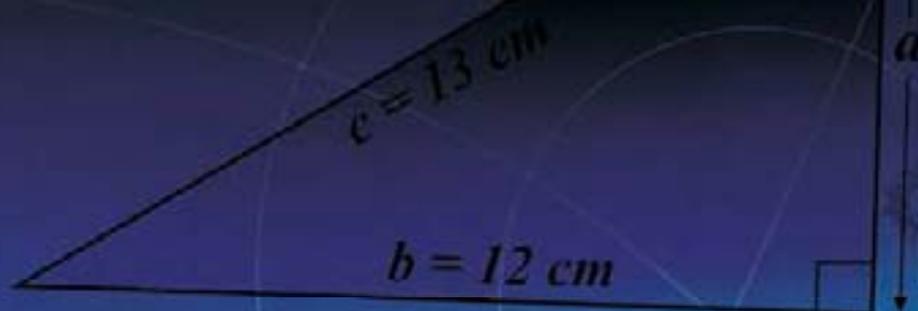


California Mathematics

Course Two



$$\begin{aligned} a^2 &= c^2 - b^2 \\ a^2 &= 169 - 144 \\ a^2 &= 25 \\ \therefore a &= 5 \text{ cm} \end{aligned}$$

Student Textbook

California Standards-Driven Program

California

Mathematics

Course Two

Student Textbook

California Standards-Driven Program



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This Textbook provides comprehensive coverage of all the California Grade 7 Standards. The Textbook is divided into eight Chapters. Each of the Chapters is broken down into small, manageable Lessons and each Lesson covers a specific Standard or part of a Standard.

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California Grade Seven Mathematics Standards

The following table lists all the California Mathematics Content Standards for Grade 7 with cross references to where each Standard is covered in this Textbook. Each Lesson begins by quoting the relevant Standard in full, together with a clear and understandable objective. This will enable you to measure your progression against the California Grade 7 Standards as you work your way through the Program.

California Standard	Number Sense	Chapter
1.0	Students know the properties of, and compute with, rational numbers expressed in a variety of forms:	2, 5, 8
1.1	Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.	2, 5
1.2	★ Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.	2, 5
1.3	Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.	2, 8
1.4	★ Differentiate between rational and irrational numbers.	2
1.5	★ Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.	2
1.6	Calculate the percentage of increases and decreases of a quantity.	8
1.7	★ Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.	8
2.0	Students use exponents, powers, and roots and use exponents in working with fractions:	2, 5
2.1	Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.	5
2.2	★ Add and subtract fractions by using factoring to find common denominators.	2
2.3	★ Multiply, divide, and simplify rational numbers by using exponent rules.	5
2.4	Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.	2
2.5	★ Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.	2
California Standard	Algebra and Functions	
1.0	Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:	1, 2, 3, 4, 5
1.1	Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).	1, 2, 3, 4
1.2	Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.	1
1.3	★ Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.	1
1.4	Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.	1, 5
1.5	Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.	1, 4
2.0	Students interpret and evaluate expressions involving integer powers and simple roots:	2, 5
2.1	Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.	2, 5
2.2	Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.	5

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3.0	Students graph and interpret linear and some nonlinear functions:	4, 5, 7
3.1	Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.	5
3.2	Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).	7
3.3	★ Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same and know that the ratio (“rise over run”) is called the slope of a graph.	4
3.4	★ Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.	4
4.0	★ Students solve simple linear equations and inequalities over the rational numbers:	1, 4
4.1	★ Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.	1, 4
4.2	★ Solve multistep problems involving rate, average speed, distance, and time or a direct variation.	4

California Standard

Measurement and Geometry

1.0	Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:	3, 4, 7
1.1	Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).	4
1.2	Construct and read drawings and models made to scale.	3, 7
1.3	★ Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.	4
2.0	Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:	3, 7
2.1	Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.	3, 7
2.2	Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.	3, 7
2.3	Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.	7
2.4	Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1 \text{ ft.}^2] = [144 \text{ in.}^2]$; 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in.}^3] = [16.38 \text{ cm}^3]$).	7
3.0	Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:	3, 7
3.1	Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.	3
3.2	Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.	3

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3.3	★ Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.	3
3.4	★ Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.	3
3.5	Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.	7
3.6	★ Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).	7

California Standard

Statistics, Data Analysis, and Probability

1.0	Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:	6
1.1	Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.	6
1.2	Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).	6
1.3	★ Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.	6

California Standard

Mathematical Reasoning

1.0	Students make decisions about how to approach problems:
1.1	Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
1.2	Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
1.3	Determine when and how to break a problem into simpler parts.
2.0	Students use strategies, skills, and concepts in finding solutions:
2.1	Use estimation to verify the reasonableness of calculated results.
2.2	Apply strategies and results from simpler problems to more complex problems.
2.3	Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
2.4	Make and test conjectures by using both inductive and deductive reasoning.
2.5	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
2.6	Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
2.7	Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
2.8	Make precise calculations and check the validity of the results from the context of the problem.
3.0	Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:
3.1	Evaluate the reasonableness of the solution in the context of the original situation.
3.2	Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
3.3	Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

Illustrated throughout Program



Published by CGP Education

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Mathematics Content Standards for California Public Schools reproduced by permission,
California Department of Education, CDE Press, 1430 N Street, Suite 3207, Sacramento, CA 95814.

ISBN 13: 978 1 60017 024 9

website: www.cgpeducation.com

Printed by Elanders Hindson Ltd, UK and Johnson Printing, Boulder, CO
Clipart sources: CoreIDRAW and VECTOR.

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Chapter 1

The Basics of Algebra

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Section 1.1 introduction — an exploration into: Algebra Tiles

You can write expressions using *algebra tiles*. This shows what an expression actually “looks like,” and how you can *simplify* it. You’ll be using two types of tile. Their *areas* represent their values —

1-tile: $1 \begin{array}{c} \square \\ 1 \end{array}$ x -tile: $1 \begin{array}{c} \square \\ x \end{array}$

Here’s the sum $4 + 3$ shown using algebra tiles: $\begin{array}{c} \square \square \square \square \\ 4 \end{array} + \begin{array}{c} \square \square \square \\ 3 \end{array} = \begin{array}{c} \square \square \square \square \\ \square \square \square \square \\ 7 \end{array}$

The example below shows how you can *simplify* algebra expressions by *rearranging the tiles*.

Example

Show $3 + x + 1 + 2x$ using algebra tiles. Rearrange the tiles to simplify the expression.

Solution

$\begin{array}{c} \square \\ \square \\ \square \end{array} + \begin{array}{c} \square \\ \square \end{array} + \begin{array}{c} \square \\ \square \end{array} + \begin{array}{c} \square \\ \square \end{array} \rightarrow \begin{array}{c} \square \square \square \\ \square \square \end{array} + \begin{array}{c} \square \\ \square \end{array}$

$3 + x + 1 + 2x = 3x + 4$ ← Simplified expression

Exercises

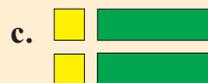
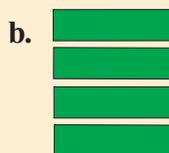
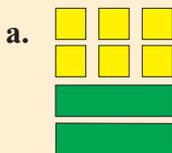
1. Use algebra tiles to model the following addition problems:

a. $5 + 2$

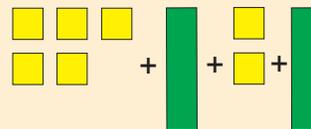
b. $2 + 1$

c. $x + 2x$

2. Write the algebra expressions that are modeled by these tiles.

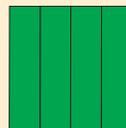


3. Rearrange these tiles to make a simplified expression:



4. Write an algebraic expression to represent the tiles in Exercise 3. Rewrite this as a simplified expression.

5. Write expressions to represent the perimeter and area of these rectangles. a.



b.



Round Up

When you’re *simplifying expressions*, you have to organize them so the bits that are the same are all *grouped together*. You can then *combine* these — so $2x + 3x = 5x$, and $1 + 2 = 3$.

What you *can’t* do is combine *different things* together. For example, you could never add $4x$ and 7 .

Lesson 1.1.1

Section 1.1 Variables and Expressions

California Standards:

Algebra and Functions 1.1

Use **variables** and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., 3 less than a number, half as large as area A).

Algebra and Functions 1.4

Use **algebraic terminology** (e.g., **variable**, equation, **term**, **coefficient**, inequality, **expression**, **constant**) correctly.

What it means for you:

You'll learn how to use a letter or symbol to stand in for an unknown number, and how it can be exchanged for the real number when you know it.

Key words:

- variable
- expression
- evaluate

Don't forget:

xy means "x multiplied by y."
It's just the same as writing $x \times y$ or $x \cdot y$.

When you write out a problem in math you often need to include **unknown numbers**. You have to use a **letter** or **symbol** to stand in for an unknown number until you figure out what it is — you did this before in earlier grades. That's what a **variable** does. And that's what this Lesson is about.

A Variable Represents an Unknown Number

In algebra you'll often have to work with numbers whose values you don't know. When you write out math problems, you can use a **letter** or a **symbol** to stand in for the number. The letter or symbol is called a **variable**.

Coefficient \rightarrow $2k + 4$ \leftarrow Constant
Variable \rightarrow

The number that the variable is being multiplied by is called the **coefficient** — like the 2 above.

Any number not joined to a variable is called a **constant** — like the 4 above. It's called that because its value doesn't **change**, even if the value of the variable changes.

A **term** is a group of **numbers** and **variables**. One or more terms added together make an **expression**. For example, in the expression above, $2k$ is one term and 4 is another term. In the expression $3 + 4x - 5wyz$, the terms are 3, $4x$, and $-5wyz$.

An Expression is a Mathematical Phrase

Expressions are **mathematical phrases** that may contain **numbers**, **operations**, and **variables**. The operations act like a set of instructions that tell you what to do with the numbers and variables. For example, $2k + 4$ tells you to double k , then add four to it.

There are two types of expressions — **numeric** and **variable**.

- **Numeric expressions** have numbers in them, and often operations — but they don't include any variables:

$\rightarrow 5 + 13$
 $\rightarrow 2 \cdot 5 - 6$
 $\rightarrow 8 + 7 \div 6$

- **Variable expressions** have variables in them, and may also include numbers and operations:

$\rightarrow 5h$
 $\rightarrow 7x - 2$
 $\rightarrow 2k + 4$

✓ Guided Practice

Say whether the expressions in Exercises 1–8 are numeric or variable.

1. y

2. $4(x - y)$

3. $10 - 7$

4. 6^2

5. $8xy$

6. $4(3 + 7)$

7. $9(5 - y)$

8. x^2

Expressions Can Be Described in Words

To show you understand an expression you need to be able to explain **what it means** in words. You can write a **word expression** to represent the numeric or variable expression.

Example 1

Write the variable expression $x + 5$ as a word expression.

Solution

In this question x is the **variable**. The rest of the expression tells you that five is being **added on** to the unknown number x .

So the expression becomes:

“ **x is increased by five**” or

“**the sum of x and five**” or

“**five more than x .**”

They all mean the same thing.

When you change a **variable expression** to a **word expression** you can say the **same thing** in several **different ways**.

+ Instead of “2 **added to** x ” you could say “ x **increased by** 2,”
“2 **more than** x ,” or “**the sum of** x and 2.”

– “2 **subtracted from** x ” means the same as “2 **less than** x ” or
“ x **decreased by** 2.”

× “ x **multiplied by** 2” means the same as “**the product of** x and 2,”
“ x **times** 2,” or “**twice** x .”

÷ And you could say either “ x **divided by** 3” or “**one third** x .”

As long as it **matches with the operation** that you’re describing, you can use any of the phrases.

Check it out:

These four groups of phrases are describing the four operations you’ll need: addition, subtraction, multiplication, and division.

Example 2

Write the variable expression $4(w - 3)$ as a word expression.

Solution

In this expression w is the **variable**. The rest of the expression tells you that three is **subtracted** from w , and the result is **multiplied** by four.

So the expression becomes:

“**four times the the result of subtracting three from w** ” or

“**three subtracted from w , multiplied by four.**”

They both mean the same thing.

