2x^2 + 8x + 6\).

Solution
\[
= 2(x^2 + 4x + 3) \quad \text{Take out the GCF first}
\]
\[
= 2(x + 3)(x + 1)
\]

Factor completely \(2kx - 3x - 2k + 3\).

Solution
\[
= (2kx - 2k) + (-3x + 3) \quad \text{Collect terms in } k
\]
\[
= 2k(x - 1) - 3(x - 1)
\]
\[
= (x - 1)(2k - 3) \quad \text{Both parts of the expression contain } (x - 1), \text{ so it can be factored out}
\]

1. Factor each of the following expressions completely.
   a. \(xy + 3y - 3 - x\)
   b. \(14b + ab - 2a - 28\)
   c. \(pq - 3q + 9 - 3p\)
   d. \(ac + bc + ad + bd\)
   e. \(ax + ay - bx - by\)
   f. \(ca - ba + cd - bd\)
   g. \(6x - 6y + rx - ry\)

2. Factor each of the following expressions. Look for a common factor first.
   a. \(2xy + 2xb + 2ay + 2ab\)
   b. \(12ar + 12af - 4br - 4bf\)
   c. \(10xy - 5x + 10y - 5\)
   d. \(3fg - 18g + 12f - 72\)
   e. \(8pq + 84 + 56p + 12q\)
3. Factor each of the following expressions completely.
   a. $3rx - 6r - 4x + 8$
   b. $15ar + 8bt + 20at + 6br$
   c. $k^2r - 4r + 2k^2 - 8$
   d. $24ax - 63ay + 42ax - 36ay$
   e. $m^2 - 9y + m^2y - 9$
   f. $3x^2 + 12x - 12xy - 48y$
   g. $70x + 35y - 14x^2 - 7xy$
   h. $5ab^2c + 5a^2b - 20bc^2 - 20ac$
   i. $3ac^2 + 3bcxy - 3adxy - 3bdy^2$
   j. $14x^2 - 4xy + 7xy - 2y^2$

4. The area of a rectangle is $15xu - 9xv + 20yu - 12yv$.
   What are the length and width in variable notation?

5. The area of a triangle is $\frac{1}{2} (6ac + 8ad + 9bc + 12bd)$.
   What is the length of the base and height in terms of $a$, $b$, $c$ and $d$?

6. Charley looked at $(2a + 3b)(2x - 3y)$ and said this product is $4a^2x^2 - 9b^2y^2$ because it is the sum and difference of the same numbers. Is Charley correct? Explain.
**Topic 7.1.1**

**Solving Quadratic Equations by Factoring**

California Standards: 11.0, 14.0

**Example**

Given the equation \(-8 + x^2 - 2x = 0\):

a. Put the equation in order from highest power to lowest power.
b. Factor the trinomial.
c. Solve the equation.

**Solution**

a. \(x^2 - 2x - 8 = 0\)  \hspace{1cm} Order the terms by powers of \(x\)
b. \((x - 4)(x + 2) = 0\) \hspace{1cm} Put the equation in the form \((x + a)(x + b)\)
where \(ab = -8\) and \(a + b = -2\)
c. \(x - 4 = 0 \Rightarrow x = 4\) \hspace{1cm} Set each of the brackets equal to 0
d. \(x + 2 = 0 \Rightarrow x = -2\) \hspace{1cm} to determine the possible values of \(x\)

1. Put each of the following quadratics in order from the highest power to the lowest power.

   a. \(x - 4 + x^2\)  

   b. \(5 + x^2 - 2x\)  

   c. \(x^2 - 3 + 2x\)  

   d. \(4x + x^2 + 3\)

2. The general form of a quadratic equation is \(ax^2 + bx + c = 0\) where \(a, b,\) and \(c\) are numbers. Identify \(a, b,\) and \(c\) in the following equations.

   a. \(2x^2 + 3x + 7 = 0\)  

   b. \(3x - 5x^2 + 2 = 0\)  

   c. \(4 - 2x + 7x^2 = 0\)  

   d. \(5x - 2x^2 - 4 = 0\)  

   e. \(2x - x^2 + 3 = 0\)  

   f. \(\frac{2 + 4x + 6x^2}{2} = 0\)  

   g. \(1 - 2x + \frac{x^2}{4} = 0\)  

   h. \(-4 - (2x)^2 - x = 0\)

3. Determine which of the values given is a solution of the following equations.

   a. \(x^2 + 3x - 10 = 0\) for \(x = 1, x = 2, x = -3\)

   b. \(2x^2 - 2x - 24 = 0\) for \(x = 2, x = -3, x = -4\)
4. A quadratic equation factors to \((x + 3)(x – 5) = 0\). Jim says that the solutions are 3 and –5. Using the zero property, explain why Jim is wrong.

5. Solve each of the following quadratic equations by factoring.
   a. \(x^2 + 7x + 12 = 0\)
   b. \(x^2 – 7x + 10 = 0\)
   c. \(x^2 + 2x – 15 = 0\)
   d. \(x^2 – 3x – 18 = 0\)

6. Solve the following equations.
   a. \(2x^2 – x – 6 = 0\)
   b. \(2x^2 – x – 15 = 0\)
   c. \(3x^2 + 11x + 6 = 0\)
   d. \(3x^2 – 13x + 12 = 0\)
   e. \(3k^2 – 15 = –4k\)
   f. \(2y^2 – 3 = –5y\)
   g. \(6p^2 – 42p = –72\)
   h. \(6x^2 + 8 = 20x – 8\)
   i. \(x^2 + 7x + 12 = 72\)
   j. \(x^2 + 2x – 15 = 33\)
   k. \(3x^2 – 11x – 20 = 22\)
   l. \(4x(x - 6) = 6(1 - 4x) + 3\)
   m. \(2x(6x - 3) = 2(2 - 7x)\)
   n. \(x(10 - 3x) + 6 = x(4x - 9)\)
   o. \(3x(2x - 14) = -4(3x + 7) - x\)
   p. \(4(9x - 7) = 2x(1 + 5x) - (1 - x)\)

7. A rectangular piece of metal has dimensions \(x + 2\) and \(x + 6\) and an area of 117 sq in. What are the dimensions, in inches, of the piece of metal?

8. The dimensions of a box are 4 in. high, \((x + 5)\) in. long and \((2x – 3)\) in. wide. If the volume of the box is 96 cu in., calculate its dimensions.
Quadratic Equations
— Taking Square Roots

California Standards: 11.0, 14.0

1. Find the square roots of the numbers below.
   a. 49                           b. 81
   c. 121                          d. \( \frac{1}{4} \)
   e. \( \frac{9}{16} \)               f. \( \frac{16}{625} \)

2. Determine the square roots of each of the following expressions.
   a. \( x^2 + 8x + 16 \) 
   b. \( 9x^2 - 30x + 25 \) 
   c. \( 25x^2 - 70x + 49 \) 

3. Solve the following equations by taking square roots.
   a. \( x^2 = 25 \) 
   b. \( y^2 = 49 \) 
   c. \( z^2 = 9 \) 
   d. \( v^2 = 36 \)

4. Solve the following equations by taking square roots.
   a. \( 4x^2 = 100 \) 
   b. \( 3x^2 - 108 = 0 \) 
   c. \( 7x^2 - 112 = 0 \) 
   d. \( 5x^2 = 180 \)

Example

Solve \( (x - 7)^2 = 75 \) by taking square roots.

Solution

\[ (x - 7)^2 = 75 \]
\[ (x - 7) = \pm \sqrt{75} \quad \text{Check whether the radical can be simplified} \]
\[ x - 7 = \pm \sqrt{25 \cdot 3} \quad 75 \text{ is } 25 \times 3 \quad \text{so } \sqrt{75} = \sqrt{25} \times \sqrt{3} \]
\[ x - 7 = \pm 5\sqrt{3} \]
\[ x = 7 \pm 5\sqrt{3} \quad \therefore x = 7 + 5\sqrt{3} \text{ or } x = 7 - 5\sqrt{3} \]

Use "or," not "and" — \( x \) can only have one value at a time.
5. Simplify the following radicals.
   a. √180  
   b. √405  
   c. √252

6. Solve each of the following equations by taking square roots. Give your answers in their simplest form.
   a. (x + 3)^2 = 25  
   b. (x - 5)^2 = 75  
   c. (z - 8)^2 = 147  
   d. (v + 6)^2 = 180

7. Solve each of the following equations by taking square roots.
   a. x^2 + 6x + 9 = 36  
   b. 4x^2 + 20x + 25 = 9  
   c. 9x^2 - 20x + 16 = 4x + 9  
   d. 4x^2 - 8x + 9 = 4x + 64

8. Solve each of the following equations by taking square roots.
   a. m^2 = 289  
   b. p^2 - 5 = 67  
   c. 4x^2 + 12x + 9 = 49  
   d. \( \left( r - \frac{1}{2} \right)^2 = \frac{9}{4} \)  
   e. 16y^2 - 24y + 9 = 225  
   f. t^2 - \( \frac{1}{2} \)t + \( \frac{1}{16} \) = \( \frac{4}{9} \)

9. A square with sides x + 3 cm long has an area of 121 sq cm. What is the numerical length of each side?

10. A circle has radius (x - 3) cm and area 225π sq cm. What is the numerical length of the radius?
Completing the Square

California Standards: 14.0

Example

Write a perfect square trinomial based on \( x^2 - hx \) and write the final result as the square of a binomial.

Solution

Complete the square on \( x^2 - hx \)

\[
x^2 - hx + \left( \frac{-h}{2} \right)^2 = x^2 - hx + \frac{h^2}{4} = \left( x - \frac{h}{2} \right)^2
\]

Factor the trinomial

You can check your answer by multiplying out the brackets.

1. Find the value of \( k \) that will make each expression below a perfect square trinomial.
   
a. \( x^2 + 12x + k \) ................................................
   
b. \( y^2 - 12y + k \) ................................................
   
c. \( z^2 + 8z + k \) ................................................
   
d. \( v^2 - 8v + k \) ................................................

2. Form a perfect square trinomial from each of the expressions below.
   
a. \( r^2 + 14r \) ................................................
   
b. \( t^2 - 8t \) ................................................
   
c. \( x^2 - dx \) ................................................
   
d. \( x^2 + 6x \) ................................................
   
e. \( x^2 + 7x \) ................................................
   
f. \( y^2 - 5y \) ................................................
   
g. \( z^2 + 9z \) ................................................
   
h. \( v^2 - 3v \) ................................................

3. Find the value of \( c \) and \( k \) in each of the following cases.
   
a. \( x^2 + 8x + c = (x + k)^2 \) ................................................
   
b. \( x^2 + 12x + c = (x + k)^2 \) ................................................
   
c. \( x^2 - 7x + c = (x + k)^2 \) ................................................
   
d. \( x^2 - 11x + c = (x + k)^2 \) ................................................
4. By finding the appropriate value for $m$, rewrite each of the following expressions in the form $a(x + k)^2$.
   
   a. $4x^2 + 40x + m$
   
   b. $2x^2 - 28x + m$
   
   c. $3x^2 + 24x + m$
   
   d. $5x^2 - 10x + m$

5. Convert each of the following to an expression of the form $a(x + k)^2$.
   
   a. $3x^2 + 18x$
   
   b. $5x^2 - 40x$
   
   c. $7x^2 + 70x$
   
   d. $4x^2 - 24x$
   
   e. $4x^2 + 144$
   
   f. $7x^2 + 1183$

6. Determine, in terms of $x$, the side length of a square with area $x^2 + 8x + c$, assuming that $x^2 + 8x + c$ is a perfect square trinomial.

7. Determine, in terms of $x$, the radius of a circle with area $\pi(x^2 + 7x + c)$, assuming that $x^2 + 7x + c$ is a perfect square trinomial.
More on Completing the Square

1. Convert each of the following into perfect square trinomials by adding a constant term.
   a. $x^2 + 8x$
   b. $x^2 - 12x$
   c. $x^2 + 9x$
   d. $x^2 - 5x$

2. Express the following binomials in the form $(x + k)^2 + m$.
   a. $x^2 + 10x$
   b. $x^2 - 14x$
   c. $x^2 + 7x$
   d. $x^2 - 13x$

Example

Write $2x^2 - 6x + 7$ in the form $a(x + h)^2 + k$.

Solution

$2x^2 - 6x + 7 = 2\left(x^2 - 3x + \frac{7}{2}\right)$ Take out a factor of 2 to get 1 for the coefficient of $x^2$.

$= 2\left(\left(\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{7}{2}\right)$ Complete the square for $x^2 - 3x$. Subtract $\left(\frac{3}{2}\right)^2$ so that the value of the expression does not change.

$= 2\left(\frac{3}{2}\right)^2 + \frac{5}{4}$ Square the fraction.

$= 2\left(\frac{3}{2}\right)^2 + \frac{5}{2}$ Simplify by adding the fractions together.

$= 2\left(\frac{3}{2}\right)^2 + \frac{5}{2}$ Simplify by multiplying $\frac{5}{4}$ by 2.
3. Write the following expressions in the form \((x + k)^2 + m\).
   
   a. \(x^2 + 6x + 3\)  
   b. \(x^2 - 12x + 5\)  
   c. \(x^2 + 8x - 5\)  
   d. \(x^2 - 16x - 22\)  
   e. \(x^2 - 4x + 5\)  
   f. \(x^2 + 4x - 7\)  
   g. \(x^2 - 3x + \frac{5}{4}\)  

4. Write the following expressions in the form \(a(x + k)^2 + m\).
   
   a. \(5x^2 + 20x + 9\)  
   b. \(3x^2 + 30x + 50\)  
   c. \(7x^2 - 56x + 100\)  
   d. \(4x^2 - 24x + 45\)  
   e. \(\frac{1}{4}x^2 + \frac{3}{5}x\)  

5. Determine, in terms of \(x\), the side length of a square with an area of \(x^2 + 20x + k\), assuming that \(x^2 + 20x + k\) is a perfect square trinomial.

6. A triangle has area \(\frac{1}{2} (x^2 + 14x + k)\). Its height and length of base are equal.

   Express the height of the triangle in terms of \(x\), assuming that \(x^2 + 14x + k\) is a perfect square trinomial.
Example

Solve \( 7x^2 - 14x - 28 = 0 \) by completing the square.

Solution

\[
7x^2 - 14x - 28 = 0
\]

\[
x^2 - 2x - 4 = 0 \quad \text{Divide by 7}
\]

\[
x^2 - 2x = 4 \quad \text{Add 4 to both sides}
\]

\[
x^2 - 2x + (-1)^2 = 4 + (-1)^2 \quad \text{Complete the square for } x^2 - 2x \text{ on the left-hand side}
\]

\[
(x - 1)^2 = 5
\]

\[
x - 1 = \pm \sqrt{5} \quad \text{Take the square root of both sides}
\]

\[
x = 1 \pm \sqrt{5}
\]

\[
x = 1 + \sqrt{5} \text{ or } x = 1 - \sqrt{5}
\]

1. Solve each of the following equations by completing the square.
   a. \( x^2 + 6x = 7 \)  
   b. \( x^2 + 10x = 11 \)

2. Solve each of the following equations by completing the square.
   a. \( x^2 + 2x - 10 = 14 \)  
   b. \( x^2 + 12x - 6 = -42 \)  
   c. \( x^2 - 20x + 19 = -56 \)  
   d. \( x^2 - 16x - 10 = 26 \)

3. Solve each of the following equations by completing the square.
   a. \( 3x^2 + 30x + 60 = 90 \)  
   b. \( 5x^2 + 35x + 60 = 75 \)  
   c. \( 4x^2 - 12x - 12 = 16 \)
4. Solve each of the following equations by completing the square.
   a. \( x^2 + 3x - 1 = 0 \)
   b. \( 2k^2 - 3k - 1 = 0 \)
   c. \( 4x^2 - 3x - 2 = 0 \)
   d. \( 3x^2 + 4x - 9 = 0 \)
   e. \( 2y^2 + y - 3 = 12 \)
   f. \( 5x^2 = 7 - 2x \)
   g. \( 5x - 3x^2 = -8 \)
   h. \( -2x^2 + 3x = \frac{1}{2} \)
   i. \( 2p - 5p^2 = -4 \)
   j. \( 5 - 2k^2 = 3k \)
   k. \( 3x + 5 = 4x^2 + 5x - 13 \)
   l. \( 4x + 5x^2 + 7 = 2x^2 + 19 - 5x \)

5. A square with sides \((2x + 7)\) in. long has an area of 361 sq in.
   Calculate the length of each side of the square.

6. A polygon with \(n\) sides has \(\frac{1}{2} n^2 - \frac{3}{2} n\) diagonals that can be drawn inside it.
   Calculate the number of sides of a polygon that can have 77 diagonals drawn inside it.
The Quadratic Formula

It’s often easier to divide an equation by the coefficient of \(x^2\) before completing the square.

1. Given that \(ax^2 + bx + c = 0\), complete the square to solve for \(x\) in terms of \(a\), \(b\), and \(c\).

Example

Use the quadratic formula to solve \(x^2 + 10x - 3 = 0\).

Solution

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

where \(a = 1\), \(b = 10\), \(c = -3\)

\[
x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(-3)}}{2 \cdot 1}
\]

Sub in values of \(a\), \(b\), and \(c\)

\[
x = \frac{-10 \pm \sqrt{100 + 12}}{2}
\]

Take care with the negative signs: \(-4 \times +1 \times -3 = +12\)

\[
x = -5 \pm \frac{\sqrt{112}}{2}
\]

Simplify

\[
x = -5 \pm \frac{\sqrt{16 \cdot 7}}{2} = -5 \pm \frac{4\sqrt{7}}{2} = -5 \pm 2\sqrt{7}
\]

Simplify the radical as much as possible

2. Solve \(x^2 - 2x - 1 = 0\) using the quadratic formula.
3. Use the quadratic formula to solve each of the following equations.
   a. \(x^2 - x - 5 = 0\)  
   b. \(x^2 + 2x - 4 = 0\)
   c. \(y^2 - 2y = 1\)  
   d. \(-m^2 + 4m + 3 = 0\)
   e. \(x^2 + 3x - 11 = 0\)  
   f. \(5x^2 + 20x + 9 = 0\)
   g. \(7y^2 - 24y = 18\)  
   h. \(-4m^2 + 14m + 3 = 0\)
   i. \(-3x^2 + 16x - 12 = 0\)  
   j. \(\frac{x - 1}{2} = \frac{1}{2x - 1}\)
   k. \(\frac{3x - 4}{3} = \frac{2}{2x + 1}\)  
   l. \(\frac{x + 3}{5} = \frac{3}{2x + 1}\)

4. A rectangle has an area of 80 sq cm. Its dimensions are \((x + 2)\) cm by \((x - 5)\) cm. Calculate the numerical dimensions of the rectangle.

5. The height above the ground of a ball thrown vertically upwards is described by the expression \(-16t^2 + vt + h\), where \(t\) is the time the object is in the air, \(v\) is the velocity the object is thrown with, and \(h\) is the height the object is thrown from.

Margaret throws a ball vertically upwards from a point 4 ft above the ground. The ball leaves Margaret's hand at a velocity of 50 ft per second. How long after Margaret throws the ball will it hit the ground?
Applications of Quadratics

California Standards: 20.0

Example

The product of two consecutive odd integers is 483. Find the numbers.

Solution

Let \( n \) = 1st odd integer

\[ \therefore n + 2 = \text{2nd odd integer} \]

Equation: \( n(n + 2) = 483 \)

\[ n^2 + 2n = 483 \]

\[ n^2 + 2n - 483 = 0 \]

Make one side equal zero so you can use the quadratic formula

\[ n = \frac{-2 \pm \sqrt{4 + 1932}}{2} = \frac{-2 \pm \sqrt{1936}}{2} \]

\[ = \frac{-2 \pm 44}{2} = -1 \pm 22 = 21 \text{ or } -23 \]

The numbers are 21 and 23, or -23 and -21. Write the solution clearly

1. Two numbers differ by 7. If the product of the numbers is 294, find all possible pairs of such numbers.

2. Find the dimensions and perimeter of the rectangle shown below if its area is 144 cm\(^2\).

   \[ (3x - 2) \text{ cm} \]

   \[ \text{A} = 144 \text{ cm}^2 \]

   \[ (2x - 3) \text{ cm} \]

Remember, you cannot have a negative length.

3. Carlos is 7 years older than Li Wei. Three years ago, the product of their ages was 294. How old is each one now?
4. Two sisters differ in age by 11 years. Three years ago the product of their ages was 276. How old is each sister now? 

5. A mother is 27 years older than her son. In five years' time, the product of their ages will be 810. Find their ages now.

6. A sticker consists of a light-colored rectangle surrounded by a darker band of uniform width \((x)\). If the light-colored rectangle measures 3 cm by 4 cm, and the total area of the sticker is 56 cm², find the width of the darker band.

7. A roll of fence 56 feet long is used to fence a rectangular garden with an area of 187 ft². Find the dimensions of the garden, given that there is no overlap in the fence.

8. A 64 ft piece of wire is bent to form the perimeter of a rectangular plot of area 240 ft². Find the length and width of the rectangular plot, if there is no overlap in the wire.

9. A wire that is 50 cm long is bent to form a rectangular figure, with no overlap. The area of the figure formed is 154 cm². Find the dimensions of the figure formed.

10. A 5 inch by 7 inch picture is enclosed in a frame with an area (including space for the picture) of 143 in². Find the length and width of the frame, if the distance between the edge of the picture and the outside edge of the frame is the same all the way round.
Topic 7.4.1  
**Graphs of Quadratic Functions**

California Standards: 21.0

1. Jasmine plots a graph of $y = ax^2$.
   Describe how the graph would compare to that of $y = x^2$ if:
   
   a. $a = 2$
   
   b. $a = -2$
   
   c. $a = \frac{1}{4}$

2. Write down the line of symmetry and the coordinates of the vertex of each of the following graphs.

   a. 
   
   b. 
   
   c. 
   
   d. 

---

**Example**

Jake plots five graphs of $y = ax^2$ for different values of $a$. Match the equations with the correct graphs on the right.

1. $y = x^2$  
2. $y = 2x^2$  
3. $y = -x^2 - 4$  
4. $2x^2 + 4$  
5. $y = \frac{1}{2}x^2 - 3$

**Solution**

1. B  
2. C  
3. E  
4. D  
5. A
3. The following graphs are transformations of the graph of \( y = x^2 \). Match the correct equation to each graph.

a. \( y = \frac{1}{3} x^2 \) \hspace{1cm} \( y = \frac{1}{2} x^2 \)

b. \( y = \frac{1}{3} x^2 - 3 \) \hspace{1cm} \( y = \frac{1}{2} x^2 + 2 \)

c. \( y = \frac{1}{2} x^2 - 2 \)

4. Describe the graphs of the quadratics below in relation to the graph of \( y = x^2 \).

a. \( x^2 + 4 \)

b. \( 2x^2 - 3 \)

c. \( -x^2 + 3 \)

d. \( \frac{1}{3} x^2 - 7 \)

e. \( -4x^2 - 2 \)

5. The graphs below are transformations of the graph \( y = x^2 \). Find the equation of each graph.

6. State the \( x \)- and \( y \)-intercepts of each of these graphs.
**Drawing Graphs of Quadratic Functions**

**California Standards:** 21.0, 22.0

1. Find the x-intercepts of each of the following by solving the quadratic equation when $y = 0$.
   - **a.** $y = x^2 + 4x - 12$  
   - **b.** $y = x^2 - 7x + 12$

2. Find the y-intercept of each of the following by setting $x = 0$.
   - **a.** $y = x^2 + 3x - 4$  
   - **b.** $y = x^2 + 6x + 8$

3. Find the vertex of each of the following:
   - **a.** $y = x^2 + 2x - 3$  
   - **b.** $y = x^2 - 7x + 10$

4. Given the quadratic function $y = 3x^2 + 4x - 7$, explain how you know that the y-intercept is at $y = -7$ without doing any calculations.

---

**Example**

Sketch the graph of $y = x^2 + 4x - 12$, using its y-intercept, x-intercepts, vertex coordinates, and line of symmetry.

**Solution**

- **y-intercept:** Let $x = 0$ and solve for $y$
  
  $y = x^2 + 4x - 12$

  Let $x = 0$ and solve for $y$

  
  $y = 0^2 + 4(0) - 12$

  $y = -12$

  The y-intercept is $(0, -12)$.

- **x-intercept:** Let $y = 0$ and solve for $x$
  
  $x^2 + 4x - 12 = 0$

  $(x + 6)(x - 2) = 0$

  $x = -6$ or $x = 2$

  The x-intercepts are $(-6, 0)$ and $(2, 0)$.

- **Vertex coordinates:** The vertex is the maximum or minimum point of the curve.
  
  The x-coordinate of the vertex = $\frac{-6}{2} = -3$

  The y-coordinate = $(-2)^2 - 8 - 12 = -16$

  Vertex: $(-2, -16)$

- **Line of symmetry:** $x = -2$

The vertex is always half way between the x-intercepts. Now use this data to sketch the graph of $y = x^2 + 4x - 12$. 

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CGP Education Algebra I — Homework Book
5. Sketch the graph of each quadratic function in exercises a. – f., using the x-intercepts, y-intercept, vertex coordinates, and the line of symmetry as guides.

**a.** \( y = x^2 - 2x + 1 \)

**b.** \( y = x^2 - 4x + 3 \)

**c.** \( y = x^2 + 6x - 7 \)

**d.** \( y = x^2 + 4x - 5 \)

**e.** \( y = 2x^2 - 10x + 8 \)

**f.** \( y = 3x^2 - 9x + 6 \)

The coefficient of \( x^2 \) determines whether the parabola is u-shaped (a positive coefficient) or n-shaped (a negative coefficient).
California Standards: 21.0, 22.0

1. By completing the square, put each of the following into the form $y = (x + k)^2 + p$.
   a. $y = x^2 + 4x + 2$
   b. $y = x^2 - 6x + 3$

Example

Write $y = x^2 - 10x + 12$ in the form $y = (x - h)^2 + k$, then state the vertex of the graph and its line of symmetry.

Solution

$y = \left(x^2 - 10x + \left(\frac{-10}{2}\right)^2\right) - \left(\frac{-10}{2}\right)^2 + 12$

$= (x - 5)^2 - 25 + 12$

$= (x - 5)^2 - 13$

Vertex: $(h, k) = (5, -13)$

The minimum value of $y$ (the vertex) occurs where $x - 5 = 0$, so, at $x = 5$.

Line of symmetry: $x = h \Rightarrow x = 5$

The graph is symmetrical about the vertex.

2. Write each equation $y = ax^2 + bx + c$ in the form $y = a(x + h)^2 + k$, then state the vertex coordinates and the line of symmetry of its graph.
   a. $y = x^2 - 4x + 6$
   b. $y = x^2 + 6x + 7$
   c. $y = 2x^2 - 4x + 12$
   d. $y = 4x^2 - 16x + 13$
   e. $y = 3x^2 - 15x + 7$
   f. $y = 5x^2 + 15x - 11$
3. Write each of the following equations \( y = ax^2 + bx + c \) in the form \( y = a(x + h)^2 + k \). State the vertex coordinates, the line of symmetry, and both the \( x \)-intercepts (if any) and \( y \)-intercept for each equation. Use your answers to plot each graph on the axes given.

\[ \begin{align*}
\text{a. } y &= x^2 + 6x + 5 \\
\text{b. } y &= x^2 - 2x - 3
\end{align*} \]

4. Sketch the following quadratic equations on the axes provided.

\[ \begin{align*}
\text{a. } y &= x^2 + 2x + 4 \\
\text{b. } y &= x^2 - 4x + 7
\end{align*} \]

5. Randy throws a baseball straight upwards, releasing it 6 ft above the ground, and giving it an initial speed of \( v = 128 \) ft/sec. Use the formula \( h = -16t^2 + vt + 6 \) ft (where \( t \) = time, \( h \) = height) to find the maximum height of the ball and time taken for the ball to reach this height.
Describe the nature of the roots of the quadratic equation \( y = 2x^2 - 7x - 3 \).
Find the values of the roots.

**The graph of \( y = 2x^2 - 7x - 3 \) crosses the \( x \)-axis at two points if \( b^2 - 4ac > 0 \),
at one point if \( b^2 - 4ac = 0 \), or at zero points if \( b^2 - 4ac < 0 \).**

**Solution**

The nature of the roots can be found by working out the value of the discriminant \( b^2 - 4ac \).

\[
b^2 - 4ac = (-7)^2 - 4(2)(-3) = 73.\]

The discriminant is > 0, so \( y = 2x^2 - 7x - 3 \) intersects the \( x \)-axis at two distinct points.

**Use the quadratic formula to find the values of the roots.**

The graph of \( y = 2x^2 - 7x - 3 \) intersects the \( x \)-axis when \( 2x^2 - 7x - 3 = 0 \).

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{where } a = 2, \; b = -7, \; \text{and } c = -3
\]

\[
x = \frac{7 \pm \sqrt{73}}{4} \Rightarrow x = \frac{7 + \sqrt{73}}{4} \quad \text{or} \quad x = \frac{7 - \sqrt{73}}{4}
\]

So the graph of \( f(x) = 2x^2 - 7x - 3 \) crosses the \( x \)-axis at two distinct points:

\[
\left( \frac{7 + \sqrt{73}}{4}, 0 \right) \quad \text{and} \quad \left( \frac{7 - \sqrt{73}}{4}, 0 \right).
\]

The discriminant tells you if the quadratic has 2 real distinct roots, 1 real double root, or no real roots.

1. For each of the following, determine the nature of the roots.

   a. \( y = x^2 - 5x - 1 \)
   b. \( y = x^2 + 2x - 5 \)
   c. \( y = 4x^2 - 4x + 1 \)

   d. \( y = x^2 - 6x + 9 \)
   e. \( y = x^2 - 3x + 4 \)
   f. \( y = x^2 - 4x + 5 \)
2. At how many points does the graph of \( y = x^2 + 3x + 7 \) intersect the \( x \)-axis?

3. At how many points does the graph of \( y = 2x^2 - 5x - 3 \) intersect the \( x \)-axis?

4. At how many points does the graph of \( y = 4x^2 + 28x + 49 \) intersect the \( x \)-axis?

5. How many solutions does \( 2x^2 - 3x + 1 = 0 \) have, and what is their nature?

6. Suppose \( x = 2 \) is a solution of \( 2x^2 - 3x - k = 0 \). Find the value of \( k \) and then the other solution.

7. Given that \( 3x^2 + kx - 10 = 0 \), and that \( x = -5 \) is one of the solutions, find the value of \( k \) and the other solution.

8. If \( y = x^2 - 5x + k \), find the possible values of \( k \) so that the function has two distinct, real roots.

9. If \( y = 2x^2 - kx + 2 \), find the possible value(s) of \( k \) so that the graph touches the \( x \)-axis at one point.

10. If \( y = kx^2 - 2x - 3 \), find the possible value(s) of \( k \) so that the graph touches the \( x \)-axis at one point.

11. For any quadratic equation \( ax^2 + bx + c = 0 \), the sum of the solutions is equal to \( -\frac{b}{a} \), and the product of the solutions is equal to \( \frac{c}{a} \). Show that this is true for \( x^2 - x - 6 = 0 \).
## Example

A baseball is hit from home base with an upward speed of 80 feet per second. The gain in height of the ball from the point it was struck is modeled by the equation $h = -16t^2 + 80t$, where $t$ is the time in seconds.

### a. How far has the ball risen after exactly 2 seconds?

#### Solution

Value of $-16t^2 + 80t$ when $t = 2$:

$$h = -16(2)^2 + 80(2)$$

Substitute 2 into the equation

$$= -64 + 160$$

Include the units in your answer

$$= 96 \text{ feet}$$

### b. After how many seconds is the ball 64 feet above its start height?

#### Solution

$$-16t^2 + 80t = 64$$

Substitute 64 into the equation

$$t^2 - 5t = -4$$

Divide equation by $-16$

$$t^2 - 5t + 4 = 0$$

Factor

$$(t - 1)(t - 4) = 0$$

$t - 1 = 0$ or $t - 4 = 0$

$t = 1 \text{ second and } t = 4 \text{ seconds}$

### c. The ball is caught at the same height that it was struck. After how many seconds is the ball caught?

#### Solution

$$-16t^2 + 80t = 0$$

The height when the ball is struck is 0 — substitute 0 into the equation

$$t^2 - 5t = 0$$

Factor

$t(t - 5) = 0$

$t = 0 \text{ or } t = 5$

$t = 0 \text{ seconds and } t = 5 \text{ seconds}$

Ignore $t = 0$ — this is when the ball is struck

---

1. A ball is dropped from the top of a 400-foot building. Its height $h$ feet in the air at any time $t$ in seconds is described by the equation $h = -16t^2 + 400$.

After how many seconds will the ball hit the ground?

---

2. A car is traveling at 120 feet per second when the driver applies the brakes. The speed, $f$, of the braking car is modeled by the equation $f = -10t^2 - 10t + 120$, where $t$ is the time in seconds. How long will the car take to stop completely?
3. A ball is thrown into the air. If its height in feet is given by \(-16t^2 + 64t\),
at what time \(t\) will the ball reach a height of 64 feet?