Don't forget:

You need to do the operations in the correct order. The PEMDAS rule helps you remember this — it tells you that you need to deal with the parentheses first. There's more on the PEMDAS rule in Lesson 1.1.3.

✓ Guided Practice

Write a word expression for each of the variable expressions in Exercises 9–14.

9. $z - 10$

10. $6b$

11. $12h + 4$

12. $5(j + 6)$

13. $6(4t)$

14. $2c + 4d$

Check it out:

There is more than one possible word expression to describe each of these mathematical phrases. As long as it means the same thing, any variation you use is fine.

When You Evaluate, You Swap Variables for Numbers

When you're given the actual numbers that the variables are standing in for you can **substitute** them into the expression. When you have substituted numbers for **all the variables** in the expression, you can work out its numerical value. This is called **evaluating the expression**.

Example 3

Evaluate the expression $6f + 4$ when $f = 7$.

Solution

$$\begin{aligned} 6f + 4 &= 6 \cdot 7 + 4 \\ &= 42 + 4 \\ &= 46 \end{aligned}$$

Substitute 7 in place of f
Perform the multiplication first
Then do the addition

Don't forget:

Multiplication and division come before addition and subtraction in the order of operations. That's why you do the multiplication first in this example.

Example 4

Evaluate the expression $11(p - q)$ when $p = 7$ and $q = 3$.

Solution

$$\begin{aligned} 11(p - q) &= 11(7 - 3) \\ &= 11(4) \\ &= 44 \end{aligned}$$

Replace p and q with 7 and 3

Do the subtraction

Do the multiplication to get the answer

✓ Guided Practice

Evaluate the expressions in Exercises 15–22, given that $j = 5$ and $t = 7$.

15. $6t - 4j$

16. $t + j$

17. $3(t - j)$

18. $3t + 7j$

19. jt

20. $2(j - t)$

21. $j + tj - 40$

22. $3jt - j + 20$

✓ Independent Practice

Write word expressions for the variable expressions in Exercises 1–3.

1. $6y + 10$

2. $3(p - 6)$

3. $18t$

4. Joe and Bonita went fishing. Joe caught j fish and Bonita caught $j + 4$ fish. Write a sentence that describes the amount of fish Bonita caught compared with Joe.

Evaluate the expressions in Exercises 5–7, given that $x = 3$.

5. $4x$

6. $1 - x$

7. x^2

8. An orchard uses the expression $0.5w$ to work out how much money, in dollars, it will make, where w = the number of apples sold. How much money does the orchard make if 250 apples are sold?

9. Given that $a = -1$, $b = 3$, and $c = 2$, evaluate the expression $a^2 + ab + c^2$.

10. A car rental company uses the expression $\$30 + \$0.25m$ to calculate the daily rental price. The variable m represents the number of miles driven. What is the price of a day's rental if a car is driven 100 miles?

Now try these:

Lesson 1.1.1 additional questions — p430

Round Up

Variables are really useful — you can use them to stand in for any unknown numbers in an expression. When you know what the numbers are you can write them in — and then evaluate the expression. In Section 1.2 you'll see how expressions are the building blocks of equations.

Lesson 1.1.2

Simplifying Expressions

California Standards:

Algebra and Functions 1.3
Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

Algebra and Functions 1.4
Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

What it means for you:

You'll learn about some math techniques that will help you to simplify expressions.

Key words:

- like terms
- distributive property
- simplify

Check it out:

You can move the terms about because of the commutative and associative properties of addition. There's more about these in Lesson 1.1.5.

Check it out:

The minus signs in the expression stay with the 5 and the y they were originally connected to.

Remember — you can write subtractions as additions:
 $x - 5 + 2y + 9 - y + 2x$
 $= x + (-5) + 2y + 9 + (-y) + 2x$

This often makes it easier to collect like terms.

When you're *evaluating an expression that's made up of many terms*, it helps to *simplify it as much as possible first*. The fewer terms you have to deal with, the *less likely* you are to make a *mistake*. This Lesson is about two ways that you can *simplify expressions*.

Simplifying an Expression Makes It Easier to Solve

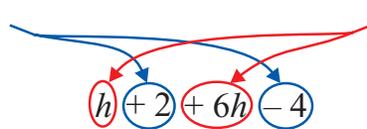
In math you'll come across some very **long expressions**.

The first step toward solving them is to **simplify** them.

The first way to simplify an expression is to **collect like terms**.

Like terms are terms that contain **exactly the same** variables.

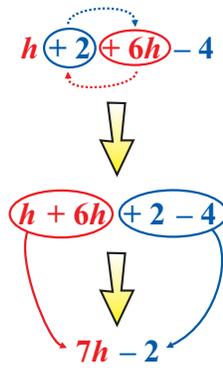
These are like terms because they contain no variables — they're both constants.



These are like terms because they both contain the same variable, h .

You need to bring **like terms** together to simplify the expression.

When you do that, the plus or minus sign in front of a term stays with that term as it moves.



First swap the positions of the "+ 2" term and the "+ 6h" term.

Now all the terms containing the variable h are together and all the constants are together.

Then simplify the grouped terms.

You can't simplify this any more because there are no longer any like terms.

Example 1

Simplify the expression $x - 5 + 2y + 9 - y + 2x$.

Solution

$$\begin{aligned} &x - 5 + 2y + 9 - y + 2x \\ &= (x + 2x) + (2y - y) + (-5 + 9) \\ &= 3x + y + 4 \end{aligned}$$

Collect together the like terms
Simplify the parentheses

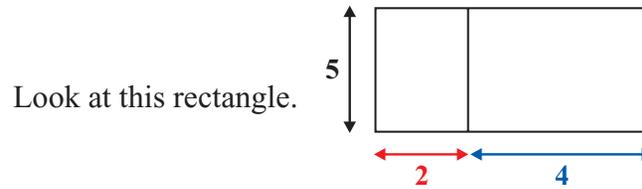
Guided Practice

Simplify the expressions in Exercises 1–6 by collecting like terms.

- $a + 4 + 2a$
- $3r + 6 - 5r$
- $c - 2 + 4 \cdot c - 3$
- $4x + 5 - x + 4 - 2x$
- $7 - k + 2k + 3 - k$
- $m + 4 - n + m - 2 - n$

In an expression like $2(3m + 4) + m$ the **parentheses** stop you from **collecting like terms**. To **simplify** it any more you first need to **remove** the parentheses. You can use a property of math called the **distributive property** to do this.

Use the Distributive Property to Remove Parentheses



You can find its **total area** in two different ways.

- You could find the **areas** of both of the **smaller rectangles** and **add** them.
Total area = $(5 \cdot 2) + (5 \cdot 4) = 10 + 20 = 30$
- Or you could find the total width of the whole rectangle by adding 2 and 4, and then multiply by the height. Total area = $5(2 + 4) = 5(6) = 30$

Whichever way you work it out you get the **same answer**, because both expressions represent the **same area**. This is an example of the **distributive property**.

So:

$$5 \cdot (4 + 2) = (5 \cdot 4) + (5 \cdot 2) = 20 + 10 = 30$$

To use the distributive property you multiply the number outside the parentheses by every term inside the parentheses.

Algebraically the property is written as:

The Distributive Property

$$a(b + c) = ab + ac$$

Don't forget:

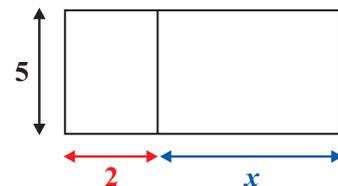
The formula for the area of a rectangle is:
Area = Length \times Width

Check it out:

No matter what numbers a , b , and c stand for, the rule will always be true.

The Distributive Property and Variable Expressions

Think about what would happen to the problem above if you didn't know one of the lengths.



Find its **total area** using the two different methods again.

- Adding the area of the two small rectangles:
Total area = $(5 \cdot 2) + (5 \cdot x) = 10 + 5x$
- Adding 2 and x to find the width and multiplying by the height:
Total area = $5(2 + x)$

Again, you know that the two expressions have the **same value** because they represent the **same area**. So you can say that: $5(2 + x) = 10 + 5x$

You can use the **distributive property** to **multiply out** the parentheses in the first expression to get the second equivalent expression.

Example 2

Write the expression $5(y - 3)$ without using parentheses.

Solution

$$\begin{aligned} & 5(y - 3) \\ &= (5 \cdot y) + (5 \cdot -3) && \text{Multiply both } y \text{ and } -3 \text{ by } 5 \\ &= 5y - 15 && \text{Simplify the multiplications} \end{aligned}$$

You don't know what number y is standing in for, so this is the most that you can simplify the expression.

Don't forget:

If you're multiplying the contents of parentheses by a negative number, remember to include the negative sign when you multiply each term.

For example:

$$\begin{aligned} -3(2 - 1) &= (-3 \cdot 2) + (-3 \cdot -1) \\ &= -6 + 3 = -3 \end{aligned}$$

Guided Practice

Write the expressions in Exercises 7–14 in a new form using the distributive property.

- | | |
|-----------------|---------------------------|
| 7. $6(w + 5)$ | 8. $4(7 - h)$ |
| 9. $-2(d + 2)$ | 10. $-3(f - 3)$ |
| 11. $-1(3 - x)$ | 12. $-2(-3 - m)$ |
| 13. $y(9 + t)$ | 14. $3(3 + k) + 2(k - 0)$ |

Independent Practice

Simplify the expressions in Exercises 1–3 by collecting like terms.

1. $2p + 5 + 2p$ 2. $3 + 4x - 5 + x$ 3. $5h + 7 - h - 3 + 2h$

4. Keon and Amy are collecting leaves for a project. On a walk Keon finds k leaves and Amy finds 6. Then Keon finds another $4k$ leaves, and Amy loses 2. Write an expression to describe how many leaves they ended up with, then simplify it fully.

Using the distributive property, write the expressions in Exercises 5–7 in a new form.

5. $8(y + 1)$ 6. $5(w - 9)$ 7. $-h(t - 4)$

8. Damian makes \$7 an hour working at the mall. Last week Damian worked for 12 hours. This week he will work for x hours.

- a) Write an expression using parentheses to describe how much money he will have made altogether?
b) Given that $x = 8$, evaluate your expression using the distributive property.

Simplify the expressions in Exercises 9–12 as far as possible.

9. $2(m + 1) + 1(3 - m)$ 10. $3(b + 2 - b)$
11. $2(h + 4) - 4(h - 1)$ 12. $x(2 + y) + 3(x + y)$

Round Up

Collecting like terms and using the distributive property are both really useful ways to simplify an expression. Simplifying it will make it easier to evaluate — and that will make solving equations easier later in this Chapter.

Lesson 1.1.3

The Order of Operations

California Standards:

Algebra and Functions 1.2

Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.

What it means for you:

You'll learn about the special order to follow when you're deciding which part of an expression to evaluate first.

Key words:

- parentheses
- exponents
- PEMDAS

Check it out:

Another way to remember the order of operations is using the GEMA rule:

Grouping — any symbol that groups things, like parentheses, fraction bars, or brackets.

Exponents.

Multiplication and **D**ivision — done from left to right.

Addition and **S**ubtraction — done from left to right.

Use either PEMDAS or GEMA — whichever one you feel happier with.

Check it out:

You'll learn about exponents in Section 2.4.

When you have a calculation with *more than one operation* in it, you need to know what *order* to do the operations in.

For example, if you evaluate the expression $2 \cdot 3 + 7$ by doing "multiply 2 by 3 and add 7," you'll get a different answer from someone who does "add 7 to 3 and multiply the sum by 2." So the order you use really matters.

There's a set of rules to follow to make sure that everyone gets the same answer. It's called *the order of operations* — and you've seen it before in grade 6.

The Order of Operations is a Set of Rules

An expression can contain lots of **operations**. When you evaluate it you need a set of rules to tell you **what order** to deal with the different bits in.

Order of operations — the PEMDAS Rule

$\{\}$	Parentheses	First do any operations inside parentheses.
$x^2 y^7$	Exponents	Then evaluate any exponents.
\times \div	Multiplication or Division	Next follow any multiplication and division instructions from left to right.
$+$ $-$	Addition or Subtraction	Finally follow any addition and subtraction instructions from left to right.