4. A ball is thrown downwards from the top of a cliff.
The distance \(d\), in feet, fallen by the ball after \(t\) seconds is modeled by the equation \(d = 16t^2 + 4t\).
How long will it take the ball to fall a distance of 420 feet?

5. The distance, in feet, from a fixed point of an object moving in a straight line
is given by \(p = -5t^2 - 15t + 100\), where \(t\) is the time in seconds.
How much time is required for the object to travel 50 feet?

6. A bus is traveling at 80 feet per second. Suddenly, the driver applies the brakes.
The speed of the braking bus after \(t\) seconds is given by \(f = 80 - 20t^2\).
How long will it take the bus to come to a complete stop?

7. A stone is thrown straight upward from the ground with a speed of 48 feet per second. The height of
the stone at any time of its flight is modeled by \(h = -16t^2 + 48t\), where \(t\) is time in seconds.
a. Find the number of seconds it takes the stone to land on the ground.

b. Find the maximum height reached by the stone, and the time it takes to reach the maximum height.

8. A ball is thrown vertically from the base of a telephone pole. The ball’s height,
in feet, is modeled by the equation \(h = -16t^2 + 64t\), where \(t\) is time in seconds.
If the ball just reaches the top of the telephone pole and goes no further,
find the height of the pole and how long it takes the ball to reach the top of the pole.

9. A ball is shot upwards. Its height in feet, \(h\), after \(t\) seconds,
is given by the equation \(h = -16t^2 + 100t\). How long will it take
the ball to reach a height of 150 feet? Explain your answer(s).

Take care — sometimes only one solution makes sense.
Remember to answer the question clearly.
Remember you can use the quadratic formula if the equation doesn’t factor easily.
See textbook Topic 7.3.1.
1. Mrs. Rodriguez has a small business with a weekly payroll of $800.00. She believes the formula 
   \[ P = t^2 + 60t + 800 \] will tell her the weekly payroll in future years (where \( P \) is the payroll in $ and \( t \) is the time in years). If this is correct, what will her weekly payroll be after 10 years?

   \[ P = t^2 + 60t + 800 \]

2. A research group modeled the sales (in millions of dollars) of cell phones in the United States by the equation \( S = 1500y^2 + 4500 \), where \( y \) is the number of years since 1995. Use their model to estimate the year that sales will reach $388,500 million.

3. Mr. Smith’s company has developed a new product. Mr. Smith believes that the equation \( S = -p^2 + 100p + 1000 \) will give the total daily sales in $, where \( p \) is the selling price of the new product in $. What price will give the maximum dollar amount of the daily sales? What is the maximum amount?

   **Example**

   Frank opened a savings account with $500.00 in 2006. He believes that the amount of money in the savings account \( (S) \) can be modeled by the formula \( S = -2t^2 + 60t + 500 \), where \( t \) is the number of years. In what year will the formula give a maximum amount? What is that maximum amount?

   **Solution**

   \[
   S = -2t^2 + 60t + 500 \\
   S = -2(t^2 - 30t) + 500 \\
   S = -2(t^2 - 30t + 225) + 500 + 450 \\
   S = -2(t - 15)^2 + 950 
   \]

   This means that after 15 years, Frank would have $950.00 in the account. So, the maximum amount would be $950.00, and this would happen in 2021.
4. The daily revenue, in thousands of dollars, for a car manufacturer is modeled by the formula
\[ R = -3p^2 + 60p + 1060, \] where \( p \) is the price of each car in thousands of dollars. At what price must
the cars be sold to receive maximum revenue? What is that maximum revenue?

5. The amount of money, in tens of thousands of dollars, deposited at the local bank since 2000, can be
modeled by the formula \( y = -7x^2 + 280x + 12,000 \), where \( x \) represents the number of years since 2000.
In what year will the maximum deposits be reached? What is this maximum amount?

6. A theater company’s annual profit is modeled by the equation \( P = 0.2x^2 - 6.8x \), where \( x \) is the number
of plays performed in a year. How many plays must the company present in order to make a profit?

7. Mr. Jones estimates his profit starting in 2006 will be given by the formula \( P = 7t^2 - 14t + 10 \) where \( t \)
is number of years and \( P \) is the profit. If this formula is correct, when will Mr. Jones have a profit of
more than $1000.00?

8. A workout gym’s annual profit is modeled by the equation \( P = 0.4x^2 - 7.2x \), where \( x \) is the number of
paying customers. How many paying customers are needed to make a profit?

9. Mrs. Franklin uses the formula \( A = -10m^2 + 60m + 300 \) to predict how the amount an average
customer spends at her store will change over time, where \( A \) is the amount in $, and \( m \) is the number
of months after the customer’s initial purchase.
   a. What is the maximum the average customer will spend according to this model and after how many
   months will it occur?

   b. The model predicts that the amount the average customer spends will eventually fall to $0.00.
   After how many months is this predicted to happen?
1. Determine the value of \( x \) where each of the following is undefined:

   a. \( \frac{5}{x + 3} \) 
   b. \( \frac{7}{x - 5} \) 
   c. \( \frac{3}{x - 2} \) 
   d. \( \frac{7}{5 + x} \) 
   e. \( \frac{3}{7 - x} \) 
   f. \( \frac{-6}{4 - x} \) 
   g. \( \frac{3x}{x - 5} \) 
   h. \( \frac{5x}{x + 4} \) 
   i. \( \frac{7x}{x + 7} \)

Example

Determine the values of \( x \) for which the fraction \( \frac{x + 1}{x^2 - 81} \) is undefined.

Solution

Any fraction \( \frac{a}{b} \) is undefined when \( b = 0 \), so \( \frac{x + 1}{x^2 - 81} \) is undefined when \( x^2 - 81 = 0 \).

\[ x^2 - 81 = 0 \]
\[ (x - 9)(x + 9) = 0 \]

Solve the equation by factoring

\[ x - 9 = 0 \] or \( x + 9 = 0 \)
\[ x = 9 \] or \( x = -9 \)

So, the expression is undefined when \( x = -9 \) or \( x = 9 \).

2. Determine the values of the variable for which each of the fractions in Exercises a. – d. is undefined.

   a. \( \frac{7}{x^2 + 2x} \) 
   b. \( \frac{4x}{x^2 - 2x - 3} \) 
   c. \( \frac{7x}{x^2 + 4x - 12} \) 
   d. \( \frac{-5x}{x^2 + 6x - 16} \)
3. Determine the values of \( x \) for which each of the following is undefined.

\[
\begin{align*}
\text{a. } & \frac{x^2 - x - 2}{x^2 + x - 6} \\
\text{b. } & \frac{2x + 5}{x^2 - 3x + 2} \\
\text{c. } & \frac{3x - 8}{x^2 - 5x + 6} \\
\text{d. } & \frac{7x + 3}{x^2 + 7x + 12} \\
\text{e. } & \frac{2x - 5}{x^2 + 10x + 21}
\end{align*}
\]

4. Determine the values of \( x \) for which each of the following is undefined.

\[
\begin{align*}
\text{a. } & \frac{3x + 4}{2x^2 + 7x + 6} \\
\text{b. } & \frac{x^2 + 3x + 7}{6x^2 + 5x - 6} \\
\text{c. } & \frac{7}{4x^2 - 36x} \\
\text{d. } & \frac{m + 1}{m^3 - 49m}
\end{align*}
\]

5. For what values of \( x \) is the function \( f(x) = \frac{x + 1}{x^2 - 2x - 99} \) undefined?

6. Sally was asked to find where the expression \( \frac{x + 5}{x + 5} \) was undefined. She said, “There is no place where it is undefined because when \( x = -5 \), the fraction becomes \( \frac{0}{0} \) and when you divide into zero you get zero.” Is Sally correct? Explain.
Example

Simplify $\frac{a^2 - a - 2}{a^2 + a - 6}$.

Solution

\[
\frac{(a+1)(a-2)}{(a-2)(a+3)}
\]

Factor the numerator and denominator
Cancel common factors

\[
\frac{a + 1}{a + 3}
\]

1. Simplify each expression in exercises a. – f. to its lowest terms.

a. $\frac{22xy}{44xd}$

b. $\frac{x + 3}{x^2 + x - 6}$

c. $\frac{4 - k^2}{k + 2}$

d. $\frac{x^2 - 9}{x - 3}$

e. $\frac{x + 7}{x^2 + 6x - 7}$

f. $\frac{x + 2}{x^2 + 7x + 10}$

2. Show that $\frac{a^2 - 3a + 2}{a^2 + a - 6} = \frac{a - 1}{a + 3}$ when $a \neq 2$.

3. Simplify $\frac{15 + 2a - a^2}{a^2 - 2a - 15}$.

4. Simplify each expression to its lowest terms:

a. $\frac{6 + m - m^2}{m^2 - 4}$

b. $\frac{10 - 3x - x^2}{15 - 2x - x^2}$
5. Show that \( \frac{2x - 2}{3 - 3x} \) is equal to \( -\frac{2}{3} \) when \( x \neq 1 \).

---

**Example**

What is \( \frac{2x^2 + 2xy - 4y^2}{2x^2 - 2y^2} \) reduced to its lowest terms?

**Solution**

\[
\frac{2(x^2 + xy - 2y^2)}{2(x^2 - y^2)} = \frac{2(x - y)(x + 2y)}{2(x - y)(x + y)} = \frac{x + 2y}{x + y}
\]

Factor the numerator and denominator
Take out any common monomial factors first
Difference of two squares: \( a^2 - b^2 = (a - b)(a + b) \)
Cancel common factors

6. Simplify each expression in exercises a. – d. to its lowest terms.

**a.** \[
\frac{3x - 9}{x^2 - 5x + 6}
\]

**b.** \[
\frac{12 - 4t}{48 - 10t - 2t^2}
\]

**c.** \[
\frac{x^3 - 36x}{x^3 + 9x^2 + 18x}
\]

**d.** \[
\frac{3a^4 - 3a^2}{6a^4 + 12a^3 + 6a^2}
\]

7. Show that \( \frac{a^2k - ak}{k - ak} = -a \) when \( a \neq 1 \).

---

8. Simplify each expression:

**a.** \[
\frac{bx + b - x^2 - x}{x^2 + 2x + 1}
\]

**b.** \[
\frac{x^2 - k^2}{x^2 + x + kx + k}
\]

**c.** \[
\frac{y^2 + 3y + 2}{y^2 + yx + 2y + 2x}
\]
## Multiplying Rational Expressions

### Example

Simplify \( \frac{x^2 - 3x + 2}{x^2 + 2x - 3} \cdot \frac{x^2 - 2x - 15}{x^2 + 2x - 8} \).

**Solution**

\[
\begin{align*}
&= \frac{(x - 2)(x - 1)}{(x + 3)(x - 1)} \cdot \frac{(x - 5)(x + 3)}{(x + 4)(x - 2)} \\
&= \frac{(x - 2)\cancel{(x - 1)}}{(x + 3)\cancel{(x - 1)}} \cdot \frac{(x - 5)(x + 3)}{(x + 4)(x - 2)} \\
&= \frac{x - 5}{x + 4}
\end{align*}
\]

### 1. Multiply and simplify the rational expressions in exercises a. – g.

a. \( \frac{72}{(x + 1)} \cdot \frac{x^2 - 1}{9(x - 1)} \)

b. \( \frac{x^2 - 3x + 2}{x^2 + 3x - 10} \cdot \frac{x^2 + 6x + 5}{x^2 - 8x + 7} \)

c. \( \frac{x^2 - 5x + 6}{x^2 + 3x - 18} \cdot \frac{x^2 + 11x + 30}{x^2 + 7x - 18} \)

d. \( \frac{x^2 - 4}{x^2 - x - 6} \cdot \frac{x^2 - 2x - 3}{x^2 + 6x + 5} \)

e. \( \frac{c^2 - 9}{c^2 + 2c - 15} \cdot \frac{c^2 + 4c - 5}{c^2 + 10c + 21} \)

f. \( \frac{3}{4k + 4} \cdot \frac{4k^2 + 12k + 8}{3k - 15} \)

g. \( \frac{x^2 + 2x - 3}{x^2 + 5x + 6} \cdot \frac{x^2 + 9x + 14}{x^2 + 4x - 21} \)
2. Multiply and simplify:
   a. \( \frac{x^2 - 3x + 2}{x^2 + 6x + 5} \cdot \frac{x^2 + 3x - 10}{x^2 + x - 6} \cdot \frac{x^2 - 9}{x^2 - 5x + 6} \)
   b. \( \frac{x^2 + 5x + 6}{x^2 - 3x - 10} \cdot \frac{x^2 - 2x - 15}{x^2 + x - 6} \cdot \frac{x^2 + 5x - 14}{x^2 - x - 12} \)
   c. \( \frac{x^2 + x - 12}{x^2 - 5x + 6} \cdot \frac{x^2 - 3x - 10}{3x^2 + 15x + 12} \cdot \frac{3x^2 - 12}{x^2 - 2x - 15} \)
   d. \( \frac{3x^3 - 27x}{x^3 - 6x^2 + 9x} \cdot \frac{x^2 + 3x + 2}{3x^2 + 15x + 18} \)

3. Multiply and simplify:
   a. \( \frac{2a^2 + ak - k^2}{2a^2 - 3ak + k^2} \cdot \frac{a^2 + 2ak - 3k^2}{-a - k} \)
   b. \( \frac{x^2 + mx - 2m^2}{x^2 - 5mx - 14m^2} \cdot \frac{mx - 7m^2}{m - x} \)

4. I have a box whose dimensions are \( \frac{x + 2}{3x + 9}, \frac{3x^2 + 12x - 15}{x^2 - 4}, \) and \( \frac{x^2 + x - 6}{3x^2 + 9x - 12} \). What is the volume of the box in terms of \( x \)?

5. You have a triangle which has a height of \( \frac{x^2 + 5x + 6}{x^2 + 2x - 15} \) and a base of \( \frac{2x^2 - 10 + 8x}{2x - 3 + x^2} \). What is the area of the triangle in terms of \( x \)?
Topic 8.2.2
Dividing Rational Expressions

California Standards: 13.0

Example

Simplify \( \frac{k^2 - 25}{k^2 + 3k - 10} \div \frac{3k - 15}{2 - k} \).

Solution

\[
\begin{align*}
\text{Rewrite the division as a multiplication} & \quad \text{by the reciprocal of the divisor.} \\
\text{Factor and cancel out common terms} & \quad \text{in the numerators and denominators.} \\
\end{align*}
\]

\[
= \frac{k^2 - 25}{k^2 + 3k - 10} \cdot \frac{2 - k}{3k - 15} \\
= \frac{(k - 5)(k + 5)}{(k + 5)(k - 2)} \cdot \frac{2 - k}{3(k - 5)} \\
= -\frac{1}{3}
\]

1. Simplify the rational expressions in exercises a. – e.

   a. \( (m^2 - 16) \div \left( \frac{m + 4}{3} \right) \)

   b. \( \frac{k^2 + 12k + 36}{k + 6} \div \frac{k^2 - 36}{-k + 6} \)

   c. \( \frac{c^2 - 1}{c^2 - 3c + 2} \div \frac{-c + 3}{c^2 - 5c + 6} \)

   d. \( \frac{k^2 + k - 6}{k^2 + 3k - 10} \div \frac{-3 - k}{k^2 + 4k - 5} \)

   e. \( \frac{3x^2 + 7x - 20}{x^2 + 3x - 4} \div \frac{3x^2 - 2x - 5}{x^2 - 2x + 1} \)

2. Divide \( \frac{x^2 - 5x + 6}{x^2 - 2x - 3} \) by \( \frac{x^2 + x - 20}{x^2 + 6x + 5} \).

3. Reduce \( \left( \frac{x - 5}{x + 7} \div \frac{x^2 - 49}{x^2 - 25} \right) \div \frac{x - 7}{x + 5} \) to its lowest terms.
4. Simplify \[ \frac{2x^2 - x - 3}{2x^2 - 5x + 3} \div \frac{3x^2 - x - 4}{x^2 - 1} \cdot \frac{6x - 8}{-x - 1}. \]

5. Reduce \[ \frac{x^2 - 4x + 3}{x^2 - 1} \cdot \frac{x^2 + 3x + 2}{x^2 - 5x + 6} \div \frac{-2x - 4}{x^2 - 3x + 2} \] to its lowest terms.

6. Simplify \[ \frac{x^2 - 3x + 2}{x^2 - 4x + 3} \div \frac{x^2 + 5x + 6}{x^2 - x - 6} \cdot \frac{x^2 - x - 12}{x^2 - 3x - 4}. \]

7. James makes a circular spinner with a circumference of \( \frac{x^2 - 4}{x^2 - 7x^2 + 12x} \). He splits the spinner into two parts, “win” and “lose”. He decides that the edge length of the “win” section should be \( \frac{x^2 + 5x + 6}{x^2 - x - 12} \). What is the probability, in terms of \( x \), that a spin of the spinner will “win”? 