When an expression contains multiplication and division, or addition and subtraction, do first whichever comes first as you read from left to right.

$$9 \div 4 \cdot 3 \quad \text{Divide first, then multiply.} \quad 9 \cdot 4 \div 3 \quad \text{Multiply first, then divide.}$$

$$9 + 4 - 3 \quad \text{Add first, then subtract.} \quad 9 - 4 + 3 \quad \text{Subtract first, then add.}$$

Following these rules means that there's only one correct answer. Use the rules each time you do a calculation to make sure you get the **right answer**.

Example 1

What is $8 \div 4 \cdot 4 + 3$?

Solution

Follow the order of operations to decide which operation to do first.

$$8 \div 4 \cdot 4 + 3 \quad \text{There are no parentheses or exponents}$$

$$= 2 \cdot 4 + 3 \quad \text{Do the division first}$$

$$= 8 + 3 \quad \text{Then the multiplication}$$

$$= 11 \quad \text{Finally do the addition to get the answer}$$

You do the division first as it comes before the multiplication, reading from left to right.

✓ Guided Practice

Evaluate the expressions in Exercises 1–6.

1. $3 - 2 + 6 - 1$

2. $6 \div 2 + 1$

3. $4 + 3 - 2 + 7$

4. $2 + 5 \cdot 10$

5. $40 - 10 \div 5 \cdot 6$

6. $5 + 10 \div 10$

Always Deal with Parentheses First

When a calculation contains **parentheses**, you should deal with any operations inside them **first**. You still need to follow the **order of operations** when you're dealing with the parts inside the parentheses.

Example 2

What is $10 \div 2 \cdot (10 + 2)$?

Solution

The **order of operations** says that you should deal with the operations in the **parentheses** first — that's the **P** in **PEMDAS**.

$$\begin{aligned} & 10 \div 2 \cdot (10 + 2) \\ &= 10 \div 2 \cdot 12 \\ &= 5 \cdot 12 \\ &= \mathbf{60} \end{aligned}$$

Do the addition in parentheses
Then do the division
Finally do the multiplication

You do the division first here because it comes first reading from left to right.

Don't forget:

Remember to show all your work step by step to make it clear what you're doing.

✓ Guided Practice

Evaluate the expressions in Exercises 7–14.

7. $10 - (4 + 3)$

8. $(18 \div 3) + (2 + 3 \cdot 4)$

9. $10 \div (7 - 5)$

10. $41 - (4 + 2 - 3)$

11. $10 \cdot (2 + 4) - 3$

12. $(5 - 7) \cdot (55 \div 11)$

13. $6 \cdot (8 \div 4) + 11$

14. $32 + 2 \cdot (16 \div 2)$

PEMDAS Applies to Algebra Problems Too

The order of operations still applies when you have calculations in **algebra** that contain a mixture of **numbers** and **variables**.

Example 3

Simplify the calculation $k \cdot (5 + 4) + 16$ as far as possible.

Solution

$$\begin{aligned} & k \cdot (5 + 4) + 16 \\ &= k \cdot 9 + 16 \\ &= \mathbf{9k + 16} \end{aligned}$$

Do the addition within parentheses
Then the multiplication

✓ Guided Practice

Simplify the expressions in Exercises 15–20 as far as possible.

15. $5 + 7 \cdot x$

16. $2 + a \cdot 4 - 1$

17. $3 \cdot (y - 2)$

18. $10 \div (3 + 2) - r$

19. $20 + (4 \cdot 2) \cdot t$

20. $p + 5 \cdot (-2 + m)$

✓ Independent Practice

1. Alice and Emilio are evaluating the expression $5 + 6 \cdot 4$. Their work is shown below.

Alice

Emilio

$$5 + 6 \cdot 4$$

$$5 + 6 \cdot 4$$

$$= 11 \cdot 4$$

$$= 5 + 24$$

$$= 44$$

$$= 29$$

Explain who has the right answer.

The local muffler replacement shop charges \$75 for parts and \$25 per hour for labor.

2. Write an expression with parentheses to describe the cost, in dollars, of a replacement if the job takes 4 hours.

3. Use your expression to calculate what the cost of the job would be if it did take 4 hours.

Evaluate the expressions in Exercises 4–7.

4. $2 + 32 \div 8 - 2 \cdot 5$

5. $4 + 7 \cdot 3$

6. $7 + 5 \cdot (10 - 6 \div 3)$

7. $3 \cdot (5 - 3) + (27 \div 3)$

8. Paul buys 5 books priced at \$10 and 3 priced at \$15. He also has a coupon for \$7 off his purchase. Write an expression with parentheses to show the total cost, after using the coupon, and then simplify it to show how much he spent.

9. Insert parentheses into the expression $15 + 3 - 6 \cdot 4$ to make it equal to 48.

Simplify the expressions in Exercises 10–12 as far as possible.

10. $x - 7 \cdot 2$

11. $y + x \cdot (4 + 3) - y$

12. $6 + (60 - x \cdot 3)$

Now try these:

Lesson 1.1.3 additional questions — p430

Round Up

If you evaluate an expression in a *different order* from everyone else, you won't get the *right answer*. That's why it's so important to follow the *order of operations*. This will feature in almost all the math you do from now on, so you need to know it. Don't worry though — just use the word *PEMDAS* or *GEMA* to help you remember it.

Lesson 1.1.4

The Identity and Inverse Properties

California Standards:

Algebra and Functions 1.3

Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

What it means for you:

You'll learn how to use math properties to show why the steps of your work are reasonable.

Key words:

- justify
- identity
- inverse
- reciprocal
- multiplicative
- additive

Don't forget:

Writing x is exactly the same as writing $1x$ or $1 \cdot x$.

When you're *simplifying* and *evaluating* expressions you need to be able to *justify* your work. To justify it means to use *known math properties* to explain why each step of your calculation is *valid*.

The math properties describe the ways that numbers and variables in expressions behave — you need to know their names so that you can say which one you're using for each step.

The Identity Doesn't Change the Number

There are two identity properties — one property for **addition** and one property for **multiplication**:

The Additive Identity = 0

For any number, a , $a + 0 = a$.

Adding **0** to a number **doesn't change it**. For example:

$$5 + 0 = 5$$

$$x + 0 = x$$

This is called the **identity property of addition**, and **0** is called the **additive identity**.

The Multiplicative Identity = 1

For any number, a , $a \cdot 1 = a$.

Multiplying a number by **1** **doesn't change it**. For example:

$$1 \cdot 7 = 7$$

$$1 \cdot x = x$$

This is called the **identity property of multiplication**, and **1** is called the **multiplicative identity**.

✓ Guided Practice

1. What do you get by multiplying 6 by the multiplicative identity?
2. What do you get by adding the additive identity to $3y$?

Complete the expressions in Exercises 3–6.

3. $x + \underline{\quad} = x$

4. $\underline{\quad} + 0 = h$

5. $k \cdot \underline{\quad} = k$

6. $1 \cdot \underline{\quad} = t$

The Inverse Changes the Number to the Identity

There are two inverse properties — one for **addition** and one for **multiplication**. Different numbers have different **additive inverses** and different **multiplicative inverses**.

The Additive Inverse Adds to Give 0

The **additive inverse** is what you add to a number to get **0**.

The additive inverse of 2 is -2 $\Rightarrow 2 + -2 = 0$

The additive inverse of -3 is 3 $\Rightarrow -3 + 3 = 0$

The additive inverse of $\frac{1}{4}$ is $-\frac{1}{4}$ $\Rightarrow \frac{1}{4} + -\frac{1}{4} = 0$

Don't forget:

Remember — adding a negative number can be rewritten as a subtraction:
 $2 + -2 = 2 - 2 = 0$

The Additive Inverse of a is $-a$.

For any number, a , $a + -a = 0$.

✓ Guided Practice

Give the additive inverses of the numbers in Exercises 7–12.

7. 6

8. 19

9. -5

10. -165

11. $\frac{1}{7}$

12. $-\frac{2}{3}$

The Multiplicative Inverse Multiplies to Give 1

The **multiplicative inverse** is what you multiply a number by to get **1**. So, a number's **multiplicative inverse** is **one divided by the number**.

The multiplicative inverse of 2 is $1 \div 2 = \frac{1}{2}$.

The multiplicative inverse of 7 is $1 \div 7 = \frac{1}{7}$.

To check, $2 \cdot \frac{1}{2} = \frac{2 \cdot 1}{2} = \frac{2}{2} = 1$ and $7 \cdot \frac{1}{7} = \frac{7 \cdot 1}{7} = \frac{7}{7} = 1$.

Don't forget:

A multiplicative inverse is sometimes called a reciprocal.

The Multiplicative Inverse of a is $\frac{1}{a}$.

For any nonzero number, a , $a \cdot \frac{1}{a} = 1$.

✓ Guided Practice

Give the multiplicative inverses of the numbers in Exercises 13–16.

13. 2

14. 10

15. -4

16. -5

Fractions Have Multiplicative Inverses Too

When you multiply two **fractions** together, you multiply their **numerators** and their **denominators** separately.

For example: $\frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$

If you **multiply** a **fraction** by its **multiplicative inverse**, the product will be **1** — because that's the definition of a multiplicative inverse.

For a fraction to equal 1, the numerator and denominator must be **the same**.

So when a fraction is multiplied by its inverse, the product of the numerators must be the same as the product of the denominators.

For example: $\frac{3}{4} \cdot \frac{4}{3} = \frac{3 \cdot 4}{4 \cdot 3} = \frac{12}{12} = 1$

You can say that for any two non-zero numbers a and b , $\frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ab} = 1$.

So, the multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$.

The **multiplicative inverse**, or **reciprocal**, of a **fraction** is just the fraction **turned upside down**.

Example 1

Give the multiplicative inverse of $\frac{1}{4}$.

Solution

$\frac{4}{1}$, or 4, is the **multiplicative inverse** of $\frac{1}{4}$.

To check your answer **multiply** it by $\frac{1}{4}$.

$\frac{1}{4} \cdot \frac{4}{1} = \frac{1 \cdot 4}{4 \cdot 1} = \frac{4}{4} = 1$.

So 4 is the multiplicative inverse of $\frac{1}{4}$.

Don't forget:

Any fraction where the numerator is the same as the denominator is equal to 1.

So,

$$\frac{12}{12} = \frac{9}{9} = \frac{4}{4} = \frac{1}{1} = 1$$

Don't forget:

Any number divided by one is equal to itself. So any whole number can be written as a fraction by putting it over 1.

$$\text{So } 6 = \frac{6}{1} \text{ and } 4 = \frac{4}{1}.$$

✓ Guided Practice

Give the multiplicative inverses of the fractions in Exercises 17–20.

17. $\frac{2}{5}$

18. $\frac{1}{10}$

19. $-\frac{3}{4}$

20. $-\frac{1}{2}$

You Can Use Math Properties to Justify Your Work

To **justify** your work you need to use **known math properties** to **explain** why each step of your calculation is **valid**.

Check it out:

To justify a step of your work you can write beside it which of the known math properties you have used.

Example 2

Simplify the expression $4(2 - \frac{1}{4}x)$. Justify your work.

Solution

$$\begin{aligned} & 4(2 - \frac{1}{4}x) \\ &= 4 \cdot 2 - 4 \cdot \frac{1}{4}x && \text{The distributive property} \\ &= 8 - 1x && \text{The inverse property of multiplication} \\ &= 8 - x && \text{The identity property of multiplication} \end{aligned}$$

✓ Guided Practice

Simplify the expressions in Exercises 21–24. Justify your work.

21. $m \cdot 1 + 6$

22. $d + 0 - d + 9$

23. $\frac{1}{3}(9 + 3f)$

24. $-5(2 - 4 \cdot \frac{1}{4}a) + 10$

✓ Independent Practice

Complete the expressions in Exercises 1–4.

1. $1 \cdot \underline{\quad} = 5$

2. $\underline{\quad} + 0 = -2$

3. $2.5 \cdot \underline{\quad} = 2.5$

4. $-0.5 + \underline{\quad} = -0.5$

Give the additive and multiplicative inverses of the numbers in Exercises 5–8.

5. 6

6. -7

7. $\frac{5}{7}$

8. $-\frac{2}{3}$

Simplify the expressions in Exercises 9–12. Justify your work.

9. $a + a - a$

10. $\frac{1}{4} \cdot 4 \cdot d$

11. $5(\frac{1}{5} + 2 - n)$

12. $2(\frac{1}{2}x + 4 + 0) - 1(5 - 5 + 7)$

Now try these:

Lesson 1.1.4 additional questions — p431

Round Up

The *identity property* and the *inverse property* are two *math properties* you'll need to use in *justifying* your work. *Justifying your work* is *explaining* how you know that each step is right. In the next Lesson you'll cover two more properties that can be used in justifying your work.

Lesson 1.1.5

The Associative and Commutative Properties

California Standards:

Algebra and Functions 1.3

Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

What it means for you:

You'll learn about some more math properties that will help you to justify your work.

Key words:

- associative property
- commutative property
- justify

There are two more properties you need to know about to help *simplify and evaluate expressions*. They're the *associative and commutative properties*. They allow you to be a little more flexible about the order you do calculations in.

You used these properties already in earlier grades. It's important to know their names and to practice using them to *justify* your work.

The Associative Properties

If you **change** the way that you group numbers and variables in a **multiplication or addition** expression, you **won't change the answer**.

$$7 + (4 + 2) = (7 + 4) + 2 = 13$$

$$4 \cdot (y \cdot 5) = (4 \cdot y) \cdot 5 = 20y$$

The numbers and variables don't move — only the parentheses do.

These are the **associative properties of addition and multiplication**. In math language they are:

The Associative Properties

Addition: $(a + b) + c = a + (b + c)$

Multiplication: $(ab)c = a(bc)$

Sometimes changing the grouping in an expression using the associative property allows you to simplify it.

Example 1

Simplify the expression $(h + 12) + 13$ using the associative property.

Solution

$$\begin{aligned} &(h + 12) + 13 \\ &= h + (12 + 13) \quad \text{The associative property of addition} \\ &= h + 25 \quad \text{Do the addition} \end{aligned}$$

Example 2

Simplify the expression $15(10y)$ using the associative property.

Solution

$$\begin{aligned} &15 \cdot (10 \cdot y) \\ &= (15 \cdot 10) \cdot y \quad \text{The associative property of multiplication} \\ &= 150y \quad \text{Do the multiplication} \end{aligned}$$

✓ Guided Practice

Simplify the expressions in Exercises 1–6. Use the associative property.

- $10 + (15 + k)$
- $5(7d)$
- $(x + 5) + 7$
- $-3(4f)$
- $(y + 13) + (20 + m)$
- $0.5(3p)$

The Commutative Properties

When you're adding numbers together it doesn't matter what order you add them in — the answer is always the same.

For example: $10 + 14 = 24$ and $14 + 10 = 24$

Also, when you're multiplying numbers it doesn't matter what order you multiply them in — the answer is always the same.

For example: $4 \cdot x \cdot 2 = 8x$ and $2 \cdot 4 \cdot x = 8x$

The numbers and variables move around, but the answer doesn't change — these are the **commutative properties**. Algebraically they're written as:

The Commutative Properties

Addition: $a + b = b + a$

Multiplication: $ab = ba$

Check it out:

You can't use the associative or commutative properties on a subtraction or a division problem. But you could use the inverse property to change it into an addition or multiplication problem first. Subtraction is the same as adding a negative number, and division is the same as multiplying by a fraction.

So $x - y = x + -y$

and $x \div y = x \cdot \frac{1}{y}$.

When they're expressed in this way, you can use the associative and commutative properties.

Example 3

Simplify the expression $18v + 9 + 2v + 4$. Justify your work.

Solution

$$\begin{aligned} & 18v + 9 + 2v + 4 \\ &= 18v + 2v + 9 + 4 && \text{The commutative property of addition} \\ &= (18v + 2v) + (9 + 4) && \text{The associative property of addition} \\ &= \mathbf{20v + 13} && \text{Do the additions} \end{aligned}$$

Example 4

Simplify the expression $4 \cdot n \cdot 9$ using the commutative property.

Solution

$$\begin{aligned} & 4 \cdot n \cdot 9 \\ &= 4 \cdot 9 \cdot n && \text{The commutative property of multiplication} \\ &= (4 \cdot 9) \cdot n && \text{The associative property of multiplication} \\ &= \mathbf{36n} && \text{Do the multiplications} \end{aligned}$$

✓ Guided Practice

Simplify the expressions in Exercises 7–12.

- $7 + j + 15$
- $11 \cdot x \cdot 20$
- $3 + 4t + 7 + t$
- $3 \cdot -y \cdot 4$
- $5 + q + 3 + -q$
- $-2 \cdot f \cdot -3$

You Can Use the Properties to Justify Your Work

Check it out:

To justify your work means to use a math property to explain why each step of your calculation is valid.

Don't forget:

The math properties that you've seen in the last two Lessons are:

- the distributive property and the
 - associative,
 - commutative,
 - identity, and
 - inverse
- properties of multiplication and addition.

And you've also seen how to collect like terms.

You can use **all the properties** together to **justify** the work you do when solving a math problem.

Example 5

Simplify the expression $3(x + 5 + 2x)$. Justify your work.

Solution

$$\begin{aligned} & 3(x + 5 + 2x) \\ &= 3x + 15 + 6x && \text{The distributive property} \\ &= 3x + 6x + 15 && \text{The commutative property of addition} \\ &= \mathbf{9x + 15} && \text{Collect like terms} \end{aligned}$$

Example 6

Simplify the expression $(x \cdot y) \cdot \frac{1}{y}$. Justify your work.

Solution

$$\begin{aligned} & (x \cdot y) \cdot \frac{1}{y} \\ &= x \cdot (y \cdot \frac{1}{y}) && \text{The associative property of multiplication} \\ &= x \cdot 1 && \text{The multiplicative inverse property} \\ &= \mathbf{x} && \text{The identity property of multiplication} \end{aligned}$$

✓ Guided Practice

Simplify the expressions in Exercises 13–16. Justify your work.

13. $2(\frac{1}{2}h)$

14. $(c + -2) + 2$

15. $5(1 + t + 3 + 2t)$

16. $p \cdot 5 \cdot \frac{1}{p} \cdot 2$

✓ Independent Practice

Simplify the expressions below using the associative properties.

1. $55 + (7 + z)$

2. $6 \cdot (10 \cdot t)$

3. $-3(7k)$

4. $(-22 + q) + (q + 30)$

Simplify the expressions below. Use the commutative properties.

5. $7 + f + 3 + f$

6. $9 \cdot y \cdot 4$

7. $2 + a + 18 + b$

8. $-3 \cdot c \cdot 4 \cdot -h$

Simplify the expressions in Exercises 9–12. Justify your work.

9. $-5 + m + 5$

10. $(r \cdot \frac{1}{2}) \cdot 2$

11. $2(x + 5 + -x)$

12. $(\frac{1}{p} \cdot 3) \cdot p \cdot 2(4 + p - 4)$

Now try these:

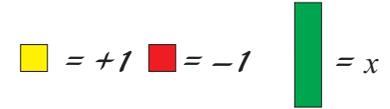
Lesson 1.1.5 additional questions — p431

Round Up

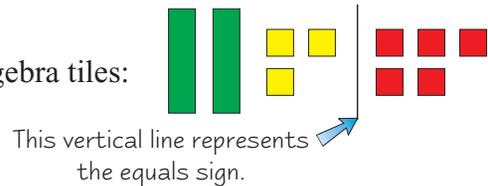
The associative and commutative properties are two more math properties. They're all important tools to use when you're simplifying expressions. By saying which property you are using in each step, you can justify your work.

Section 1.2 introduction — an exploration into: Solving Equations

You can use *algebra tiles* to model algebra equations. You can also use them to model how you *solve equations* to find the *value of x*. The tiles on the right will be used to represent algebra equations:



Here is the algebra equation $2x + 3 = -5$ modeled using algebra tiles:



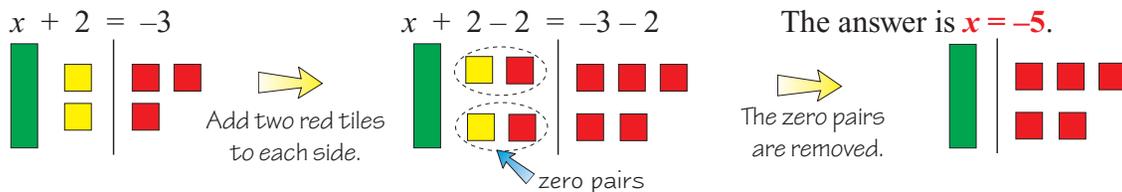
To solve an equation, you need to get **one green “x” tile** by itself on one side of the equation, and only **red and yellow tiles** on the other. This way you’ll know the value of **one green “x” tile**.

You’ll often need to use the idea of **“zero pairs.”** A yellow and a red tile together make **zero**. This is called a **zero pair**. \rightarrow $1 + (-1) = 0$

Example

Use algebra tiles to model and solve the equation $x + 2 = -3$.

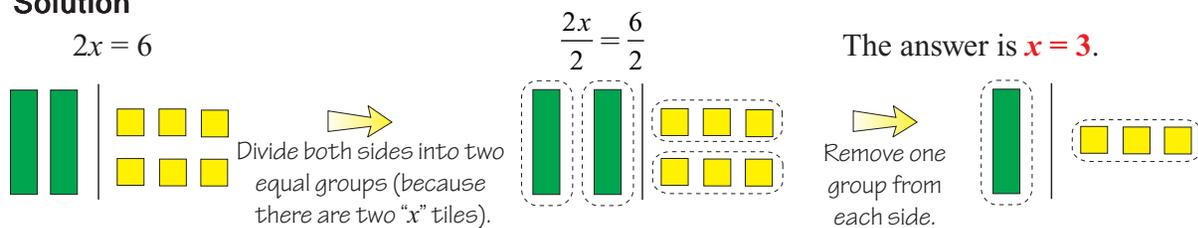
Solution



Example

Use algebra tiles to model and solve the equation $2x = 6$.

Solution



Exercises

- Use algebra tiles to model these equations.
 - $x + 3 = 4$
 - $x - 5 = 3$
 - $3x = 9$
 - $4x = -8$
- Use algebra tiles to solve the equations above.

Round Up

When you *solve* an equation, the aim is always to get the *variable on its own*, on one side of the equation. In this Exploration, the variable was the green tile “x.” To get it on its own, you have to do *exactly the same to both sides* of the equation — or else the equation won’t stay balanced.

Lesson 1.2.1

Section 1.2 Writing Expressions

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

What it means for you:

You'll learn how to change word descriptions into math expressions.

Key words:

- addition
- subtraction
- product
- quotient

Don't forget:

There are lots of different phrases that describe the four operations. You could come across any of them, so you need to remember which of the operations each phrase refers to. For a reminder, see Lesson 1.1.1.

You can use both a word expression and a math expression to describe the same situation. In Section 1.1 you practiced changing numeric and variable expressions into word expressions. This Lesson is all about doing the reverse — changing word expressions into math expressions.

Variable Expressions Describe Word Expressions

To write an expression to represent a word sentence you need to figure out what the word sentence actually means.

Example 1

Write a variable expression to describe the sentence
“A number, x , is increased by five.”

Solution

The phrase “a number, x ” is telling you that x is the variable being used. The words “increased by” are telling you that something is being added on to the variable x . In this case it is the number five.

So the sentence “A number, x , is increased by five” translates as $x + 5$.

Example 2

Write a variable expression to describe the sentence
“Nine is multiplied by a number, k .”

Solution

The operation phrase being used is “multiplied by.” The rest of the sentence tells you that it is the number nine and the variable k that are being multiplied together.