Remember, the probability of a fair spinner landing on a certain section is the edge length of that section divided by the entire edge length of the spinner.

8. Ann is watching birds fly through a glassless window in a barn. The entire window is rectangular in shape and has a width \( a \) of \( \frac{x + 5}{3x + 9} \) and a height \( b \) of \( \frac{x + 3}{x - 4} \). The window is divided into smaller rectangular panes, one of which has a width \( c \) of \( \frac{x + 5}{6x - 24} \) and height \( d \) of \( \frac{x - 1}{x + 2} \). Assuming that the likelihood of the bird entering through a particular pane is directly proportional to the area of that pane, what is the probability, in terms of \( x \), of the next bird flying in through the pane measuring \( c \) by \( d \)?
Topic 8.3.1 Fractions with Identical Denominators

California Standards: 13.0

Example

Simplify the expression \( \frac{2x + 3}{5} + \frac{3x + 7}{5} \).

Solution

\[
\frac{2x + 3}{5} + \frac{3x + 7}{5} = \frac{(2x + 3) + (3x + 7)}{5}
= \frac{2x + 3 + 3x + 7}{5}
= \frac{5x + 10}{5}
= x + 2
\]

Both fractions have an identical denominator, so can be placed in the same fraction.

Remove the parentheses

Group all like terms

Cancel any common factors

1. Simplify each of the following:
   
   a. \( \frac{a}{2} + \frac{b}{2} \)
   
   b. \( \frac{r}{5} + \frac{s}{5} \)
   
   c. \( \frac{2v}{7} + \frac{5v}{7} \)
   
   d. \( \frac{13q}{5} - \frac{3q}{5} \)

2. Simplify each of the following:
   
   a. \( \frac{x + 3}{7} + \frac{x + 2}{7} \)
   
   b. \( \frac{x - 4}{5} - \frac{x + 2}{5} \)
   
   c. \( \frac{4x + 3}{8} + \frac{4x + 5}{8} \)
   
   d. \( \frac{7x - 3}{3} - \frac{4x + 9}{3} \)
   
   e. \( \frac{2x + 3}{3x} + \frac{3x + 5}{3x} \)
   
   f. \( \frac{4x + 3}{5x} - \frac{2x - 4}{5x} \)
   
   g. \( \frac{6x - 5}{4x} + \frac{2x + 1}{4x} \)
   
   h. \( \frac{7x + 4}{3x} - \frac{4x - 11}{3x} \)
3. Simplify each of the following:
   a. \( \frac{3x + 7}{x + 1} + \frac{2x + 5}{x + 1} \)  
   b. \( \frac{5x + 3}{x - 5} - \frac{2x + 7}{x - 5} \)  
   c. \( \frac{3x - 2}{4x + 1} + \frac{5x + 4}{4x + 1} \)  
   d. \( \frac{7x - 6}{x - 1} - \frac{3x - 2}{x - 1} \) 

4. Simplify each of the following:
   a. \( \frac{2x^2 + 5x + 6}{(x + 3)(x - 4)} - \frac{-x^2 + 2x + 6}{(x + 3)(x - 4)} \)  
   b. \( \frac{3x^2 + 2x + 5}{(x - 1)(x - 2)} - \frac{-2x^2 - 6x - 2}{(x - 1)(x - 2)} \)  
   c. \( \frac{5x^2 - 3x - 9}{(x + 2)(x + 3)} - \frac{4x^2 - 2x - 3}{(x + 2)(x + 3)} \)  
   d. \( \frac{4x^2 - 2x + 3}{(x - 2)(2x + 3)} - \frac{2x^2 - 9x - 3}{(x - 2)(2x + 3)} \)  

5. Simplify each of the following:
   a. \( \frac{3x^2 + 4x + 8}{x^2 - 4} - \frac{-2x^2 - 5x - 2}{x^2 - 4} \)  
   b. \( \frac{4x^2 - 5x + 2}{x^2 - x - 12} - \frac{3x^2 - 6x + 8}{x^2 - x - 12} \)  
   c. \( \frac{x^2 - 3x + 3}{x^2 - 5x + 6} + \frac{2x - 9}{x^2 - 5x + 6} \)  
   d. \( \frac{8x^2 + 3x - 5}{2x^2 - 5x - 12} - \frac{2x^2 + 2x + 7}{2x^2 - 5x - 12} \)  

6. You are given the three numbers: \( \frac{2x + 3}{x} \), \( \frac{5x - 2}{x} \), and \( \frac{3x + 4}{x} \). Find the average, in terms of \( x \), of these three numbers.
Fractions with Different Denominators

California Standards: 13.0

1. Simplify the expressions in exercises a. – f.
   a. \( \frac{a-1}{2} + \frac{a+1}{3} \)
   b. \( \frac{x-3}{4} - \frac{x+1}{5} \)
   c. \( \frac{1}{x-1} - \frac{2}{x+1} \)
   d. \( \frac{3}{h-1} - \frac{2}{h+1} \)
   e. \( \frac{x-1}{x+1} - \frac{5}{2} \)
   f. \( \frac{7}{x-2} - \frac{5}{x+2} \)

Example

Simplify \( \frac{2}{x-2} - \frac{3}{x-5} + \frac{x-1}{x^2-7x+10} \).

Solution

\[
\begin{align*}
\frac{2}{x-2} - \frac{3}{x-5} + \frac{x-1}{x^2-7x+10} \\
= & \quad \frac{2}{x-2} - \frac{3}{x-5} + \frac{x-1}{(x-5)(x-2)} \\
= & \quad \frac{2(x-5) - 3(x-2) + (x-1)}{(x-5)(x-2)} \\
= & \quad \frac{2x-10 - 3x + 6 + x - 1}{(x-5)(x-2)} \\
= & \quad \frac{5}{(x-5)(x-2)} \\
= & \quad \frac{5}{x^2 - 7x + 10}
\end{align*}
\]

Factor the denominators and find their least common multiple (LCM).

Convert each fraction into an equivalent fraction with the LCM as its denominator.

Simplify the expression.
2. Simplify the expressions in Exercises a. – i.

a. \[
\frac{x + 2}{x - 5} - \frac{x - 1}{x^2 - 2x - 15}
\]

b. \[
\frac{2x}{x + 1} - \frac{1}{x - 1} - \frac{2}{x^2 - 1}
\]

c. \[
\frac{y - 2}{y + 3} - \frac{y + 3}{y - 4} + \frac{y^2 - 1}{y^2 - y - 12}
\]

d. \[
\frac{x + 2}{3} - 2 - \frac{3x - 2}{5} + \frac{2}{3}
\]

e. \[
\frac{x^2 + 1}{x + 2} - \frac{x^2 + 2}{x - 2} + \frac{5}{x^2 - 4}
\]

f. \[
\frac{2x - 5}{x + 4} - \frac{x + 3}{x + 5} + \frac{2x - 1}{x^2 + 9x + 20}
\]

g. \[
\frac{5}{x - 3} - \frac{1}{x + 5} + \frac{(x + 1)(x + 3)}{x^2 + 2x - 15}
\]

h. \[
\frac{2x^2 - 7x - 15}{(x + 5)(x^2 - 4x - 5)} + \frac{2x - 3}{x^2 - 25} - \frac{2x + 3}{x^2 + 6x + 5}
\]

i. \[
\frac{21x^2 + 17x + 2}{x(3x + 2)} + (3x - 1) - \frac{3x^2}{(7x + 1)}
\]

3. Three sisters, Joan, Jan, and Jane, keep their money in one account. Joan had \(\frac{3x + 2}{x + 2}\) dollars in the account, and Jan had \(\frac{2x + 3}{x + 5}\) dollars in the account. Jane had no money in the account, but took out \(\frac{3x^2 + 23x + 22}{x^2 + 7x + 10}\) of her sisters’ money. How much money, in terms of \(x\), remains in the account?

4. Alan went from San Francisco to Los Angeles, about 400 miles, averaging \((x + 5)\) mph. On his journey from Los Angeles to Redding, about 500 miles, he averaged \((x - 5)\) mph. Then he returned to San Francisco, about 200 miles, averaging \((x^2 - 25)\) mph. How long did it take him, in terms of \(x\), to make the round trip?

The following equation might be useful in this question if you have forgotten it.

Distance = Speed \times Time
Topic
8.4.1 Solving Fractional Equations

California Standards: 13.0

Example

Solve \( \frac{y+1}{2y-3} - 2 = -\frac{y+3}{2y+3} \) for \( y \). Find any restrictions on \( y \).

Solution

The restrictions on \( y \) are: \( 2y - 3 \neq 0 \) and \( 2y + 3 \neq 0 \). The denominators cannot equal zero

Therefore, \( y \neq \frac{3}{2} \) and \( y \neq -\frac{3}{2} \). Solve each inequality to give the restrictions on \( y \)

\[
\begin{align*}
(y + 1)(2y + 3) &- 2(4y^2 - 9) = -(y + 3)(2y - 3) \\
2y^2 + 3y + 2y + 3 - 8y^2 + 18 &- (2y^2 - 3y + 6y - 9) \\
-6y^2 + 5y + 21 &- 2y^2 - 3y + 9 \\
4y^2 - 8y - 12 &- 0
\end{align*}
\]

Simplify

\( y^2 - 2y - 3 = 0 \)

\( (y - 3)(y + 1) = 0 \)

\( y = 3 \) or \( y = -1 \)

Checking:

\[
\begin{align*}
y &= 3 \\
\frac{y+1}{2y-3} - 2 &= -\frac{y+3}{2y+3} \\
\frac{4}{3} - 2 &= -\frac{2}{3}
\end{align*}
\]

\[
\begin{align*}
y &= -1 \\
\frac{y+1}{2y-3} - 2 &= -\frac{y+3}{2y+3} \\
\frac{0}{-5} - 2 &= -\frac{2}{1}
\end{align*}
\]

1. For each exercise, find any restrictions on the variable and then solve the equation.

a. \( h - 2 = \frac{8}{h} \)

b. \( \frac{1}{x+1} = \frac{1}{x} - \frac{1}{6} \)
c. \[ \frac{3}{x} - \frac{x}{x-2} = -2 \]

d. \[ \frac{x}{x+1} - \frac{x-4}{x-3} = \frac{1}{8} \]

e. \[ \frac{1}{3} + \frac{3}{y+1} = \frac{5}{2y-2} \]

f. \[ \frac{2}{x+1} + \frac{1}{x+2} = \frac{5}{x-1} \]

g. \[ \frac{x}{x+2} - \frac{x-2}{3x-3} = \frac{1}{2} \]

h. \[ \frac{x}{x+1} + \frac{2x}{x+4} = \frac{9}{10} \]

i. \[ \frac{1}{y+2} - \frac{5}{y-1} = -\frac{2}{y+1} \]

You can check that your solutions work by putting them back into the original equation.

2. When 2 is subtracted from a number \( x \), the result is equal to \( \frac{3}{x} \). Find all possible values for \( x \).

3. Joe and Jim are picking oranges in an orange grove. Joe can fill his sack with oranges in 20 minutes. Jim can fill his sack with oranges in 30 minutes. They need one more sack at the end of the day. Working together, how long will it take them to fill the sack?
Applications of Fractional Equations

California Standards: 13.0

1. The sum of a number and its reciprocal is \( \frac{29}{10} \). Find all possible values for the number.

2. The difference between a number and its reciprocal is \( \frac{45}{14} \). Find all possible values for the number.

3. When the reciprocal of a number is subtracted from the number, the result is \( \frac{15}{4} \). Find all possible values for the number.

Example

The denominator of a fraction is three more than its numerator. A second fraction is formed by reducing both the numerator and denominator of the first fraction by five. If the new fraction is subtracted from the original fraction, the result is \( \frac{3}{10} \). Find the original fraction, given that both the numerator and denominator are positive.

Solution

Let \( x \) = numerator of 1st fraction
\( x + 3 \) = denominator of 1st fraction
\( x - 5 \) = numerator of 2nd fraction
\( x + 3 - 5 \) = denominator of 2nd fraction

Equation: \( \frac{x}{x+3} - \frac{x-5}{x-2} = \frac{3}{10} \)

LCM of the denominators is \( 10(x + 3)(x - 2) \).

\[
10x(x - 2) - 10(x - 5)(x + 3) = 3(x + 3)(x - 2)
\]

\[
10x^2 - 20x - 10(x^2 - 2x - 15) = 3(x^2 + x - 6)
\]

\[
10x^2 - 20x - 10x^2 + 20x + 150 = 3x^2 + 3x - 18
\]

\[
150 = 3x^2 + 3x - 18
\]

\[
3x^2 + 3x - 168 = 0
\]

\[
x^2 + x - 56 = 0
\]

\[
(x + 8)(x - 7) = 0
\]

\[
x = -8 \quad \text{or} \quad x = 7
\]

\[
x = 7 \quad \text{The numerator is positive, so} \quad x = 7.
\]

The original fraction is \( \frac{x}{x+3} = \frac{7}{10} \). Answer the question clearly.
4. The denominator of a fraction is one more than its numerator. A second fraction has a denominator which is twice the numerator of the first fraction, and a numerator which is one less than the numerator of the first fraction. If the second fraction is subtracted from the first, the result is \( \frac{5}{12} \). Find all possible values for the numerator of the first fraction.

5. Two rational numbers have a sum of \( \frac{17}{12} \).
The second number is formed by reducing both the numerator and denominator of the first number by one. If the denominator of the first number is one more than its numerator, find the two fractions.

6. Ann and Melissa are picking grapes in the Napa Valley of California. Ann can fill her sack in 30 minutes and Melissa can fill hers in 45 minutes. They have 15 minutes until quitting time so decide to work together and try to fill one sack before quitting time. Will they fill the sack?

7. Frank can wash the car in \( \frac{2}{3} \) the time his younger brother, Joe, can wash the car. They worked together and washed the car in 40 minutes. How long would it have taken each boy working separately?

Frank ________________ Joe ________________

8. The school bought a new copy machine which makes 20 copies per minute. The old machine makes 12 copies per minute. Francine needs 1000 copies, so she uses both machines. How long will it take to have the copies?

9. John bought a number of notepads for $70. Four of the notepads got spoiled. He sold the remaining notepads at $3 more per pad than he paid, and made a total profit of $10. How many notepads did John buy?
Relations

California Standards: 16.0, 17.0

Example

Set A is defined as \{(3, 4), (5, 3), (4, 4), (7, 3), (a, b), (c, b)\}. Does Set A represent a relation? If so, explain why and give the domain and range. If not, explain why not.

Solution

Yes, this is a relation. Any set of ordered pairs is a relation. The domain is \{3, 5, 4, 7, a, c\}, the range is \{4, 3, b\}.

Remember, every set of ordered pairs, even \{(3, 5)\} is a relation. The domain is the "x" values and the range is the "y" values. Notice that \((x, y)\) and \((\text{domain}, \text{range})\) are both in alphabetical order. That should help remembering that domain and x go together and range and y go together.

1. State the domain and range of the relation \{(-5, 11), (3, 10), (2, -12)\}.

2. This table shows John's age (in years) and height (in inches) at that age, in the form of an input-output table. Does this table show a relation? If so, which column represents the domain and which represents the range?

<table>
<thead>
<tr>
<th>Age</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
</tr>
<tr>
<td>6</td>
<td>37</td>
</tr>
<tr>
<td>8</td>
<td>47</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
</tr>
<tr>
<td>12</td>
<td>60</td>
</tr>
</tbody>
</table>

3. Does the following table show a relation? If so, give the range and domain.

<table>
<thead>
<tr>
<th>a</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

4. Look at the following mapping diagram. Give the range and the domain.
5. Look at the following mapping diagram.
   Give the range and the domain.
   Give the rule which connects the range and the domain in this case.

6. Look at the points on the coordinate plane. Do they represent a relation? If so, give the domain and the range.

7. Find the range of \( f(x) = 2x - 5 \), if its domain is \{-1, 0, 7\}.

8. A domain is given by \{-5, -3, 0, 2, 5\}. The rule connecting the domain to the range is \( x^2 - 5 \). What is the range?

9. Given that the range is \{-11, -5, 1, 7, 16\} and that the rule connecting the domain to the range is \( 3x + 1 \), determine the domain.

10. Roger looked at the set \{(0, 5), (5, 0)\}. He said it was not a relation because all that happened was that the ordered pairs were reversed. Is Roger correct? Explain.