So “Nine is multiplied by a number, k ” translates as $9 \cdot k$ or $9k$.

✓ Guided Practice

Write variable expressions to describe the phrases in Exercises 1–5.

1. Six more than a number, h .
2. Seven is decreased by a number, m .
3. A number, g , divided by 11.
4. The product of a number, w , and 10.
5. A number, k , divided into four equal parts.

You Need to Sort Out the Important Information

You'll often need to write a **math expression** as the first step toward solving a **word problem**. That might include **choosing variables** as well as working out what **operations** the words are describing.

Example 3

Carla and Bob have been making buttons to sell at a fund-raiser. Carla made four more than Bob. Write an expression to describe how many buttons they made between them.

Solution

First you need to work out what the expression you have to write must describe. In this case the expression must describe the **total number of buttons made by Carla and Bob**.

See if there is an unknown number in the question: **you don't know how many buttons Bob made**. You don't know how many Carla made either, but you can say how many she made compared with Bob, so you only need one variable.

Assign a letter or symbol to the unknown number: **let b = the number of buttons Bob made**.

Then you need to identify any operation phrases: **“more than” is an addition phrase**.

Carla made four more buttons than Bob, so she made $b + 4$ buttons. Which means that together they made $b + 4 + b$ buttons.

This expression can be simplified to $2b + 4$.

Don't forget:

To simplify this expression you can collect like terms, as you did in Lesson 1.1.2. If you collect the two b terms together you get $2b$.

✓ Guided Practice

Write variable expressions to describe the sentences in Exercises 6–9. Use x as the variable in each case, and say what it represents.

6. A rectangle has a length of 2 inches. What is its area?
7. Jenny has five fewer apples than Jamal. How many apples does Jenny have in total?
8. The student council is selling fruit juice at the prom for \$0.75 a glass. How much money will they take?
9. A gym charges \$10 per month membership plus \$3 per visit. What is the cost of using the gym for a month?

Some Expressions Describe More than One Operation

You can also translate sentences with **multiple operations** in the same way. You just need to spot all the separate operations and work out **what order** to write them in.

Example 4

Write a variable expression to describe the phrase “ten decreased by the product of a number, y , and two.”

Solution

In this question the phrase contains **two different operation phrases**, so you need to work out which operation is carried out **first**.

The two operations here are “**decreased by**,” which is a **subtraction** phrase, and “**product**,” which is a **multiplication** phrase. You’re told to subtract the product from 10. So you need to work out the **product** first — this is the product of y and 2, which is $2y$.

Now you have to subtract this product, $2y$, from 10.

So the phrase “ten decreased by the product of a number, y , and 2.” translates as $10 - 2y$.

Don't forget:

You might need to include parentheses in your expression to show which operation needs to be done first.

Guided Practice

Write variable expressions to describe the phrases in Exercises 10–14.

10. Five more than twice a number, q .
11. Sixteen divided by the sum of a number, m , and 7.
12. Twenty decreased by a quarter of a number, j .
13. The product of 7 and six less than a number, t .
14. The product of a number, k , and the sum of 5 and a number, x .

Independent Practice

Write variable expressions to describe the phrases in Exercises 1–5.

1. The product of six and a number, h .
2. A number, y , decreased by eleven.
3. A fifteenth of a number, p .
4. Nine more than twice a number, w .
5. Sixteen increased by the product of a number, k , and three.
6. A pen costs half as much as a ruler. Write an expression to describe how much the pen costs, using r as the cost of the ruler.
7. Peter has three fewer cards than Neva. Write an expression to describe how many cards they have together, using c as the number of cards Neva has.

Now try these:

Lesson 1.2.1 additional questions — p431

Round Up

Changing word expressions into algebra expressions is all about spotting the operation phrases and working out what order the operations need to be written in. Writing expressions is the first step toward writing equations — a skill that you'll use when solving problems later in this Section.

Lesson 1.2.2

Equations

California Standards:

Algebra and Functions 1.4

Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

What it means for you:

You'll learn what an equation is, and how it's different from an expression.

Key words:

- equation
- expression
- formula

In Lesson 1.1.2 and Lesson 1.2.1 you learned how to write expressions. Writing equations takes writing expressions one step further — equations are made up of two expressions joined by an equals sign.

An Equation Has an Equals Sign

An **equation** is made up of **two expressions** joined together by an **equals sign**. The equals sign is **really important** — it tells you that the expressions on each side of the equation have **exactly the same value**.

$$5x + 8 = 10x - 12$$

Expression 1... ...is equal to... ...Expression 2.

Numeric Equations Contain Only Numbers

Numeric equations contain **only numbers** and **operations**. For example, $3 + 2 = 5$ and $(6 \cdot 4) + 3 = 31 - 4$ are both numeric equations.

If both sides of the equation do have the **same value**, then the equation is said to be **true**, or **balanced**.

Example 1

Prove that $(10 \cdot 4) + 4 = (8 \cdot 11) \div 2$ is a true equation.

Solution

To show that this is a true equation, you need to **evaluate** both sides. Treat them as **two expressions**, and **evaluate** them both according to the **order of operations**.

$$\begin{aligned}(10 \cdot 4) + 4 &= (8 \cdot 11) \div 2 && \text{First simplify the parentheses} \\ 40 + 4 &= 88 \div 2 && \text{Then complete the work} \\ 44 &= 44\end{aligned}$$

Both sides equal 44, so the equation is true.

Don't forget:

The order of operations is Parentheses, Exponents, Multiplication and Division, then Addition and Subtraction — PEMDAS.

The **numbers** and **operations** are **different** on the **left-hand** and **right-hand sides** of the equation. But the **value** of both sides is the **same**.

✓ Guided Practice

Prove that the equations in Exercises 1–6 are true by evaluating both sides.

1. $4 + 5 = 9 + 0$

2. $4 \cdot 5 = 60 \div 3$

3. $4 + 2 \cdot 3 = 3 \cdot 2 + 4$

4. $6 - 4 \div 2 = 12 \div 2 - 2$

5. $8 + 6 \cdot 3 = 2(10 + 3)$

6. $20 \div 4 - 5 = 14 \div 2 - 7$

Variable Equations Contain Numbers and Variables

Variable equations contain **variables** as well as numbers and operations. There may be variables on **either or both sides** of the equation.

For example, $2x + 2 = 6$ and $3x = 2y$ are both variable equations.

The **same rules** that apply to numeric equations also apply to variable equations. The expressions that make up the two sides of the equation still have to be **equal in value**.

The two equations above are both true if $x = 2$ and $y = 3$.

$$\begin{array}{rcl} 2x + 2 = 6 & & 3x = 2y \\ 2 \cdot 2 + 2 = 6 & & 3 \cdot 2 = 2 \cdot 3 \\ 6 = 6 & & 6 = 6 \end{array}$$

Don't forget:

x is a variable — that means that it could stand for any number. But this equation is only true when it stands for 2. So $x = 2$ is called a solution of the equation.

Writing Equations Involves Writing Expressions

To **write an equation** you write **two expressions** that have the same value and join them with an **equals sign**. One of the expressions will often just be a number.

Example 2

Write an equation to describe the sentence “Eight increased by the product of a number, k , and two is equal to twenty-four.”

Solution

The phrase “**is equal to**” represents the **equals sign**. It also **separates** the **two expressions** that make up the two sides of the **equation**.

One expression is “Eight increased by the product of a number, k , and two.” This turns into the expression $8 + 2k$.

The other expression is just a number, 24 .

So the sentence “Eight increased by the product of a number, k , and two is equal to twenty-four” turns into the equation $8 + 2k = 24$.

✓ Guided Practice

Write an equation to describe each of the sentences in Exercises 7–11.

7. Five less than a sixth of m is equal to 40.
8. Five more than the product of six and d is equal to ten.
9. Four increased by the product of three and t is equal to 40.
10. Nine less than the product of six and r is equal to 11.
11. Two times y is equal to y divided by four.

To Write an Equation, Identify the Key Information

When math problems are described using words, you'll often be given lots of **extra information** as part of the question. You need to be able to extract the **important** information and use it to **set up an equation** — just like you set up an expression.

Example 3

Sarah has been selling lemonade. The lemonade cost her \$9 to make, and she sold each glass for \$0.75. She made \$20 profit, which she is going to use to buy a necklace. Write an equation to describe this information. Use x to represent the number of glasses she sold.

Solution

Sarah made \$20 profit. So the price of one glass (\$0.75) multiplied by the number of glasses she sold (x), minus the amount the lemonade cost her to make (\$9), is equal to 20.

So you can write $0.75x - 9 = 20$.

Check it out:

It doesn't matter exactly what Sarah is selling here, or what she buys with her profit. The important thing is how the numbers in the situation relate to each other — and that's what the equation is describing.

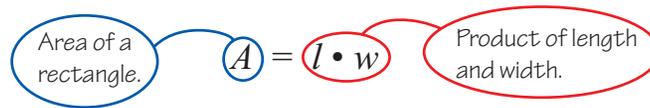
✓ Guided Practice

Write an equation to describe each of the situations in Exercises 12–15.

12. Javier spent \$20 at the gas station. He bought a drink for \$3 and spent the rest, \$ d , on gas.
13. Jane is wrapping a parcel. She needs 15 feet of string to tie it up. A roll of string is p feet long. She uses exactly three rolls.
14. Sam takes 12 sheets of paper to write an essay on. The essay is $2h$ pages long. He has k spare sheets left to put back.
15. A telephone company charges \$0.05 a minute for local calls and \$0.10 a minute for long-distance calls. Asuncion makes one local and one long-distance call. Each call is y minutes long. Her calls cost a total of \$4.

A Formula is an Equation That States a Rule

A **formula** is a specific type of **equation** that sets out a **rule** for you. It explains how some **variables** are related to each other. For example:


$$A = l \cdot w$$

Check it out:

You can use any letters or symbols you like for the variables. But make sure you use the same ones all through your work, and remember to say what they stand for.

Example 4

Write a formula for the perimeter of a square.

Solution

To calculate the **perimeter** of a square you multiply its **side length** by **4**.

Choose variables to use:

Let P = perimeter of the square, and let s = side length.

So the formula becomes $P = 4 \cdot s$.

The formula shows the **relationship** between a **square's side length and perimeter**. The formula works for **any square** at all — if you are given the value of **one** of the variables you can always **calculate the other**.

Guided Practice

Use the formula to calculate the missing values in Exercises 16–18.

16. Rectangle area = length \cdot width length = 4 cm, width = 0.5 cm

17. Speed = distance \div time distance = 8 miles, time = 2 h

18. Length in cm = 2.54 \cdot (length in in.) length in in. = 10

Independent Practice

1. Which of a) and b) is an expression? Which is an equation? How do you know? a) $2w - 6 = 21$ b) $5c + 3$

2. Given that Distance = Speed \cdot Time, calculate the distance traveled when a car goes 55 mi/h for 8 hours.

Write an equation to describe each of the sentences in Exercises 3–5.

3. Six more than x is equal to four.

4. The product of h and two is equal to 40 decreased by h .

5. Ten increased by the result of dividing five by t is equal to nine.

6. Mike earned \$100 working for h hours in a restaurant. He earns \$10 an hour, and received \$30 in tips. Write an equation using this information.

Now try these:

Lesson 1.2.2 additional questions — p432

Round Up

The **equals sign** in an equation is very important — it tells you that both sides of the equation have exactly the same **value**. In the next Lesson you'll see how to solve an equation that you've written.

Lesson 1.2.3

Solving One-Step Equations

California Standards:

Algebra and Functions 4.1

Solve two-step **linear equations** and inequalities **in one variable over the rational numbers**, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

What it means for you:

You'll learn how to solve an equation to find out the value of an unknown variable.

Key words:

- solve
- isolate
- inverse

Don't forget:

This is using the inverse properties of addition and multiplication.

$$a + -a = 0$$

$$a \cdot \frac{1}{a} = 1$$

You came across these in Lesson 1.1.4.

*Solving an equation containing a variable means finding the **value** of the variable. It's all about **changing** the equation around to get the variable **on its own**.*

Do the Same to Both Sides and Equations Stay True

The **equals sign** in an equation tells you that the two sides of the equation are of **exactly equal value**. So if you do the **same thing** to both sides of the equation, like add five or take away three, they will still have the same value as each other.

$$\begin{array}{l} 4 + 6 = 9 + 1 \\ \text{Add 5 to} \quad \downarrow \quad \text{both sides.} \\ 4 + 6 + 5 = 9 + 1 + 5 \\ \text{Then} \quad \downarrow \quad \text{simplify.} \\ 15 = 15 \end{array}$$

All three are **balanced equations**.

You Can Use This to Find the Value of a Variable

To get a variable in an equation on its own you need to do the **inverse operation** to the operation that has already been performed on it.

- If a variable has had a number **added** to it, **subtract** the same number from both sides. $+$ \rightarrow $-$
- If a variable has had a number **subtracted** from it, **add** the same number to both sides. $-$ \rightarrow $+$
- If a variable has been **multiplied** by a number, **divide** both sides by the same number. \times \rightarrow \div
- If a variable has been **divided** by a number, **multiply** both sides by the same number. \div \rightarrow \times

For example:

$$\begin{array}{l} y - 5 = 33 \\ \downarrow \\ y - 5 + 5 = 33 + 5 \\ \downarrow \\ y + 0 = 38 \\ \downarrow \\ y = 38 \end{array}$$

y has had 5 subtracted from it, so add 5 to both sides.

You've got the variable alone on one side of the equation, so now you know its value.

Reverse Addition by Subtracting

When a variable has had something added to it, you can **undo the addition** using **subtraction**.

Example 1

Find the value of x when $x + 15 = 45$.

Solution

$$\begin{aligned}x + 15 &= 45 \\x + 15 - 15 &= 45 - 15 && \text{Subtract 15 from both sides} \\x &= 30 && \text{Simplify to find } x\end{aligned}$$

Reverse Subtraction by Adding

When a variable has had something taken away from it, you can **undo the subtraction** using **addition**.

Example 2

Find the value of k when $k - 17 = 10$.

Solution

$$\begin{aligned}k - 17 &= 10 \\k - 17 + 17 &= 10 + 17 && \text{Add 17 to both sides} \\k &= 27 && \text{Simplify to find } k\end{aligned}$$

Example 3

Find the value of g when $-10 = g - 9$.

Solution

$$\begin{aligned}-10 &= g - 9 \\-10 + 9 &= g - 9 + 9 && \text{Add 9 to both sides} \\-1 &= g && \text{Simplify to find } g\end{aligned}$$

Check it out:

Here the variable is on the right-hand side of the equals sign. But that doesn't matter — as long as it's on its own. You've still found its value by isolating it.

✓ Guided Practice

Find the value of the variable in Exercises 1–8.

1. $x - 7 = 14$

2. $70 = t + 41$

3. $f + 13 = 9$

4. $g - 3 = -54$

5. $y - 14 = 30$

6. $22 = 14 + d$

7. $4.5 = 9 + v$

8. $-6 = b - 4$

Reverse Multiplication by Dividing

When a variable in an equation has been multiplied by a number, you can **undo the multiplication** by **dividing** both sides of the equation by the same number.

$$\begin{array}{l} 2y = 18 \\ \downarrow \\ 2y \div 2 = 18 \div 2 \\ \text{Then } \downarrow \text{ simplify.} \\ y = 9 \end{array}$$

y has been multiplied by 2, so start by dividing both sides by 2.

Example 4

Find the value of b when $20b = 100$.

Solution

$$\begin{array}{l} 20b = 100 \\ 20b \div 20 = 100 \div 20 \\ \mathbf{b = 5} \end{array}$$

Divide both sides by 20
Simplify to find b

Reverse Division by Multiplying

When a variable in an equation has been divided by a number, you can **undo the division** by **multiplying** both sides of the equation by the same number.

$$\begin{array}{l} \frac{d}{2} = 50 \\ \downarrow \\ \frac{d}{2} \cdot 2 = 50 \cdot 2 \\ \text{Then } \downarrow \text{ simplify.} \\ d = 100 \end{array}$$

d has been divided by 2, so start by multiplying both sides by 2.

Example 5

Find the value of t when $t \div 4 = 6$.

Solution

$$\begin{array}{l} t \div 4 = 6 \\ t \div 4 \cdot 4 = 6 \cdot 4 \\ \mathbf{t = 24} \end{array}$$

Multiply both sides by 4
Simplify to find t

Check it out:

Fractions represent divisions.

So $1 \div 2$ and $\frac{1}{2}$ mean exactly the same thing. You can write either.

✓ Guided Practice

Find the value of the variables in Exercises 9–16.

9. $3k = 18$

10. $b \div 3 = 4$

11. $h \div 5 = -3$

12. $-9y = 99$

13. $q \div 8 = \frac{1}{2}$

14. $10t = -55$

15. $d \div -2 = -4$

16. $240 = 8m$

✓ Independent Practice

1. The Sears Tower in Chicago is 1451 feet tall, which is 405 feet taller than the Chrysler Building in New York. Use the equation $C + 405 = 1451$ to find the height of the Chrysler Building.

Find the value of the variable in Exercises 2–7.

2. $k + 7 = 10$

3. $c + 10 = -27$

4. $s + 4 = -7$

5. $70 = 5 + b$

6. $h + 0 = 14$

7. $32 = 11 + a$

8. The Holland Tunnel in New York is 342 feet longer than the 8216-foot-long Lincoln Tunnel. Use the equation $H - 342 = 8216$ to find the length of the Holland Tunnel.

Find the value of the variable in Exercises 9–14.

9. $x - 7 = 13$

10. $41 = m - 35$

11. $p - 13 = -82$

12. $t - 27 = 37$

13. $100 = g - 18$

14. $-7 = y - 2$

15. Marlon buys a sweater for \$28 that has \$17 off its usual price in a sale. Write an equation to describe the cost of the sweater in the sale compared with its usual price. Then solve the equation to find the usual price of the sweater.

Find the value of the variable in Exercises 16–21.

16. $5c = 80$

17. $v \div 7 = 3$

18. $22x = -374$

19. $h \div -2 = 4$

20. $-3k = -24$

21. $-27 = f \div 3$

22. The tallest geyser in Yellowstone Park is the Steamboat Geyser. Reaching a height of 380 feet, it is twice as high as the Old Faithful Geyser. Use the equation $2F = 380$ to find the height reached by the Old Faithful Geyser.

Now try these:

Lesson 1.2.3 additional questions — p432

Round Up

Solving an equation tells you the value of the unknown number — the variable.

To solve an equation all you need to do is the reverse of what's already been done to the variable.

That way you can isolate the variable. Just remember that you need to do the same thing to both sides. That's what keeps the equation balanced.

Lesson 1.2.4

Solving Two-Step Equations

California Standards:

Algebra and Functions 4.1

Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

What it means for you:

You'll learn how to solve an equation that involves more than one operation to find out the value of an unknown variable.

Key words:

- solve
- isolate
- inverse

Don't forget:

You need to remember to think of PEMDAS or GEMA. That way you'll know what order the operations have been done in — and what order to go in to reverse them.

When you have an equation with *two operations* in it, you need to do *two inverse operations* to isolate the variable. But other than that, the process is just the same as for solving a one-step equation.

Two-Step Equations Have Two Operations

A **two-step equation** is one that involves **two different operations**.

$$7 \cdot x + 3 = 17$$

first operation  second operation 

You need to perform two inverse operations to isolate the variable.

It's easiest to **undo** the operations in the **opposite order** to the way that they were done. It's like taking off your shoes and socks. You normally put on your socks first and then your shoes. But when you're removing them you go in the **reverse order** — you take your shoes off first, and then your socks.

Here x is first multiplied by *seven*, and then the product has *three* added to it. $7x + 3 = 17$

So to isolate the variable, first subtract *three* from both sides... $7x + 3 - 3 = 17 - 3$
 $7x = 14$

...and then divide both sides by *seven*. $7x \div 7 = 14 \div 7$
 $x = 2$

Example 1

Find the value of d when $4d + 6 = 38$.

Solution

In the equation $4d + 6 = 38$ the variable d has first been **multiplied** by 4, and 6 has then been **added** to the product. So to isolate the variable you must first **subtract** 6 from both sides, and then **divide** both sides by 4.

$$\begin{aligned} 4d + 6 &= 38 \\ 4d + 6 - 6 &= 38 - 6 && \text{Subtract 6 from both sides} \\ 4d &= 32 \\ 4d \div 4 &= 32 \div 4 && \text{Divide both sides by 4} \\ d &= 8 \end{aligned}$$

Example 2

Find the value of h when $3h - 11 = 25$.

Solution

In the equation $3h - 11 = 25$ the variable h has first been **multiplied** by 3, and 11 has then been **subtracted** from the product. So to isolate the variable you must first **add** 11 to both sides, and then **divide** them both by 3.

$$\begin{aligned}
 3h - 11 &= 25 \\
 3h - 11 + 11 &= 25 + 11 && \text{Add 11 to both sides} \\
 3h &= 36 \\
 3h \div 3 &= 36 \div 3 && \text{Divide both sides by 3} \\
 \mathbf{h} &= \mathbf{12}
 \end{aligned}$$

Guided Practice

Find the value of the variables in Exercises 1–6.

1. $2x + 8 = 12$

2. $10 + 5y = 25$

3. $5t - 6 = 34$

4. $7f - 19 = 30$

5. $60 = 8b + 12$

6. $34 = 4p - 10$

Follow the Same Procedure with All the Operations

You can use this method for any **two-step equation**. Just perform the **inverse** of the two operations in the **opposite order** to the order in which they were done.

Sometimes the order in which the operations are performed is less obvious, and you'll need to think more carefully about it.

Example 3

Find the value of r when $r \div 4 - 6 = 13$.

Solution

In the equation $r \div 4 - 6 = 13$, the variable r has first been **divided** by 4, and 6 has then been **subtracted** from the quotient. So to isolate the variable you must first **add** 6 to both sides, and then **multiply** both sides by 4.

$$\begin{aligned}
 r \div 4 - 6 &= 13 \\
 r \div 4 - 6 + 6 &= 13 + 6 && \text{Add 6 to both sides} \\
 r \div 4 &= 19 \\
 r \div 4 \cdot 4 &= 19 \cdot 4 && \text{Multiply both sides by 4} \\
 \mathbf{r} &= \mathbf{76}
 \end{aligned}$$

Check it out:

This equation could have been written as $(r \div 4) - 6 = 13$. But division takes priority over subtraction, so the parentheses aren't needed.

Example 4

Find the value of v when $(v + 2) \div 7 = 3$.

Solution

In the equation $(v + 2) \div 7 = 3$, the variable v and 2 have first been **added** together, and then their sum has been **divided** by 7. So to isolate the variable you must first **multiply** both sides by 7, and then **subtract** 2 from both sides.

$$\begin{aligned} (v + 2) \div 7 &= 3 \\ (v + 2) \div 7 \cdot 7 &= 3 \cdot 7 && \text{Multiply both sides by 7} \\ v + 2 &= 21 \\ v + 2 - 2 &= 21 - 2 && \text{Subtract 2 from both sides} \\ v &= 19 \end{aligned}$$

Check it out:

The parentheses are needed here because addition doesn't take priority over division. If the parentheses were left out, this equation would have a different solution — because of the order of operations rules.

✓ Guided Practice

Find the value of the variables in Exercises 7–12.

7. $x \div 2 + 8 = 9$

8. $d \div 7 + 4 = 6$

9. $k \div 3 - 15 = 30$

10. $y \div 4 - 3 = 12$

11. $9 = g \div 2 - 6$

12. $(j + 20) \div 5 = 3$

✓ Independent Practice

In Exercises 1–4, say which order you should undo the operations in.

1. $x \div 3 + 7 = 20$

2. $21x - 12 = 44$

3. $11 = x \div 10 - 5$

4. $14 = 2 \cdot (2 + x)$

Find the value of the variables in Exercises 5–16.