11. A store manager bought a bunch of briefcases, at a cost of $2000, to sell just before the start of school. The manager calculated the profits using the equation \( P = 12b - 2000 \), where \( P \) is the profit and \( b \) is the number of briefcases sold.
   a. Does this rule represent a relation? If so, which variable represents the domain and which the range? If not, explain why not.
   b. How many briefcases must be sold before there is a profit?
Determine whether the relation $v = \{(-2, 3), (3, 5), (4, 10)\}$ is a function or not. Explain your reasoning.

**Solution**

$v$ is a function since no two ordered pairs have the same first entry ($x$-value) but a different second entry ($y$-value).

1. Determine whether each relation in exercises a. – d. is a function or not. Explain your reasoning.

   a. $f = \{(1, 5), (2, 5), (3, 5)\}$

   b. $k = \{(c, 2), (c, 11), (c, 9)\}$

   c. $h = \{(x, y): x \in \{-2, -1, 0, 1\} \text{ and } y = x + 1\}$

   d. $m = \{(x, \pm \sqrt{x})\}$

2. Use the domain $\{0, 1, 4, 9\}$ to generate sets of ordered pairs for each relation in exercises a. – d. State whether each relation is a function or not.

   a. $r = \{(x, 2x - 1)\}$

   b. $t = \{(x, x^2 - 2)\}$

   c. $h = \{(x, \sqrt{x})\}$

   d. $m = \{(x, \pm \sqrt{x})\}$
3. Determine whether the mapping shown represents a function:

Example

State the domain and range of the mapping illustrated on the right. Determine whether or not the mapping represents a function.

Solution

Domain = \{-3, -1, 1, 5, 6\}
Range = \{cat, hen, chick, duck, dog\}

The mapping does not represent a function since the ordered pairs (1, chick) and (1, dog) have the same first entry but different second entries.

4. In exercises a. – d., determine the domain and range of the relation.
State whether the relation is a function or not.

a. \(h = \{(0, 2), (-2, 0), (2, 0), (0, -2)\}\)

b. 

You can use the vertical line test to see whether a graph represents a function.

c. \(k(x) = \{(x, y): y = \pm \sqrt{x - 1}, x \in \{1, 5, 10, 17, 26\}\}\)

d. 

5. Explain the difference between a relation and a function.
Topic 8.5.3

Function Notation

California Standards:  16.0, 17.0

Example

Given the functions \( f(x) = x + 4 \), and \( g(x) = 2x - 3 \), find \( f(3) + g(4) \).

Solution

Since \( f(x) = x + 4 \), then \( f(3) = 3 + 4 = 7 \)

Since \( g(x) = 2x - 3 \), then \( g(4) = (2 \times 4) - 3 = 8 - 3 = 5 \)

Hence \( f(3) + g(4) = 7 + 5 = 12 \)

1. Given \( f(x) = 3x \), determine each of the following:
   a. \( f(7) \)  
   b. \( f(-5) \)  
   c. \( f(3) \)  
   d. \( f(-2) \)

2. Given \( h(x) = 2x + 3 \), determine each of the following:
   a. \( h(4) \)  
   b. \( h(-3) \)  
   c. \( h(0) \)  
   d. \( h(-7) \)

3. Given \( f(x) = x^2 + 3x \), \( g(x) = 4x + 2 \), and \( h(x) = 2x^2 - 5 \), determine each of the following:
   a. \( f(3) - g(2) \)  
   b. \( g(5) \times h(2) \)  
   c. \( \frac{f(4)}{x} \)  
   d. \( f(6) + h(6) \)

4. Given \( f(x) = 3x + 4 \), and \( k(x) = x^2 - 3 \), determine \( f(r + 4) + k(r - 3) \).
5. Given \( m(x) = 3x^2 - 2x + 5 \) and \( n(x) = 2x^2 - 3x + 4 \), determine \( m(s - 3) - n(s + 4) \).

6. Determine \( f(a + b) + g(a + b) \) for \( f(x) = x^2 + 3x + 5 \) and \( g(x) = 2x^2 - 3x + 8 \).

7. Joan’s work for determining \( g(r^2 + 2) \) for the function \( g(x) = x^2 + 3 \) is as follows:

\[
g(r^2 + 2) = r^2 + 2 + 3
\]

Was Joan correct? Explain.

8. The function \( m(w) = \frac{1}{7} \times w \) gives your approximate weight on the Moon as compared with Earth, where \( w \) is your weight on Earth.
   a. If you weighed 133 pounds on Earth, what would your Moon-weight be?
   b. If you weighed 22 pounds on the Moon, what would your Earth-weight be?

9. You design a toy that you are going to manufacture and sell. It costs you $250 for the manufacturing equipment. Your costs for each toy are $5 in labor and $4 in materials. What is your cost function for manufacturing the toys? Using this function, how much will it cost to build 100 toys?
More on Functions

California Standards: 16.0, 17.0

1. Joe is given the two functions, \( f(x) = 4x^2 - 9 \) and \( g(x) = (2x + 3)(2x + 3) \), where \( x \in \{-3, -1, 2, 4, 5\} \) in each case. Are the two functions equal? Joe thinks they are, because they have the same domain. Is Joe correct? Explain.

2. Use the function \( f(x) = \frac{x^3 - x + 1}{x^2 - 3x - 40} \) to answer exercises a. – c.
   
   a. Evaluate \( f(-2) \).
   
   b. Determine any restrictions on the domain of \( f(x) \).
   
   c. Evaluate \( f(a) \).

3. In each exercise, let \( h(x) = x^2 - 2x - 8 \) and \( g(x) = x - 4 \). Evaluate each function as indicated.
   
   a. \( h(-1) \)
   
   b. \( g(a + 4) \)
   
   c. \( h(2) - g(-2) \)
   
   d. \( h(x) + g(x) \)
   
   e. \( g(c) - h(c - 1) \).
4. Given \( f(x) = \frac{x^4 - 4}{x^2 + 2} \), find \( f(-2) \).

5. Given \( h(x) = \frac{1}{x^2 - 5} \), find \( h(2) \).

6. If \( f(x) = 3x - 7 \), find \( f(h + 1) - f(1) \).

7. Given \( f(x) = 2x - 3 \), find \( \frac{f(x + h) - f(x)}{h} \).

8. Determine the domain of \( h(x) = \sqrt{3x - 9} \), given that the domain contains all real values for which the function is defined.

9. Examine the graph of the function. Describe the domain and range of the function.

10. Determine the domain of the following function.

\[
f(x) = \frac{\sqrt{2x^2 - 32}}{x - 9}
\]
Section 1.1

Topic 1.1.1
1. a. D = {3, 6, 9}
   b. 8 ∈ N

Topic 1.1.2
1. a. 11
   b. 0, 11
   c. −7, −1, 0, 11

Topic 1.1.3
1. a. {–5, 0, 1, 1, e}
   b. {1, 0, −5, 1, 2, e}
   c. {–5, 0, 1, 2, e}

Topic 1.1.4
1. a. 834 ÷ 6 = 139
   b. 17(3) = 51
   c. 456 + 44 = 500
   d. 400 − 220 = 180

Topic 1.1.5
1. a. 3
   b. 1
   c. −2
   d. 10

Section 1.2

Topic 1.2.1
1. a. Reflexive property
   b. Symmetric property
   c. Transitive property

Topic 1.2.2
1. a. x = 1
   b. x = 1
   c. x = 0
   d. x = 0

Topic 1.2.3
1. a. 5
   b. 1
   c. 8
   d. −1
   e. −7

Topic 1.2.4
1. a. Positive because both addends are positive. 5 + 18 = 23
   b. Negative because both addends are negative. −5 + (−16) = −21
   c. Negative because [−4]>[2], (−4) + 2 = −2
   d. Positive because |9| > |−1|.

Topic 1.2.5
1. a. −6 + 7
   b. 18 + (−9)
   c. 25 + 14

Topic 1.2.6
1. a. 11 − 9 = 2
   b. −4 + (−4) = −8
   c. 4 − 10 = −6
   d. 8 + (−1) − 12 = −5
   e. 7 + [2 + (−2) − 5] = 7 − 5 = 2
   f. [12 + 2] − 2 = 14 − 2 = 12
   g. 16 − {7 + 9} = 16 − 16 = 0

Topic 1.2.7
1. a. Commutative Property of Multiplication
   b. Associative Property of Addition
   c. Distributive Property
   d. Commutative Property of Addition

Topic 1.2.8
1. a. 6
   b. 27y
   c. −(18 + u) = −18 − u
   d. −(39 − 8h) = 39 + 8h
   e. −(12k + 27) = −12k − 27
   f. −(x + 2y) = x − 2y

Topic 1.2.9
1. a. Distributive property of multiplication over addition
   b. Associative property of multiplication
   c. Inverse property of multiplication
   d. Commutative property of addition
   e. Inverse property of addition / definition of subtraction
   f. Commutative property of multiplication
   g. Commutative property of addition
   h. Distributive property of multiplication over addition
   i. Identity property of addition
   j. Associative property of addition
   k. Inverse property of addition
   l. Identity property of multiplication

Section 1.3

Topic 1.3.1
1. a. 5 × 5 × 5 × 5 × 5 = 5^5
   b. 2 × 2 × 2 × 2 × 2 × 2 = 2^6
   c. 7 × p × 7 × p × 7 × p = 7^3p^3 OR (7p)^3
   d. 8 × w × y × x × y × x × y = 8^3w^3y^3
   e. 6 × r × 6 × r × r × r = 6^2r^3
   f. 10 × m × n × m × 10 × m = 10^2m^3n
Topic 1.3.2
1. a. sixth, \(w\)
   b. \(x + y\)
   c. 15
   d. \(\frac{1}{5}\)

Topic 1.3.3
1. a. \(\sqrt{36} = 6\sqrt{2}\)
   b. \(\sqrt{16} = 4\sqrt{3}\)
   c. \(\sqrt{25} = 5\sqrt{2}\)
   d. \(\sqrt{4} = 2\sqrt{3}\)
   c. \(\sqrt{36} = 6\sqrt{3}\)

Topic 1.3.4
1. a. 36
   b. 16
   c. 25
   d. 42
   e. 36

Topic 1.3.5
1. a. \[\left(\frac{16}{3}\right) \cdot \left(\frac{3}{4}\right) \cdot \left(\frac{3}{2}\right) \cdot \frac{1}{3} = \frac{1}{3}\]
   b. \[\frac{\sqrt{18} \cdot \sqrt{6} \cdot 3}{\sqrt{8} \cdot \sqrt{2} \cdot \sqrt{2}} = \frac{5}{6} = \frac{5}{18}\]
   c. \[\frac{5}{8} \cdot \frac{\sqrt{11}}{\sqrt{8}} = \frac{21}{8} = 88\]
   d. \[\frac{\sqrt{3} \cdot \sqrt{2} \cdot \sqrt{1}}{\sqrt{8} \cdot \sqrt{4} \cdot \sqrt{2}} = \frac{1}{3}\]
   e. \[\frac{\sqrt{11} \cdot \sqrt{5}}{\sqrt{24} \cdot \sqrt{10}} = \frac{11}{24} = \frac{11}{20}\]

Topic 1.3.6
1. a. \(\frac{3 + 9}{14} = \frac{12}{14} = \frac{6}{7}\)
   b. \(\frac{4 + 8}{9} = \frac{12}{9} = \frac{4}{3}\)
   c. \[\frac{18 - 25}{27} = \frac{7}{27}\]
   d. \[\frac{43 - 31}{40} = \frac{12}{40} = \frac{3}{10}\]
   e. \[\frac{2 - 5}{14} = \frac{10 + 13}{14} = \frac{23}{14}\]

Section 1.4

Topic 1.4.1
1. a. The missing answers are:
   - Given equation
   - Associative property of multiplication
   - Dividing
   - Identity property of multiplication
   b. The missing answers are:
   - Adding
   - Adding
   - Adding

Topic 1.4.2
1. a. \(x = 3n\)
   b. \(x = n + 7\)
   c. \(x = 5n - 2\)

Topic 1.4.3
1. a. \(x = 9\) or \(x = -9\)
   b. \(x = -9\)
   c. \(x = 9\)
   d. \(x = 2\) or \(x = -2\)

Section 2.1

Topic 2.1.1
1. a. 4
   b. 5
   c. 1
   d. 2

Topic 2.1.2
1. a. \(2(x + 3) = 2x + 6\)
   b. \(5(x + 3) + 2(6 + x) = 5x + 15 + 2x = 7x + 27\)
   c. \(4(x - 2) + (x + 6) = 4x - 8 + x + 6 = 5x - 2\)

Topic 2.1.3
1. a. \(x(x + 3) = x^2 + 3x\)
   b. \(-4x(x + 1) = -4x^2 - 4x\)
   c. \(5x(x + 2) - x(7 + x) = 5x^2 + 10x - 7x - x^2 = 4x^2 + 3x\)
   d. \(x(5 + x) - 2x(x - 3) = 5x + x^2 - 2x^2 + 6x = 11x - x^2\)
   e. \(2(x - 4) + 2x(5 - x) = 2x^2 - 8x + 10x - 2x^2 = 2x\)
   f. \(-3x(-4x) + x(x - 1) = 12x^2 + x^2 - x = 13x^2 - x\)

Section 2.2

Topic 2.2.1
1. a. \(a + 6 = 12, a = 12 - 6, a = 6\)
   b. \(a - 17 = 4 + 2, a - 17 = 6, a = 6 + 17, a = 23\)
   c. \(7 - a = 16, -a = 16 - 7, -a = 9, a = 9\)
   d. \[\frac{1}{4} = a - \frac{2}{4}, \frac{1 + 2}{4} = a, \frac{3}{4} = a, a = \frac{3}{4}\]
Section 2.3

Topic 2.3.1
1. a. Prime factorizations of 5, 3, 6, and 2 are 5, 3, 3 \times 2, and 2.
   So LCM is $5 \times 3 \times 2 = 30$
   b. Prime factorizations of 7 and 3 are 7 and 3.
   So LCM is $7 \times 3 = 21$
   c. Prime factorizations of 9, 3 and 5 are $3 \times 3$, 3, and 5.
   So LCM is $3 \times 3 \times 5 = 45$
   d. Prime factorizations of 2, 7, 6, and 4 are 2, 7, $2 \times 3$, and $2 \times 2$.
   So LCM is $2 \times 7 \times 3 \times 2 = 84$

Topic 2.3.2
1. a. Substitute $x = 2$ into the equation:
   \[
   \frac{2}{3} - \frac{1}{7} = \frac{3}{7} \cdot 2 + \frac{1}{3}
   \]
   Multiply both sides by 21.
   \[
   7 \cdot 4 - 3 \cdot 1 = 3 \cdot 6 + 7 \cdot 1
   \]
   $28 - 3 = 18 + 7$
   $25 = 25$
   LHS = RHS, so $x = 2$
   is a solution of the equation

b. Substitute $x = 30$ into the equation:
   \[
   \frac{30 + 7}{4} \cdot \frac{30 - 3}{3} = \frac{1}{4}
   \]
   Multiply both sides by 12.
   \[
   3 \cdot 37 - 4 \cdot 27 = 3
   \]
   $111 - 108 = 3$
   $3 = 3$
   LHS = RHS, so $x = 30$
   is a solution of the equation

c. Substitute $x = -1$ into the equation:
   \[
   \frac{-6(-1) + 1}{6} + \frac{(-1) + 5}{3} = \frac{9 - (-1)}{4}
   \]
   Multiply both sides by 12.
   \[
   2 \cdot 7 + 4 \cdot 4 = 3 \cdot 10
   \]
   $14 + 16 = 30$
   $30 = 30$
   LHS = RHS, so $x = -1$
   is a solution of the equation

Topic 2.3.3
1. a. $10^0$
   b. $10^0$
   c. $10^0$

Section 2.4

Topic 2.4.1
1. $4x + 11 = -17$
   $4x = -28$
   $x = -7$

Topic 2.4.2
1. Let $n$ be the number of nickels and $d$ the number of dimes.
   \[
   n + d = 23
   0.05n + 0.10d = 1.65
   \]
   $n = 23 - d$
   $0.05(23 - d) + 0.10d = 1.65$
   $5(23 - d) + 10d = 165$
   $115 - 5d + 10d = 165$
   $5d = 50$
   $d = 10$
   $n = 23 - 10$
   $n = 13$
   Andres has 13 nickels and 10 dimes.

Section 2.5

Topic 2.5.1
1. a. Let $x$ = first integer
   $x + 1$ = second integer
   $x + (x + 1) = 91$
   $2x + 1 = 91$
   $2x = 90$
   $x = 45$
   $45, 46$

   b. Let $x$ = first integer
   $x + 1$ = second integer
   $x + 2$ = third integer
   $x + (x + 1) + (x + 2) = -51$
   $3x + 3 = -51$
   $3x = -54$
   $x = -18$
   $-18, -17, -16$

   c. Let $x$ = first integer
   $x + 1$ = second integer
   $x + 2$ = third integer
   $x + 3$ = fourth integer
   $x + (x + 1) + (x + 2) + (x + 3) = 162$
   $4x + 6 = 162$
   $4x = 156$
   $x = 39$
   $39, 40, 41, 42$

   d. Let $x$ = first integer
   $x + 1$ = second integer
   $x + 2$ = third integer
   $x + 3$ = fourth integer
   $x + (x + 1) + (x + 2) = 0$
   $3x + 3 = 0$
   $3x = -3$
   $x = -1$
   $-1, 0, 1$
**Topic 2.5.2**

1. **Present ages:**
   - Let \( x = \) Jeremiah’s age
   - \( x + 8 = \) Raymond’s age
   - In 15 years:
     - \( x + 15 = \) Jeremiah’s age
     - \( x + 23 = \) Raymond’s age

   **Equation:**
   \[
   x + 15 + x + 23 = 96
   \]
   \[
   2x + 38 = 96
   \]
   \[
   2x = 58
   \]
   \[
   x = 29 \text{ years old}
   \]
   Jeremiah is 29 years old, Raymond is 37 years old (29 + 8).

**Topic 2.5.3**

1. **a. speed = distance ÷ time**
   \[
   = 420 ÷ 7
   \]
   \[
   = 60 \text{ mph}
   \]
   **b. distance = speed × time**
   \[
   = 45 × 2
   \]
   \[
   = 90 \text{ miles}
   \]
   **c. time = distance ÷ speed**
   \[
   = 880 ÷ 55
   \]
   \[
   = 16 \text{ hours}
   \]

**Section 2.6**

**Topic 2.6.1**

1. **a. \( I = pr \)**
   \[
   = 2500 × 0.09
   \]
   \[
   = 225
   \]
   The return would be $225.
   **b. \( I = pr \)**
   \[
   = 1750 × 0.12
   \]
   \[
   = 210
   \]
   The return would be $210.
   **c. \( I = pr \)**
   \[
   = 1400 × 0.045
   \]
   \[
   = 63
   \]
   The return would be $63.

**Topic 2.6.2**

1. Let \( x = \) percent of brightening agent
   \[
   x = 0.72 + 3.6
   \]
   \[
   x = 0.2
   \]
   \[
   x = 20\%
   \]

**Topic 2.6.3**

1. Let \( x = \) volume of 30% phosphoric acid solution
   then \((10 - x) = \) volume of water

   **Equation:**
   \[
   0.3x + 0(10 - x) = 0.15(10)
   \]
   \[
   0.3x = 1.5
   \]
   \[
   x = 5
   \]
   5 liters of the 30% solution and 5 liters of water.

**Section 2.7**

**Topic 2.7.1**

1. **a. \( \frac{1}{3} \) = work rate for Machine A**
   **b. \( \frac{1}{4} \) = work rate for Machine B**
   **c. \( \frac{1}{20} \) = Henry’s work rate**
   **d. \( \frac{1}{60} \) = Guadalupe’s work rate**
   **e. \( \frac{1}{80} \) = Adiella’s work rate**
   **f. \( \frac{1}{50} \) = Mother’s work rate**

   **Equation:**
   \[
   \frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}
   \]
   \[
   1 ÷ \frac{7}{12} = 1 \frac{5}{7} \text{ hours}
   \]
   for the machines to drain the well together.

   **Section 2.7**

   **Topic 2.7.1**

   1. **a. \( \frac{1}{3} \) = work rate for Machine A**
   **b. \( \frac{1}{20} \) = Henry’s work rate**
   **c. \( \frac{1}{60} \) = Guadalupe’s work rate**
   **d. \( \frac{1}{80} \) = Adiella’s work rate**
   **e. \( \frac{1}{50} \) = Mother’s work rate**

   **Equation:**
   \[
   \frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}
   \]
   \[
   1 ÷ \frac{7}{12} = 1 \frac{5}{7} \text{ hours}
   \]
   for the machines to drain the well together.

   **b. \( \frac{1}{20} \) = Henry’s work rate**
   **c. \( \frac{1}{60} \) = Guadalupe’s work rate**
   **d. \( \frac{1}{80} \) = Adiella’s work rate**
   **e. \( \frac{1}{50} \) = Mother’s work rate**

   **Equation:**
   \[
   \frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}
   \]
   \[
   1 ÷ \frac{7}{12} = 1 \frac{5}{7} \text{ hours}
   \]
   for the machines to drain the well together.
# Topic 2.7.2

1. a. Let $x$ = time for the two to paint the mural together

   \[
   \frac{1}{7} = \text{Miguel’s work rate} \\
   \frac{1}{4} = \text{Alfonso’s work rate} \\
   \frac{1}{x} = \text{combined work rate}
   \]

   Equation:
   \[
   \frac{1}{7} + \frac{1}{4} = \frac{1}{x}
   \]
   \[
   28x\left(\frac{1}{7} + \frac{1}{4}\right) = \frac{28x}{x}
   \]
   \[
   4x + 7x = 28
   \]
   \[
   11x = 28
   \]
   \[
   x = \frac{28}{11} = 2 \frac{6}{11}
   \]

   It would take Miguel and Alfonso $2 \frac{6}{11}$ days to paint the mural together.

b. Let $x$ = time for the tank to fill if both pipes are open

   \[
   \frac{1}{30} = \text{Pipe A’s work rate} \\
   \frac{1}{5} = \text{Pipe B’s work rate} \\
   \frac{1}{x} = \text{combined work rate}
   \]

   Equation:
   \[
   \frac{1}{30} - \frac{1}{5} = \frac{1}{x}
   \]
   \[
   30x\left(\frac{1}{30} - \frac{1}{5}\right) = \frac{30x}{x}
   \]
   \[
   6x - x = 30
   \]
   \[
   5x = 30
   \]
   \[
   x = 6
   \]

   It would take 6 hours to fill the tank.

c. Let $x$ = time for the two to clean the house and yard together

   \[
   \frac{1}{2} = \text{Lorraine’s work rate} \\
   \frac{1}{3} = \text{Andrew’s work rate} \\
   \frac{1}{x} = \text{combined work rate}
   \]

   Equation:
   \[
   \frac{1}{2} + \frac{1}{3} = \frac{1}{x}
   \]
   \[
   6x\left(\frac{1}{2} + \frac{1}{3}\right) = \frac{6x}{x}
   \]
   \[
   3x + 2x = 6
   \]
   \[
   5x = 6
   \]
   \[
   x = \frac{6}{5}
   \]

   It would take $1 \frac{1}{5}$ hours to clean the house and yard.

d. Let $x$ = time for the two to mow the grass

   \[
   \frac{1}{6} = \text{James’ work rate} \\
   \frac{1}{10} = \text{Sophie’s work rate} \\
   \frac{1}{x} = \text{combined work rate}
   \]

   Equation:
   \[
   \frac{1}{6} + \frac{1}{10} = \frac{1}{x}
   \]
   \[
   60x\left(\frac{1}{6} + \frac{1}{10}\right) = \frac{60x}{x}
   \]
   \[
   10x + 6x = 60
   \]
   \[
   16x = 60
   \]
   \[
   x = \frac{60}{16} = 3 \frac{3}{4}
   \]

   It would take $3 \frac{3}{4}$ hours to mow the grass.

# Section 2.8

## Topic 2.8.1

1. \( W = 5, \ X = 11, \ Y = 6, \ Z = 8 \)

## Topic 2.8.2

1. a. \( 3x + 5 = 14 \) or \( 3x + 5 = -14 \)

   \[
   3x = 9 \quad \text{or} \quad 3x = -19
   \]

   \[
   x = 3 \quad \text{or} \quad x = -\frac{19}{3}
   \]

b. \( \frac{4}{7}x = 4 \) or \( \frac{4}{7}x = -4 \)

   \[
   4x = 28 \quad \text{or} \quad 4x = -28
   \]

   \[
   x = 7 \quad \text{or} \quad x = -7
   \]

c. \( 2x - 13 = 5 \) or \( 2x - 13 = -5 \)

   \[
   2x = 18 \quad \text{or} \quad 2x = 8
   \]

   \[
   x = 9 \quad \text{or} \quad x = 4
   \]

d. \( |4x + 3| + 1 = 10 \)

   \[
   |4x + 3| = 9
   \]

   \[
   4x + 3 = 9 \quad \text{or} \quad 4x + 3 = -9
   \]

   \[
   4x = 6 \quad \text{or} \quad 4x = -12
   \]

   \[
   x = \frac{3}{2} \quad \text{or} \quad x = -3
   \]

e. \( |2x - 7| - 3 = 8 \)

   \[
   |2x - 7| = 11
   \]

   \[
   2x - 7 = 11 \quad \text{or} \quad 2x - 7 = -11
   \]

   \[
   2x = 18 \quad \text{or} \quad 2x = -4
   \]

   \[
   x = 9 \quad \text{or} \quad x = -2
   \]

# Section 3.1

## Topic 3.1.1

1. a. C 2

   b. E 4

   c. B 1

   d. A 3

   e. D 5
**Topic 3.1.2**

1. a. D
   b. E
   c. B
   d. A
   e. C

**Topic 3.1.3**

1. a. $\frac{x}{6} < -4$
   
   $x < -4 \times 6$
   
   $x < -24$

   b. $\frac{2}{3}x \geq 8$

   $x \geq \frac{(8 \times 3)}{2}$

   $x \geq \frac{24}{2}$

   $x \geq 12$

   c. $10 < \frac{1}{5}x$

   $10 \times 5 < x$

   $50 < x$

   d. $\frac{x}{9} > \frac{1}{27}$

   $x > \frac{9}{27}$

   $x > \frac{1}{3}$

   e. $\frac{4x}{3} \geq 0$

   $4x \geq 0$

   $x \geq 0$

   f. $\frac{x}{(2 + 3)} \leq \frac{3}{10}$

   $\frac{x}{5} \leq \frac{3}{10}$

   $x \leq \frac{15}{10}$

   $x \leq \frac{3}{2}$

**Section 3.2**

**Topic 3.2.1**

1. Solution Set: $x > 3$

   Interval Notation: $(3, \infty)$

   $4x + 3 > 15$

   $4x > 12$

   $x > \frac{12}{4}$

   $x > 3$

**Topic 3.2.2**

1. $2r - 5 \leq 15$

   $2r \leq 20$

   $r \leq 10$

**Section 3.3**

**Topic 3.3.1**

1. a. $5 \leq 3a - 4 < 17$

   b. $-3 < 12b - 1 < -2$

   c. $5 < 6 - 3c \leq 21$

   d. $-12.3 \leq 2.5d - 1.1 \leq -3$

**Section 3.4**

**Topic 3.4.1**

1. a. $-2 < m < 2$ (conjunction)

   b. $x \leq -7$ or $x \geq 7$ (disjunction)

**Section 4.1**

**Topic 4.1.1**

1. a. $x$-axis, $y$-axis (either order)

   b. $y$-axis

   c. coordinates

   d. all

**Topic 4.1.2**

1. F = quadrant III

   G = $x$-axis

   H = quadrant IV

   J = quadrant II

   K = quadrant IV

   L = $y$-axis

   M = quadrant I

**Topic 4.1.3**

1. a. two

   b. all

   c. no

**Topic 4.1.4**

1. a. $x$-axis

   $y$-axis

   $z$-axis
Section 4.2

Topic 4.2.1
1. a. linear
   b. linear
   c. nonlinear; the y term is squared
   d. nonlinear; x and y are multiplied together
   e. linear

Topic 4.2.2
1. a. e.g. (4, 3): 2x – 8y = –16
   \[2(4) - 8(3) = -16\]
   \[8 - 24 = -16\]
   \[-16 = -16\]
   b. e.g. (1, 5): x + y = 6
   \[1 + 5 = 6\]
   \[6 = 6\]

Topic 4.2.3
1. \[-2x - 3y = -14\]
   \[-2(-k) - 3(5) = -14\]
   \[2k - 15 = -14\]
   \[2k = 1\]
   \[k = \frac{1}{2}\]

Topic 4.2.4
1. | Line  | x-intercept | y-intercept |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>-7</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>-3</td>
<td>-6</td>
</tr>
<tr>
<td>D</td>
<td>doesn’t exist</td>
<td>-3</td>
</tr>
</tbody>
</table>

Topic 4.2.5
1.

Section 4.3

Topic 4.3.1
1. Line A: positive
   Line B: negative
   Line C: negative
   Line D: positive

Topic 4.3.2
1. a. \[y - y_1 = m(x - x_1)\]
   \[y + 3 = 2(x - 4)\]
   \[y + 3 = 2x - 8\]
   \[-2x + y = -11\]
   Equation: \[2x - y = 11\]
   b. \[y - y_1 = m(x - x_1)\]
   \[y - 7 = -5(x - 0)\]
   \[y - 7 = -5x\]
   Equation: \[5x + y = 7\]
   c. \[y - y_1 = m(x - x_1)\]
   \[y - 6 = 0(x - 5)\]
   \[y - 6 = 0\]
   Equation: \[y = 6\]
Section 4.4

Topic 4.4.1
1. Two lines with slopes \( m_1 \) and \( m_2 \) are parallel if \( m_1 = m_2 \).
   Rewrite each equation in the form \( y = mx + b \) to find \( m_1 \) and \( m_2 \).
   
   \[
   6y = 5x - 27 \quad \text{and} \quad 6y = 5x + 11
   \]
   \[
   y = \frac{5}{6}x - \frac{27}{6} \quad \text{and} \quad y = \frac{5}{6}x + \frac{11}{6}
   \]
   \[
   \therefore m_1 = \frac{5}{6} \quad \text{and} \quad m_2 = \frac{5}{6}
   \]
   Since \( m_1 = m_2 \), the lines are parallel.

Topic 4.4.2
1. a. \(-\frac{1}{5}\)
   b. \(\frac{5}{4}\)
   c. \(-\frac{3z}{t}\)
   d. \(-3\)
   e. \(\frac{1}{4}\)

Topic 4.4.3
1. a. no
   b. no
   c. yes
   d. yes
   e. no
   f. yes

Topic 4.4.4
1. a. \( m_1 = 4, \) y-intercept = (0, 8)
   \( m_1 = -4, \) y-intercept = (0, 8)
   Neither
   b. \( m_1 = \frac{1}{5}, \) y-intercept = (0, -15)
   \( m_2 = -5, \) y-intercept = (0, 18)
   Perpendicular
   c. \( m_1 = 4, \) y-intercept = (0, -8)
   \( 2y = 8x - 16 \)
   \( y = 4x - 8 \)
   \( m_2 = 4, \) y-intercept = (0, -8)
   Collinear
   d. \( m_1 = \frac{6}{7}, \) y-intercept = (0, 0)
   \( m_2 = \frac{6}{7}, \) y-intercept = (0, -12)
   Parallel
   e. \( m_1 = \frac{8}{9}, \) y-intercept = (0, -1)
   \( m_2 = \frac{8}{9}, \) y-intercept = (0, 8)
   Perpendicular

Section 4.5

Topic 4.5.1
1. a. Yes
   b. No
   c. No
   d. Yes

Topic 4.5.2
1. A solid line includes the points on the line in the solution set, whereas a dashed line does not.

Topic 4.5.3
1. a. 
   b. 
   c. 
   d. 
   e. 
   f. 

264 Selected Answers
Section 5.1

Topic 5.1.1
1. a. Lines intersect at (0, –5), so solution is $x = 0$ and $y = –5$.
b. Lines intersect at (5, 9), so solution is $x = 5$ and $y = 9$.
c. Lines intersect at (–3, 4), so solution is $x = –3$ and $y = 4$.
d. Lines intersect at (–5, –2), so solution is $x = –5$ and $y = –2$.

Topic 5.1.2
1. a. $y = x – 5$ and $2y + x = 2$
   
   $2(x – 5) + x = 2$
   
   $2x – 10 + x = 2$
   
   $3x = 12$
   
   $x = 4$
   
   $\Rightarrow y = x – 5 = 4 – 5$
   
   $y = –1$

   b. $y = 2x + 1$ and $3y – x = 13$
   
   $3(2x + 1) – x = 13$
   
   $6x + 3 – x = 13$
   
   $5x + 3 = 13$
   
   $5x = 10$
   
   $x = 2$
   
   $\Rightarrow y = 2x + 1 = 2(2) + 1$
   
   $y = 5$

   c. $y = \frac{2}{3}x$ and $6 – 9y = 12x$
   
   $6 – 9\left(\frac{2}{3}x\right) = 12x$
   
   $6 – 6x = 12x$
   
   $6 = 18x$
   
   $x = \frac{1}{3}$
   
   $\Rightarrow y = \frac{2}{3}x = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$
   
   $y = \frac{2}{9}$

   d. $y = 4x – 1$ and $2x – 3y = 8$
   
   $2x – 3(4x – 1) = 8$
   
   $2x – 12x + 3 = 8$
   
   $–10x + 3 = 8$
   
   $10x = –5$
   
   $x = \frac{1}{2}$
   
   $\Rightarrow y = 4x – 1 = 4 \times \frac{1}{2} – 1 = –3$
   
   $y = –3$

Topic 5.1.3
1. $2x + y = 5$ and $x – 3y = –15$
   
   $x – 3y = –15$
   
   $2(3y – 15) + y = 5$
   
   $6y – 30 + y = 5$
   
   $7y = 35$
   
   $y = 5$
   
   $\Rightarrow x = 3y – 15 = (3 \times 5) – 15 = 0$
   
   So the solution is $x = 0, y = 5$.