5. $4h + 2 = 22$

6. $2r + 11 = -13$

7. $10b - 5 = 55$

8. $5w - 15 = 10$

9. $14 = 2 + 3c$

10. $-10 = 2n - 2$

11. $m \div 4 + 6 = 11$

12. $d \div 2 + 9 = -9$

13. $p \div 7 - 4 = 2$

14. $f \div 3 - 17 = -20$

15. $10 = 5 + a \div 10$

16. $-20 = q \div 2 - 12$

Now try these:

Lesson 1.2.4 additional questions — p432

Round Up

Solving a *two-step* equation uses the same techniques as solving a *one-step* equation. The important thing to remember with two-step equations is to do the *inverse operations* in the *reverse* of the *original order*. This same method applies to *every equation*, no matter how many steps it has. Later in this Section you'll use this technique to solve *real-life* problems.

Lesson 1.2.5

More Two-Step Equations

California Standards:

Algebra and Functions 4.1

Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and **verify the reasonableness of the results**.

What it means for you:

You'll learn how to deal with fractions in equations, and how to check that your answer is right.

Key words:

- fraction
- isolate
- check

When you have a *fraction* in an equation, you can think of it as being *two different operations* that have been *merged together*. That means it can be solved in the same way as any other *two-step equation*.

Fractions Can Be Rewritten as Two Separate Steps

Fractions can be thought of as a combination of **multiplication** and **division**. You might see what is essentially the same expression written in several different ways. For example:

$$\begin{array}{ccc}
 \frac{3}{4}x & \xrightarrow{\quad} & \frac{3x}{4} \\
 & \searrow \quad \swarrow & \\
 & (3 \cdot x) \div 4 & \\
 \frac{1}{4} \cdot 3x & \xrightarrow{\quad} & 3 \cdot \frac{1}{4} \cdot x
 \end{array}$$

All five expressions are the same.

Deal with a Fraction in an Equation as Two Steps

Because a fraction can be split into **two steps**, an equation with a fraction in it can be solved using the two-step method.

Using the example above:

$$\begin{array}{l}
 \frac{3}{4}x = 6 \\
 3x \div 4 = 6 \\
 3x = 24 \\
 x = 8
 \end{array}
 \begin{array}{l}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{First split the expression into two separate} \\
 \text{operations: here } x \text{ is first multiplied by 3, and} \\
 \text{then divided by 4.} \\
 \leftarrow \text{Then solve as a two-step equation.}
 \end{array}$$

Check it out:

Another way to do this is to multiply both sides by the reciprocal of the fraction. Multiplying a fraction by its reciprocal gives a product of 1 — so it “gets rid of” the fraction.

To find the reciprocal of a fraction you invert it.

So the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\frac{2}{3}a = 6$$

$$\frac{3}{2} \cdot \frac{2}{3}a = 6 \cdot \frac{3}{2}$$

$$a = 9$$

For more on reciprocals see Lesson 1.1.4.

Example 1

Find the value of a when $\frac{2}{3}a = 6$.

Solution

$$\frac{2}{3}a = 6$$

$$2a \div 3 = 6$$

$$2a = 18$$

$$a = 9$$

Split the expression into two operations
Solve as a two-step equation

Don't forget:

The number on top of a fraction is called the numerator.



The number on the bottom of a fraction is called the denominator.

Here is another example — this one has a more complicated numerator.

Example 2

Find the value of h when $\frac{h+2}{4} = 3$

Solution

$$\frac{h+2}{4} = 3$$

$$(h+2) \div 4 = 3$$

$$h+2 = 12$$

$$h = 10$$

The whole expression $h+2$ is being divided by 4 — the fraction bar “groups” it. Put it in parentheses here to show that this operation originally took priority.

Split the expression into two operations
Solve as a two-step equation

✓ Guided Practice

Find the value of the variables in Exercises 1–6.

1. $\frac{1}{2}a = 2$

2. $\frac{3}{4}q = 33$

3. $\frac{2}{3}v = 4$

4. $\frac{4}{1}r = -8$

5. $6 = \frac{2}{5}s$

6. $\frac{2c}{3} = 6$

Check Your Answer by Substituting It Back In

When you've worked out the value of a variable you can **check your answer** is right by **substituting** it into the original equation.

Once you've substituted the value in, **evaluate** the equation — if the equation is still **true** then your calculated value is a **correct solution**.

$$\begin{aligned} 3x + 2 &= 14 \\ 3x + 2 - 2 &= 14 - 2 \\ 3x &= 12 \\ 3x \div 3 &= 12 \div 3 \end{aligned}$$

First solve the equation to find the value of x .

$$x = 4$$

Now substitute the calculated value back into the equation.

$$\begin{aligned} 3x + 2 &= 14, & x = 4 \\ 3(4) + 2 &= 14 \\ 12 + 2 &= 14 \end{aligned}$$

Then evaluate the equation using your calculated value.

As both sides are the same, the value of x is correct. $\rightarrow 14 = 14$

Don't forget:

If the equation isn't true when you've substituted in your solution, look back through your work to find the error.

Example 3

Check that $c = 8$ is a solution of the equation $10c + 15 = 95$.

Solution

$$\begin{aligned} 10c + 15 &= 95 \\ 10(8) + 15 &= 95 \\ 80 + 15 &= 95 \\ \mathbf{95} &= \mathbf{95} \end{aligned}$$

Substitute 8 into the equation

The equation is still true, so $c = 8$ is a solution of the equation $10c + 15 = 95$.

Check it out:

It might seem like needless extra work to check your solution, but it's always worth it just to make sure you've got the right answer.

Guided Practice

Solve the equations below and check your answers are correct.

7. $12m + 8 = 56$

8. $22 + 3h = 34$

9. $56 = 18 + 19v$

10. $16 - 4g = -28$

11. $3 - 6x = 9$

12. $5y - 12 = 28$

Independent Practice

Find the value of the variables in Exercises 1–6.

1. $\frac{3}{4}d = 24$

2. $\frac{4}{5}k = 8$

3. $-\frac{2}{3}b = 14$

4. $27 = \frac{3}{2}w$

5. $22 = n \cdot \frac{2}{5}$

6. $\frac{5t}{10} = 4$

Solve the equations in Exercises 7–10 and check your solution.

7. $2x + 4 = 16$

8. $3r - 6 = -12$

9. $6 = v \div 4 + 2$

10. $\frac{3}{4}c = 15$

11. For each of the equations, say whether a) $y = 3$, or b) $y = -3$, is a correct solution.

Equation 1: $10 - 2y = 16$

Equation 2: $-\frac{2}{3}y = -2$

For each equation in Exercises 12–14, say whether the solution given is a correct one.

12. $x \div 2 + 4 = 9$, $x = 10$.

13. $3x - 9 = 12$, $x = 4$.

14. $8 = 5x - 7$, $x = 3$.

Now try these:

Lesson 1.2.5 additional questions — p433

Round Up

You can think of a *fraction* as a *combination* of *two operations*. So a fraction in an equation can be treated as *two steps*. And don't forget — when you've found a solution, you should always *substitute it back into the equation to check that it's right*.

Lesson 1.2.6

Applications of Equations

California Standards:

Algebra and Functions 4.1

Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

Mathematical Reasoning 2.1

Use estimation to verify the reasonableness of calculated results.

What it means for you:

You'll see how to use equations to help solve real-life math problems, and how to check if your answer is sensible.

Key words:

- model
- check
- reasonable
- sensible

Check it out:

When you've solved your equation you'll need to decide if the solution needs units. In this example you're figuring out the price of a pen in dollars, so your answer is \$2. You'll see more about how to find the right units for your answer in Lesson 1.2.7.

Equations can be really useful in helping you to *understand* real-life situations. Writing an equation can help you sort out the *information* contained in a *word problem* and turn it into a *number problem*.

Equations Can Describe Real-Life Situations

An equation can help you to **model** a real-life situation — to **describe** it in math terms. For example:

- You've just had your car repaired. The bill was \$280.
- You know the parts cost \$120.
- You know the mechanic charges labor at \$40 per hour.
- You want to know how long the mechanic worked on your car.



Let h = number of hours worked by mechanic. ← 1. Choose a variable.

$$40h + 120 = 280 \quad \leftarrow 2. \text{ Write an equation.}$$

$$40h = 160 \quad \leftarrow 3. \text{ Solve the equation.}$$

$$h = 4$$

So you know the mechanic must have worked on your car for **4 hours**.

You can use an equation to help you describe almost any situation that involves numbers and **unknown numbers**.

Example 1

At the school supply store, Mr. Ellis bought a notebook costing \$3 and six pens. He spent \$15 in total. Find the price of one pen, p .

Solution

First **write out** the **information** you have:

Total spent = \$15

Cost of notebook = \$3

Cost of six pens = $6p$

You know that **six pens** and the **notebook** cost a total of **\$15**. So you can write an **equation** with the cost of each of the items bought on one side, and the total spent on the other.

$$6p + 3 = 15$$

Now you have a **two-step equation**. You can find the cost of one pen by **solving** it.

$$6p + 3 = 15$$

$$6p = 12$$

$$p = 2$$

One pen costs \$2.

✓ Guided Practice

Write an equation to describe each of the situations in Exercises 1–3. Then solve it to find the value of the variable.

1. Emily is seven years older than Ariela. The sum of their ages is 45. How old is Ariela?
2. A sale rack at a store has shirts for \$9 each. Raul has \$50 and a coupon for \$4 off any purchase. How many shirts can he buy?
3. The price for renting bikes is \$15 for half a day, then \$3 for each additional hour. How many hours longer than half a day can you keep a bike if you have \$24?

You Need to Check That Your Answer is Reasonable

When you've **solved** an equation that describes a real-life problem, you need to look at your answer carefully and see if it is **reasonable**. Here are two important things to think about:

1) Does Your Answer Make Sense?

You must always check that the answer **makes sense** in the context of the question. For example:

An orchard charges \$1.10 for **a pound** of apples. You have \$8.25. How many pounds of apples can you buy?

- Set up an equation to describe the problem:

$$\text{Number of pounds} = 8.25 \div 1.10 = 7.5$$

→ **This is a reasonable answer as the orchard will happily sell you half a pound of apples.**

But if you change the problem slightly:

A store charges \$1.10 for **a bag** of apples. You have \$8.25. How many bags of apples can you buy?

- **Number of bags = $8.25 \div 1.10 = 7.5$.**

→ **This is no longer a reasonable answer — the store wouldn't sell you half a bag of apples. You could only buy 7 bags.**

2) Is Your Answer About the Right Size?

The **size of your answer** has to make sense in relation to the question that is being asked. For example:

→ If you're finding the height of a mountain, and your answer is **5 feet**, it's **not reasonable**.

→ If you're finding the height of a person, and your answer is **5000 feet**, that's **not reasonable** either.

If the size of your answer doesn't seem **reasonable** then it's really important to go back and **check your work** to see if you've made an error somewhere.

Don't forget:

You might need to round your answer up or down, depending on the question.

In this example, you calculate that you can afford 7.5 bags — but you can only buy **whole bags**. You can't afford to buy 8 bags — so it's sensible to round your answer down to 7.

If instead you were working out how many bags you needed for a recipe and your answer came out as 7.5, then you would round it up. You would buy 8 bags, because you want **at least** 7.5.

There's a lot more about rounding, and how to round reasonably, in Section 8.3.

Example 2

Kea is going to walk 1.5 miles at a steady speed of 3 miles per hour. She works out how long it will take using the work shown. Is her answer reasonable?

Distance = 1.5 miles
Speed = 3 mi/hour
Time = $1.5 \times 3 = 4.5$ hours

Check it out:

If Kea walked at 3 miles per hour for 1 hour she'd go 3 miles. So to cover half that distance would take her half the time — 0.5 hours.

Solution

Given that Kea's walk is only 1.5 miles long and she walks at 3 mi/h, **4.5 hours is not a reasonable answer** — it is much too long.