Topic 5.1.4
1. a. $y = 2x + 4$ and $2y + 4x = 8$
   
   $y = 2x + 4$
   
   $y = 2(4) + 4$
   
   $y = 12$
   
   The lines intersect at (–5, 2), so the solution is $x = –5$ and $y = 2$.

   b. $y = 7$ and $x = 1$
   
   $y = 7$
   
   The lines do not intersect and are parallel so will never intersect. The system is inconsistent.

Topic 5.1.5
1. a. infinite
   
   b. coincide

Topic 5.1.6
1. a. $y + 2 = x$ and $x – y = 1$
   
   $y + 2 = x$
   
   $y = x – 2$
   
   There are infinitely many solutions, so the system is dependent.

   b. $y = 2x$ and $-2x + y = 5$
   
   $y = 2x$
   
   The lines are parallel, so there are no solutions. The system is inconsistent.
1. a. \[ 2y + x = 3 \]
\[ 3y - x = 7 \]
\[ 5y = 10 \]
\[ y = 2 \]
\[ 2y + x = 3 \]
\[ 2(2) + x = 3 \]
\[ x = -1 \]
Solution is \( x = -1, y = 2 \).

b. \[ 3y - 2x = -21 \]
\[ 7y + 2x = -29 \]
\[ 10y = -50 \]
\[ y = -5 \]
\[ 3y - 2x = -21 \]
\[ 3(-5) - 2x = -21 \]
\[ -2x = -21 + 15 \]
\[ x = 3 \]
Solution is \( x = 3, y = -5 \).

c. \[ 3y - 2x = 8 \]
\[ 4y + 2x = 13 \]
\[ 7y = 21 \]
\[ y = 3 \]
\[ 3y - 2x = 8 \]
\[ 3(3) - 2x = 8 \]
\[ -2x = 8 - 9 \]
\[ x = \frac{1}{2} \]
Solution is \( x = \frac{1}{2}, y = 3 \).

d. \[ 9y - 2x = -6 \]
\[ + 7y + 2x = -26 \]
\[ 16y = -32 \]
\[ y = -2 \]
\[ 9y - 2x = -6 \]
\[ 9(-2) - 2x = -6 \]
\[ -2x = -6 - 18 \]
\[ x = -6 \]
Solution is \( x = -6, y = -2 \).

Section 5.3

Topic 5.3.1
1. a. Let \( b \) = price per book
Let \( c \) = price per CD
\[ 2b + 5c = 110.50 \quad \text{①} \]
\[ 4b + 3c = 98.50 \quad \text{②} \]
\[ 2(2b + 5c = 110.50) \Rightarrow 4b + 10c = 221.00 \quad \text{③} \]
Subtract ② from ③ to eliminate \( b \):
\[ 4b + 10c = 221.00 \]
\[ -4b + 3c = 98.50 \]
\[ 7c = 122.50 \]
\[ c = 17.50 \]
\[ 2b + 5c = 110.50 \Rightarrow 2b + 5(17.50) = 110.50 \]
\[ 2b = 23.00 \]
\[ b = 11.50 \]
Each book costs $11.50 and each CD costs $17.50.

b. Let \( c \) = price per cup
Let \( s \) = price per spoon
\[ 2c + 3s = 5.55 \quad \text{①} \]
\[ 10c + 12s = 25.20 \quad \text{②} \]
\[ 5(2c + 3s = 5.55) \Rightarrow 10c + 15s = 27.75 \quad \text{③} \]
Subtract ② from ③ to eliminate \( c \):
\[ 10c + 15s = 27.75 \]
\[ -10c + 12s = 25.20 \]
\[ 3s = 2.55 \]
\[ s = 0.85 \]
\[ 2c + 3s = 5.55 \Rightarrow 2c + 3(0.85) = 5.55 \]
\[ 2c = 3 \]
\[ c = 1.50 \]
Each cup costs $1.50 and each spoon costs $0.85.

c. Let \( t \) = price per T-shirt
Let \( s \) = price per pair of socks
\[ 3t + 6s = 23.10 \quad \text{①} \]
\[ 7t + 10s = 45.50 \quad \text{②} \]
\[ 5(3t + 6s = 23.10) \Rightarrow 15t + 30s = 115.50 \quad \text{③} \]
\[ 3(7t + 10s = 45.50) \Rightarrow 21t + 30s = 136.50 \quad \text{④} \]
Subtract ③ from ④ to eliminate \( s \):
\[ 21t + 30s = 136.50 \]
\[ -15t + 30s = 115.50 \]
\[ 6t = 21.00 \]
\[ t = 3.50 \]
\[ 3t + 6s = 23.10 \Rightarrow 3(3.50) + 6s = 23.10 \]
\[ 6s = 12.60 \]
\[ s = 2.10 \]
Each T-shirt costs $3.50 and each pair of socks costs $2.10.
1. Let $e = \text{price per electric sharpener}$
   
   Let $d = \text{price per dictionary}$
   
   $8e + 5d = 555$ ①
   $3e + 2d = 215$ ②
   $2(8e + 5d = 555) \Rightarrow 16e + 10d = 1110$ ③
   $5(3e + 2d = 215) \Rightarrow 15e + 10d = 1075$ ④

   Subtract ④ from ③ to eliminate $d$:
   
   \[
   \begin{align*}
   16e + 10d &= 1110 \\
   -15e + 10d &= 1075
   \end{align*}
   \]

   \[
   e = 35
   \]

   $3e + 2d = 215 \Rightarrow 3(35) + 2d = 215$
   
   $2d = 110$
   
   $d = 55$

   Each sharpener costs $35 and each dictionary costs $55.

**Topic 5.3.2**

1. a. Let $x =$ first number
   
   Let $y =$ second number
   
   $x + y = 116$
   
   $y = 3x$
   
   $x + (3x) = 116$
   
   $4x = 116$
   
   $x = 29$
   
   $y = 3x = 3(29)$
   
   $y = 87$

   The numbers are 29 and 87.

b. Let $x =$ larger number
   
   Let $y =$ smaller number
   
   $x + y = 64$
   
   $x = 26 \Rightarrow x = 26 + y$
   
   $(26 + y) + y = 64$
   
   $2y = 64 - 26$
   
   $y = 19$
   
   $x = 26 + y = 26 + 19$
   
   $x = 45$

   The numbers are 19 and 45.

c. Let $x =$ smaller number
   
   Let $y =$ larger number
   
   $3x + 4y = 233$
   
   $2x - y = 16 \Rightarrow y = 2x - 16$
   
   $3x + 4(2x - 16) = 233$
   
   $11x - 64 = 233$
   
   $11x = 297$
   
   $x = 27$
   
   $y = 2x - 16 = 2(27) - 16$
   
   $y = 38$

   The numbers are 27 and 38.

**Topic 5.3.3**

1. Let $x =$ liters of 20% solution
   
   Let $y =$ liters of 45% solution
   
   $x + y = 20$ ①
   
   $0.2x + 0.45y = 0.3(20) \Rightarrow 0.2x + 0.45y = 6$ ②
   
   $0.2(0.2x + 0.45y = 6) \Rightarrow x + 2.25y = 30$ ③

   Subtract ① from ③:
   
   \[
   \begin{align*}
   x + 2.25y &= 30 \\
   - x + y &= 20
   \end{align*}
   \]

   \[
   1.25y = 10
   \]

   $y = 8$

   $x + y = 20 \Rightarrow x + 8 = 20$

   $x = 12$

   Anne needs 12 liters of the 20% solution and 8 liters of the 45% solution.

**Topic 5.3.4**

1. Let $x =$ the boat’s speed in still water
   
   Let $y =$ water current speed

   Upriver: $222 = (x - y) \times 6$
   
   $\Rightarrow 37 = x - y$

   Downriver: $222 = (x + y) \times 4$
   
   $\Rightarrow 55.5 = x + y$

   Add the two equations:

   \[
   \begin{align*}
   37 + 55.5 &= x - y + x + y \\
   &\Rightarrow 92.5 = 2x
   \end{align*}
   \]

   $x = 46.25$

   $37 = x - y \Rightarrow 46.25 - y = 37$

   $y = 9.25$

   The speed of the boat is 46.25 mph and the speed of the water current is 9.25 mph.

**Section 6.1**

**Topic 6.1.1**

1. a. binomial
   
   b. none of the three — it is a five term polynomial
   
   c. monomial
   
   d. trinomial
   
   e. monomial
   
   f. trinomial
   
   g. binomial
   
   h. monomial

**Topic 6.1.2**

1. a. $-4w - 8$
   
   b. $-3x^2 + 5x - 7$
   
   c. $4y^2 - 3y + 7$
   
   d. $4p + 3 - 2p^2$
   
   e. $-\frac{1}{2}x^2 + 4y^2 - 3z^2$
   
   f. $-w^4 + \frac{2}{3}w^2 - 6w - 2$

**Topic 6.1.3**

1. a. $(x^2 + 5) - (x^2 + 2) = x^2 - x^2 + 5 - 2 = 3$
   
   b. $(x^2 - 12) - (x^2 + 3) = x^2 - x^2 - 12 - 3 = -15$
   
   c. $(x^2 - 8x + 2) - (2x + 5) = x^2 - 8x - 2x + 2 - 5$
   
   $= x^2 - 10x - 3$
   
   d. $(x^3 + 3x - 1) - (x^2 + 2) = x^3 - x^2 + 3x - 1 - 2$
   
   $= x^3 - x^2 + 3x - 3$
   
   e. $(4x^3 - 2x + 14) - (x^2 + x)$
   
   $= 4x^3 - x^2 - 2x - x + 14 = 4x^3 - x^2 - 3x + 14$
   
   f. $(-6x^3 + 5x - 4) - (3x^3 - x) = -6x^3 - 3x^2 + 5x + x - 4$
   
   $= -9x^3 + 6x - 4$

**Topic 6.1.4**

1. a. $(4x + 3) - (2x - 5)$
   
   $= 4x + 3 - 2x + 5$
   
   $= 2x + 8$
   
   b. $(4y - 7) + (-2y + 5)$
   
   $= 4y - 7 - 2y + 5$
   
   $= 2y - 2$
c. \((3z - 4) - (-2z + 5) = 3z - 4 + 2z - 5 = 5z - 9\)

**Section 6.2**

**Topic 6.2.1**

1. a. \((3x^2y^3z^4)(4x^3y^2z^5)\)  
   \[= (3)(4)(x^2)(x^3)(y^3)(y^2)(z^4)(z^5)\]  
   \[= 12x^{2+3}y^{3+2}z^{4+5}\]  
   \[= 12xy^5z^9\]

   b. \((5a^2b^3c)(3a^5b^4c^5)\)  
   \[= 5(3)(a^2)(a^5)(b^3)(b^4)(c)(c^5)\]  
   \[= 15a^{2+5}b^{3+4}c^{1+5}\]  
   \[= 15a^7b^7c^6\]

   c. \((2r^7s^4)(3rs^3t^2)\)  
   \[= (2)(3)(r^7)(r)(s^4)(s^3)(t^2)\]  
   \[= 6r^{7+1}× s^{4+3}× t^2\]  
   \[= 6r^8s^7t^2\]

   d. \((5x^4y^3z^4)(3x^5y^3z^4)\)  
   \[= (5)(3)(x^4)(x^5)(y^3)(y^3)(z^4)(z^4)\]  
   \[= 15× x^{4+5}× y^{3+3}× z^{4+4}\]  
   \[= 15x^9y^6z^8\]

**Topic 6.2.2**

1. a. \[\frac{2x}{5x + 2} \times \frac{5x + 2}{4x} = \frac{10x^2}{10x^2 + 4x}\]

   b. \[\frac{6x}{8x + 2} \times \frac{8x + 2}{12x} = \frac{48x^2}{48x^2 + 12x}\]

   c. \[\frac{4x + 6}{x + 3} \times \frac{x + 3}{12x + 18} = \frac{4x^2 + 6x}{4x^2 + 18x + 18}\]

   d. \[\frac{2x + 2}{2x - 2} \times \frac{2x - 2}{-4x - 4} = \frac{4x^2 + 4x}{4x^2 - 4}\]

**Topic 6.2.3**

1. \(A = (3x + 7)(2x + 3)\)  
   \[= 6x^2 + 9x + 14x + 21\]  
   \[= (6x^2 + 23x + 21) \text{ cm}^2\]

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**Section 6.3**

**Topic 6.3.1**

1. a. \((6c^3y^z) + (2xy^z)\)  
   \[= 3(c^3y^z)(x^{n-3})(z^{n-2})\]  
   \[= 3xy^2e^0\]

   b. \((12a^7b^9c^3d^2) + (3a^2b^2d)\)  
   \[= 4(a^7b^9c^3)(d^{n-1})\]  
   \[= 4ab^3c^4d\]

   c. \((8r^7x^y) + (4r^5x^4)\)  
   \[= 2(r^7x^y)(r^{n-3})(x^{n-4})\]  
   \[= 2r^2x^4\]

   d. \(\frac{2r^7x^2}{s^3}\)

**Topic 6.3.2**

1. a. \[\frac{1}{x^2} + \frac{1}{xy}\]

   b. \[\frac{x}{2a + 5b}\]

   c. \[x^y\]

   d. \[3d^2e^3\]

**Topic 6.3.3**

1. a. \[\frac{x + 3}{x + 7}\]  
   \[\frac{3x + 1}{x + 5}\]

   \[= x + 3\]

   b. \[\frac{x + 2}{x + 2}\]  
   \[\frac{3x + 1}{x + 5}\]

   \[= \frac{1}{x - 2}\]

   c. \[\frac{x + 3}{2x - 3}\]  
   \[\frac{3x + 1}{3x + 4}\]

   \[= 2x - 5\]

   d. \[\frac{5x + 3}{2x - 5}\]  
   \[\frac{3x + 3}{5x + 3}\]

   \[= 1\]

**Topic 6.3.4**

1. \[2x - 3\]  
   \[x + 1\]  
   \[2x^3 - x - 3\]  
   \[= (2x^2 + 2x)\]  
   \[= -3x - 3\]  
   \[= (-3x - 3)\]  
   \[= 0\]
Section 6.4

Topic 6.4.1

1. a. $(k + 1)^2 = (k + 1)(k + 1)$
   \[ = k^2 + k + 1 \]
   b. $(3a + 2)^2 = (3a + 2)(3a + 2)$
   \[ = 9a^2 + 6a + 4 \]
   c. $(x^2 - 1) = (x^2 - 1)(x^2 - 1)$
   \[ = 2x^2 - 5x + 1 \]
   d. $(x^2 + r^2) = (x^2 + r^2)(x^2 + r^2)$
   \[ = x^4 + 2x^2r^2 + r^4 \]

Section 6.5

Topic 6.5.1

1. a. 1, 3, 9
   b. 1, 2, 3, 4, 6, 12
   c. 1, 3, 5, 15
   d. 1, 2, 3, 6, 9, 18
   e. 1, 2, 4, 5, 10, 20
   f. 1, 2, 4, 7, 14, 28

Topic 6.5.2

1. a. $6y^2 = 2 \times 3 \times y \times y$
   \[ GCF = 3y \]
   b. $12x^2 = 2 \times 2 \times 3 \times x \times x \times x \times x$
   \[ 18x^2 = 2 \times 3 \times 3 \times x \times x \times x \]
   \[ GCF = 6x \]
   c. $15x^3 = 3 \times 5 \times x \times x \times x$
   \[ 12x^2 = 2 \times 2 \times 3 \times x \times x \times x \]
   \[ 6x = 2 \times 3 \times x \]
   \[ GCF = 3x \]
   d. $24x^4y^2 = 2 \times 2 \times 2 \times 3 \times x \times x \times x \times x \times x \times x \times y \times y \times y \times y \times y$
   \[ 32x^4y^2 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times x \times x \times x \times x \times x \times x \times y \times y \times y \times y \times y$
   \[ GCF = 8x^3y^2 \]

Section 6.6

Topic 6.6.1

1. a. $3 - t = 1, 3(t) = 6, t = 2$
   b. $3 + t = 8, 3(t) = 15, t = 5$
   c. $4 - t = 2, 4(t) = 8, t = 2$
   d. $5 - t = 4, 5(t) = 5, t = 1$
   e. $-8 - t = -11, 8(t) = 24, t = 3$

Topic 6.6.2

1. a. $2x^2 + 8x + 6 = 2(x^2 + 4x + 3)$
   \[ = 2(x + 3)(x + 1) \]
   b. $\pi r^2 + 2\pi rh = \pi(r + 2h)$
   c. $2x^2k = 6xk + 4k = 2k(x^2 - 3x + 2)$
   \[ = 2k(x - 2)(x - 1) \]
   d. $4mx^2 + 6mx - 18m = 2m(2x^2 + 3x - 9)$
   \[ = 2m(2x - 3)(x + 3) \]
   e. $3t^2p - 3mp = 126p = 3p(m^2 - m - 42)$
   \[ = 3p(m - 7)(m + 6) \]

Section 6.7

Topic 6.7.1

1. a. $x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$
   b. $x^2 + 3kx + 2k^2 = (x + 2k)(x + k)$
   c. $x^2 + mx - 6m^2 = (x - 3m)(x + 2m)$
   d. $k^2 - 2kr + r^2 = (k + r)(k + r)$
   e. $x^2 - 2xy + y^2 = (x - y)^2$
   f. $a^2 - ab - 2b^2 = (a - 2b)(a + b)$
   g. $r^2 - 5rs + 6s^2 = (r + 2s)(r + 3s)$

Topic 6.7.2

1. a. $x^2 + 6x^2 + 8x = x(x^2 + 6x + 8)$
   \[ = x(x + 4)(x + 2) \]
   b. $x^2 + x^2 - 6x = x(x^2 + 3x - 6)$
   \[ = x(x + 3)(x - 2) \]
   c. $x^2 + 9x^2 + 20x = x(x^2 + 9x + 20)$
   \[ = x(x + 4)(x + 5) \]

Section 6.8

Topic 6.8.1

1. a. $k^2 - 1 = (k - 1)(k + 1)$
   b. $m^2 - 36 = (m - 6)(m + 6)$
   c. $r^2 - r^2 = (r - t)(r + t)$
   d. $4 - x^2 = (2 - x)(2 + x)$
   e. $100 - c^2 = (10 - c)(10 + c)$
   f. $r^2 - 81 = (r - 9)(r + 9)$
   g. $x^2 - y^2 = (x + y)(x - y)$
   h. $r^2 - 16 = (r + 4)(r - 4)$
   i. $25 - z^2 = (5 + z)(5 - z)$
   j. $4b^2 - c^2 = (2b + c)(2b - c)$
   k. $r^2 - s^2 = (r^2 + s)(r^2 - s)$
Topic 7.2.2
1. a. \((x + 4)^2 = x^2 + 8x + 16\)
   b. \((x - 6)^2 = x^2 - 12x + 36\)
   c. \((x + \frac{9}{2})^2 = x^2 + 9x + \frac{81}{4}\)
   d. \((x - \frac{5}{2})^2 = x^2 - 5x + \frac{25}{4}\)

Topic 7.2.3
1. a. \(x^2 + 6x = 7 \Rightarrow (x + 3)^2 - 9 = 7\)
   \(\Rightarrow (x + 3)^2 = 16 \Rightarrow x + 3 = \pm 4\)
   \(\Rightarrow x = -7 \text{ or } x = 1\)
   b. \(x^2 + 10x = 11 \Rightarrow (x + 5)^2 - 25 = 11\)
   \(\Rightarrow (x + 5)^2 = 36 \Rightarrow x + 5 = \pm 6\)
   \(\Rightarrow x = -11 \text{ or } x = 1\)

Section 7.3

Topic 7.3.1
1. \(ax^2 + bx + c = 0\)
   \(x^2 + \frac{b}{a} x + \frac{c}{a} = 0\)
   \(x^2 + \frac{b}{a} x + \frac{c}{a} = 0\)
   \(x^2 + \frac{b}{a} x + \frac{c}{a} = 0\)
   \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
   \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
   \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
   \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
   \(\therefore x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ or } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}\)

Topic 7.3.2
1. Let \(x\) = 1st number
   \(\therefore x + 7 = \text{ 2nd number}\)
   Equation: \(x(x + 7) = 294\)
   \(x^2 + 7x = 294\)
   \(x^2 + 7x - 294 = 0\)
   \(x = \frac{-7 \pm \sqrt{49 + 1176}}{2} = \frac{-7 \pm \sqrt{1225}}{2}\)
   \(= \frac{-7 \pm 35}{2}\)
   \(x = 14 \text{ or } x = -21\)
   So the numbers are 14 and 21, or -21 and -14.
Section 7.4

Topic 7.4.1
1. a. Narrower than \( y = x^2 \).
   b. Narrower than \( y = x^2 \) and reflected about the \( x \) axis.
   c. Wider than \( y = x^2 \).

Topic 7.4.2
1. a. \( y = x^2 + 4 \)
   \( y = (x - 2)(x + 2) = 0 \), so either \( x = 2 \) or \( x = -2 \).
   So the \( x \)-intercepts are \((2, 0)\) and \((-2, 0)\).
   b. \( y = x^2 - 6x + 3 \)
   \( y = (x - 3)(x - 1) = 0 \), so either \( x = 3 \) or \( x = 1 \).
   So the \( x \)-intercepts are \((3, 0)\) and \((1, 0)\).

Topic 7.4.3
1. a. \( y = x^2 + 4x + 2 \)
   \( y = (x + 2)^2 - 4 + 2 \)
   \( y = (x + 2)^2 - 2 \)
   b. \( y = x^2 - 6x + 3 \)
   \( y = (x - 3)^2 - 9 + 3 \)
   \( y = (x - 3)^2 - 6 \)

Section 7.5

Topic 7.5.1
1. a. \( x^2 - 5x - 1 = 0 \)
   \( a = 1, b = -5, c = -1 \)
   \( b^2 - 4ac = (-5)^2 - 4(1)(-1) \)
   \( b^2 - 4ac = 29 > 0 \)
   The discriminant > 0, so there are two distinct real roots:
   \[ x = \frac{5 \pm \sqrt{29}}{2} \]
   b. \( x^2 + 2x - 5 = 0 \)
   \( a = 1, b = 2, c = -5 \)
   \( b^2 - 4ac = (2)^2 - 4(1)(-5) \)
   \( b^2 - 4ac = 24 > 0 \)
   The discriminant > 0, so there are two distinct real roots:
   \[ x = \frac{-2 \pm \sqrt{24}}{2} = -1 \pm \sqrt{6} \]
   c. \( 4x^2 - 4x + 1 = 0 \)
   \( a = 4, b = -4, c = 1 \)
   \( b^2 - 4ac = (-4)^2 - 4(1)(4) \)
   \( b^2 - 4ac = 0 \)
   The discriminant = 0, so there is one real double root:
   \[ x = \frac{4 \pm 0}{8} = \frac{1}{2} \]
   d. \( x^2 - 6x + 9 = 0 \)
   \( a = 1, b = -6, c = 9 \)
   \( b^2 - 4ac = (-6)^2 - 4(1)(9) \)
   \( b^2 - 4ac = 0 \)
   The discriminant = 0, so there is one real double root:
   \[ x = \frac{6 \pm 0}{2} = 3 \]

Section 7.6

Topic 7.6.1
1. \( -16t^2 + 400 = 0 \)
   \( t^2 = 25 \)
   \( (t - 5)(t + 5) = 0 \)
   \( t = 5 \) or \( t = -5 \)
   The ball will hit the ground after 5 seconds.

Topic 7.6.2
1. \( t = 10: P = t^2 + 60t + 800 \\
P = 10^2 + 60(10) + 800 \\
P = 100 + 600 + 800 \\
P = 1500 \\
Her weekly payroll after 10 years will be $1500. \\

Section 8.1

Topic 8.1.1
1. a. \( x = -3 \)
   b. \( x = 5 \)
   c. \( x = 2 \)
   d. \( x = -5 \)
   e. \( x = 7 \)
   f. \( x = 4 \)
   g. \( x = 5 \)
   h. \( x = -4 \)
   i. \( x = -7 \)

Topic 8.1.2
1. a. \( \frac{22xy}{44xd} = \frac{2x \cdot x \cdot y}{22 \cdot 2 \cdot x \cdot d} = \frac{xy}{2d} \)
   b. \( x + 3 \\
   \frac{x + 3}{x^2 + x - 6} = \frac{(x + 3)}{(x - 2)(x + 3)} = \frac{1}{x - 2} \)
   c. \( 4 - k^2 \\
   \frac{4 - k^2}{k + 2} = \frac{(2 + k)(2 - k)}{(k + 2)} = 2 - k \)
Section 8.2

Topic 8.2.1

1. a. \( \frac{x^2 - 9}{x - 3} = \frac{(x + 3)(x - 3)}{(x - 3)} = x + 3 \)

b. \( \frac{x^2 - 3x + 2}{x^3 - 10x^2 - 8x + 7} = \frac{(x - 2)(x - 1)}{x^3 - 7x - 2(x - 1)} = \frac{x + 1}{x - 7} \)

c. \( \frac{x^2 - 5x + 6}{x^2 + 11x + 30} = \frac{(x - 2)(x - 3)}{x^2 + 7x - 18} = \frac{x + 5}{x + 9} \)

d. \( \frac{x^2 - 4}{x^2 - x - 6} = \frac{(x - 2)(x + 2)}{(x - 3)(x + 2)} = \frac{x - 2}{x + 5} \)

e. \( \frac{c^2 - 9}{c^2 + 2c - 15} = \frac{(c - 3)(c + 3)}{(c + 5)(c - 3)} = \frac{c - 1}{c + 7} \)

Section 8.2

Topic 8.2.2

1. a. \( \frac{m^2 - 16}{m + 4} \)

\( \frac{(m - 4)(m + 4)}{m + 4} = 3(m - 4) = 3m - 12 \)

b. \( \frac{k^2 + 12k + 36}{k + 6} \)

\( \frac{k^2 + 12k + 36}{k + 6} = \frac{(k + 6)^2}{k + 6} = k + 6 = -1 \)

c. \( \frac{c^2 - 1}{c^2 - 3c + 2} \)

\( \frac{c^2 - 1}{c^2 - 3c + 2} = \frac{c - 1}{c - 2} \)

d. \( \frac{k^2 + k - 6}{k^2 + 3k - 10} \)

\( \frac{k^2 + k - 6}{k^2 + 3k - 10} = \frac{(k - 2)(k + 3)}{(k + 5)(k - 1)} = \frac{k - 1}{k - 1} = 1 - k \)

e. \( \frac{3x^2 + 7x - 20}{x^2 - 2x - 5} \)

\( \frac{3x^2 + 7x - 20}{x^2 - 2x - 5} = \frac{(3x - 5)(x + 4)}{(x - 4)(x + 1)} = \frac{x - 1}{x + 1} \)
Section 8.3

Topic 8.3.1

1. a. \(\frac{a}{2} + \frac{b}{2} = \frac{a+b}{2}\)

b. \(\frac{r}{5} + \frac{s}{5} = \frac{r+s}{5}\)

c. \(\frac{2v}{7} + \frac{5v}{7} = \frac{2v+5v}{7} = \frac{7v}{7} = v\)

d. \(\frac{13q}{5} - \frac{3q}{5} = \frac{13q-3q}{5} = \frac{10q}{5} = 2q\)

Topic 8.3.2

1. a. \(\frac{a-1}{3} + \frac{a+1}{3} = \frac{3(a-1) + 2(a+1)}{6} = \frac{3a - 3 + 2a + 2}{6} = \frac{5a - 1}{6}\)

b. \(\frac{x - 3}{4} + \frac{x + 1}{5} = \frac{5(x - 3) - 4(x + 1)}{20} = \frac{5x - 15 - 4x - 4}{20} = \frac{x - 19}{20}\)

c. \(\frac{1}{x - 1} = \frac{2}{x + 1} = \frac{1(x + 1) - 2(x - 1)}{(x - 1)(x + 1)} = \frac{x + 1 - 2x + 2}{x^2 - 1} = \frac{3 - x}{x^2 - 1}\)

d. \(\frac{3}{h - 1} - \frac{2}{h + 1} = \frac{3(h + 1) - 2(h - 1)}{(h - 1)(h + 1)} = \frac{3h + 3 - 2h + 2}{h^2 - 1} = \frac{h + 5}{h^2 - 1}\)

Section 8.4

Topic 8.4.1

1. a. Restrictions: \(h \neq 0\)
   Multiply equation by \(h\).
   \(h^2 - 2h = 8\)
   \(h^2 - 2h - 8 = 0\)
   \((h + 2)(h - 4) = 0\)
   \(h = -2\) or \(h = 4\)

b. Restrictions: \(x \neq 0\) and \(x \neq -1\)
   Multiply equation by \(6(x + 1)\).
   \(6x = 6(x + 1) - x(x + 1)\)
   \(6x = 6x + 6 - x^2\)
   \(6x = 5x + 6 - x^2\)
   \(x = 6 - x^2\)
   \(x^2 + x - 6 = 0\)
   \((x - 2)(x + 3) = 0\)
   \(x = 2\) or \(x = -3\)

c. Restrictions: \(x \neq 0\) and \(x \neq 2\)
   Multiply the equation by the LCM of the denominators, \(x(x - 2)\).
   \(3(x - 2) - x(x) = -2(x)(x - 2)\)
   \(3x - 6 - x^2 = -2x^2 + 4x\)
   \(x^2 - x - 6 = 0\)
   \((x - 3)(x + 2) = 0\)
   \(x = 3\) or \(x = -2\)

d. Restrictions: \(x \neq -1\) and \(x \neq 3\)
   Multiply equation by \(8(x + 1)(x - 3)\).
   \(8x(x - 3) - 8(x - 4)(x + 1) = (x + 1)(x - 3)\)
   \(8x^2 - 24x - 8(x^2 - 3x - 4) = x^2 - 2x - 3\)
   \(8x^2 - 24x - 8x^2 + 24x + 32 = x^2 - 2x - 3\)
   \(32 = x^2 - 2x - 3\)
   \(35 - x^2 + 2x = 0\)
   \(x^2 - 2x - 35 = 0\)
   \((x - 7)(x + 5) = 0\)
   \(x = 7\) or \(x = -5\)

e. Restrictions: \(y \neq -1\) and \(y \neq 1\)
   LCM of the denominators is \(3(y + 1)(2y - 2)\).
   \((y + 1)(2y - 2) + 9(2y - 2) = 15(y + 1)\)
   \(2y^2 - 2 + 18y - 18 = 15y + 15\)
   \(2y^2 + 3y - 35 = 0\)
   \((2y - 7)(y + 5) = 0\)
   \(y = \frac{7}{2}\) or \(y = -5\)

Topic 8.4.2

1. Let \(x\) be the number, therefore \(\frac{1}{x}\) is the reciprocal.
   Equation: \(x + \frac{1}{x} = \frac{29}{10}\)
   \[10x(x) + \frac{10x}{x} = 10x \left(\frac{29}{10}\right)\]
   \[10x^2 + 10 = 29x\]
   \[10x^2 - 29x + 10 = 0\]
   \[(5x - 2)(2x - 5) = 0\]
   \(x = \frac{2}{5}\) or \(x = \frac{5}{2}\)
Section 8.5

**Topic 8.5.1**
1. Domain = \{-5, 3, 2\},  
   Range = \{11, 10, -12\}.

**Topic 8.5.2**
1. a. \(f\) is a function since no two ordered pairs have the same  
   first entry but a different second entry.
   b. \(k\) is a not a function since it has three ordered pairs  
   with the same first entry but different second entries.
   c. \(h = \{(−2, −1), (−1, 0), (0, 1), (1, 2)\}\) Therefore \(h\) is a  
   function since no two ordered pairs have the same first  
   entry and a different second entry.
   d. The relation is a function since each first entry is  
   unique and is mapped to only one second entry.

**Topic 8.5.3**
1. a. \(f(7) = 3 \times 7 = 21\)
   b. \(f(−5) = 3 \times (−5) = −15\)
   c. \(f(3) = 3 \times 3 = 9\)
   d. \(f(−2) = 3 \times (−2) = −6\)

**Topic 8.5.4**
1. No, Joe is not correct because \((2x + 3)(2x + 3) \neq 4x^2 - 9\).  
   For the functions to be equal, the domains must be the  
   same and the equations representing the functions must  
   be equal.