(Kea multiplied the distance of the walk by her speed. She should **divide** the distance by the speed instead: **Time = $1.5 \div 3 = 0.5$ hours.**)

Guided Practice

- Pete is buying trading cards. One card costs 20¢. He says 10 cards will cost \$20. Is this a sensible answer? Explain why or why not.
- Six friends earn \$87 washing cars. How much will each one get if they split it evenly? Is your answer reasonable in the context of the question?
- A yard has a 150-foot perimeter. Fencing is sold in 40-foot rolls. Write an equation to describe the number of rolls, n , you need to buy to fence the yard. Solve the equation. Is your answer reasonable in the context of the question?
- Ana is $\frac{5}{6}$ as tall as T.J., who is 174 cm tall. Write an equation to describe Ana's height, A . Solve it. Is the size of your answer reasonable?

Independent Practice

Write an equation to describe each situation in Exercises 1–2, and solve the equation to answer the question.

- Don has spent \$474 ordering sticks for his hockey team. A stick costs \$50. Shipping costs \$24. How many did he buy?
- Tiana is saving up to buy a fishing rod. The rod costs \$99 with tax. She already has \$27, and can afford to save another \$12 each week. How long will it take her to save enough for the rod?
- Joy went to the fabric store to buy ribbon. She got f feet, and spent \$5. The ribbon cost 80¢ a foot. Write an equation to describe how much she got. Solve it. Is your answer reasonable in the context of the question?
- Mike is asked to multiply 5 by $\frac{1}{2}$. He says the answer is 10. Is this reasonable in the context of the question? Explain why or why not.
- Two friends run a dog walking service, each walking the same number of dogs. Write and solve an equation to show how many dogs, d , each friend walks if they walk nine dogs between them. Is your answer reasonable?

Round Up

Equations can help you to *understand* situations. They can also help you to *describe* a real-life math problem involving an unknown number and come up with a *solution*. But don't forget to always think carefully about whether the answer is a *reasonable* one in relation to the question.

Lesson 1.2.7

California Standards:

Algebra and Functions 4.1

Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

Mathematical Reasoning 1.1

Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

What it means for you:

You'll learn how to spot which pieces of information are important in answering a question, and how to check that your answer has the correct units.

Key words:

- relevant
- irrelevant
- unit

Understanding Problems

Math problems are full of all kinds of details. The challenge is to work out which bits of information you need and which bits you don't need. To be able to do this you need to understand exactly what the question is asking.

You Can't Solve a Problem with Information Missing

Sometimes a piece of **information** needed to solve a real-life problem will be **missing**. You need to be able to read the question through and **identify** exactly what vital piece of information is missing.

Example 1

Brian's mechanic charged \$320 to fix his car. The bill for labor was \$157.50. How many hours did the mechanic work on the car?

Solution

The question tells you that Brian's total bill for labor was \$157.50. But to use this piece of information to work out how many hours the mechanic worked on the car you would also need to know what the **mechanic's hourly rate** was, as **hours worked = bill for labor ÷ hourly rate**.

You can't solve the problem as **the mechanic's hourly rate is missing**.

✓ Guided Practice

In Exercises 1–4 say what piece of information is missing that you need to solve the problem.

1. Samantha is 20 inches taller than half Adam's height. How tall is Samantha?
2. A coffee bar charges \$2 for a smoothie. Sol buys a smoothie and a juice. How much is his check?
3. Erin has \$36 and is going to save a further \$12 a week. How many weeks will it take her to save enough for a camera?
4. A box contains 11 large tins and 17 small tins. A large tin weighs 22 ounces. What is the weight of the box?

Some Information in a Question May Not Be Relevant

You will often come across real-life problems that contain **more information** than you need to find a solution. Information that you **don't need** to solve a problem is called **irrelevant information**.

You need to be able to **sort out** the information you do need from the information you don't. A good example of this is a question where you have to **pick out** the information that you need from a **table**.

Example 2

At the hardware store Aura spent \$140 on paint. She bought four cans of blue paint and spent the rest of the money on green paint. Use the table below to calculate how many liters of green paint she bought.

Aura only bought blue paint and green paint. So you only need the circled data in these two rows to answer the question.

Color of paint	Volume of can (l)	Price of can (\$)
Blue	1	20
Yellow	2	35
Red	1	20
Green	1.5	30

Solution

To answer the question you need the **price of a can of blue paint**, and the **volume and price of a can of green paint**. The volume of cans of blue paint is **irrelevant**, as is the information about red and yellow paint.

- First work out how much Aura spent on **blue paint**. You know that she bought four cans of blue paint that cost \$20 each. So she spent \$80 on blue paint. That means she spent $\$140 - \$80 = \$60$ on green paint.
- Each can of green paint is \$30. So she bought $\$60 \div \$30 = 2$ cans.
- A can of green paint is 1.5 liters. So she bought $1.5 \cdot 2 = 3$ liters.

Guided Practice

Use the table from Example 2 in Exercises 5–7.

5. Eduardo bought one can of yellow paint and three liters of blue paint. How much did he spend?

6. Lamarr bought 2 cans of green paint and some yellow paint. He spent \$165. How many liters of yellow paint did he buy?

7. Amber spent \$120. She bought twice as much red paint as blue paint. How many cans of red paint did she buy?

Answers Should Always Have the Correct Units

When you work out the answer to a problem, you need to think about the right **units** to use.

If you apply the **same operations** to the units as you do to the numbers, you'll find out what units your answer should have.

Example 3

Laura drives her car 150 km in 2 hours. Use the formula $\text{speed} = \text{distance} \div \text{time}$ to calculate her average speed.

Solution

$$\text{speed} = \text{distance} \div \text{time}$$

$$\text{speed} = 150 \div 2 = 75$$

Now do the same operations to the units of the numbers:

$$\text{km} \div \text{hours} = \mathbf{\text{km/hour}}$$

So the average speed of the car is **75 km/hour**.

Check it out:

When you're writing units, remember that km/hour means the same as (km \div hours), and person-days means the same as (persons \times days).

You can do this with any calculation to find the **correct units** for the answer.

Example 4

The power consumption of a computer is 0.5 kilowatts.
If the computer is running for 4 hours, how much energy will it use?
Use the equation: Power Consumption • Time Used = Energy Used.

Solution

First do the numerical calculation.
Power Consumption • Time Used = Energy Used
 $0.5 \cdot 4 = 2$

Then work out the units.
kilowatts • hours = kilowatt-hours

The computer will use 2 kilowatt-hours of energy.

Check it out:

A kilowatt-hour is a measure of energy consumption.

Check it out:

You can use the / symbol to mean “divided by” when you are writing units.

Now try these:

Lesson 1.2.7 additional questions — p433

✓ Guided Practice

Say what units the answers will have in Exercises 8–11.

8. 40 miles ÷ 2 hours = 20 ?
9. 5 newtons • 3 meters = 15 ?
10. 6 persons • 4 days = 24 ?
11. \$25 ÷ 5 hours = 5 ?

✓ Independent Practice

1. The sale bin at a music store has CDs for \$4 each. Eric buys four CDs and some posters, and uses a coupon for \$2 off his purchase. He pays \$26. How many posters did he buy? Say what information is missing from the question that you would need to solve the problem.
2. Liz meets Ana to go ice-skating at 7 p.m. Admission is \$8 and coffee costs \$1.50. Liz has \$14 and wants to buy some \$2 bottles of water for her and Ana to drink afterwards. Calculate how many bottles of water Liz can buy. What information are you given that isn't relevant ?
3. Sean has \$60 to buy books for math club. A book costs \$9.95. He orders them on a Monday. Shipping costs \$10 an order. How many books could he buy? What information are you given that isn't relevant?

Say what units the answers will have in Exercises 4–7.

4. 4 persons • 4 hours = 16 ?
5. 100 trees ÷ 10 acres = 10 ?
6. 6 meters • 7 meters = 42 ?
7. 21 meters/second ÷ 7 seconds = 3 ?

Round Up

*When you're solving a math problem, you need to be able to pick out the **important information**. Then you can use the **relevant bits** to write an **equation** and find the **solution**. Always remember to check what **units** your answer needs to be written in too.*

Lesson 1.3.1

Section 1.3 Inequalities

California Standards:

Algebra and Functions 1.4

Use algebraic terminology (e.g., variable, equation, term, coefficient, **inequality**, expression, constant) correctly.

Algebra and Functions 1.5

Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

What it means for you:

You'll learn what an inequality is, and how to show one on a number line.

Key words:

- inequality
- greater than
- less than
- equal to

Inequalities are a lot like equations. But where an equation has an equals sign, an inequality has an inequality symbol. It tells you that the two sides may not be equal or are not equal — that's why it's different from an equation.

An Inequality Does Not Have to Balance

In the last Section you saw that an **equation** is a **balanced** math sentence. The expressions on each side of the equals sign are **equal in value**.

An **inequality** is a math sentence that **doesn't have to be balanced**.

The expression on one side **does not** have to have the **same value** as the expression on the other.

$x \leq 5$, $10 > 3y$, $4h \geq 19$, and $k < 5$ are all inequalities.

Inequalities are made up of **two expressions** that are separated by one of the four **inequality symbols**:

The Inequality Symbols

- $<$ means "Less than."
- $>$ means "Greater than."
- \leq means "Less than or equal to."
- \geq means "Greater than or equal to."

The **symbol** that you use explains how **the two expressions relate** to each other.

So $2 < 10$ means "two is less than ten"
and $x \geq 5$ means "x is greater than or equal to five."

The **smaller end** of the symbol always points to the **smaller number**. So $x < 2$ and $2 > x$ are telling you the same thing — that the variable x is a number less than 2.

✓ Guided Practice

Fill in the blanks in the statements in Exercises 1–4.

1. If $a < b$ then b a .
2. If $m \geq n$ then n m .
3. If $c > d$ then d c .
4. If $j \leq k$ then k j .

Plot the Solutions to an Inequality on a Number Line

An inequality has an **infinite number** of solutions. When you solve an inequality you are describing a group or **set of solutions**.

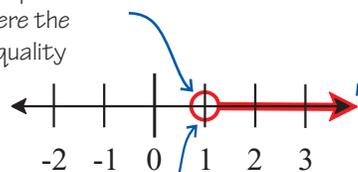
For example, for the inequality $x < 9$, **any** number that is **less than 9** is a solution of the inequality. The solutions are **not** limited to just whole numbers or positive numbers.

All the possible solutions of an inequality can be shown on a **number line**.

The following number line shows the solution of the inequality $c > 1$.

The set of possible solutions for c is all the numbers greater than 1.

The circle shows the point on the number line where the solution to the inequality starts.



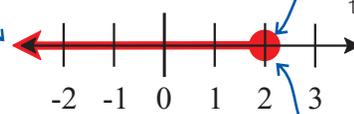
The shading stretches away to infinity in the direction of the arrow. This is showing that c can be any number that's greater than 1.

Here the circle is empty (open), because 1 is not included in the solution.

The number line below shows the solution of the inequality $y \leq 2$.

The set of possible solutions for y is 2 and all the numbers less than 2.

The shading stretches away to infinity in the direction of the arrow. This is showing that y can be any number that's less than or equal to 2.



The circle shows the point on the number line where the solution to the inequality starts.

Here the circle is filled-in (closed), because 2 is included in the solution.

Check it out:

A number line is a line that represents every number, with the numbers increasing in value along the line from left to right. To give the line a scale, some numbers are shown as labeled points spaced out evenly along the line.

Don't forget:

It's important to remember that there are an infinite amount of decimal numbers between each labeled point in the solution set. All of these are solutions too.

Don't forget:

A ray is just a straight line that begins at a point and goes on forever in one direction.

To plot an inequality on a number line:

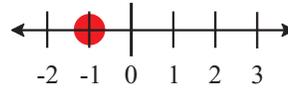
- 1) Draw a circle on the number line around the point where the set of solutions starts. It should be a **closed circle** if the point is **included** in the solution set, but an **open circle** if it **isn't**.
- 2) Draw a **ray** along the number line in the direction of the numbers in the solution set. Add an **arrowhead** at the end to show its direction (and that it goes on forever).

Example 1

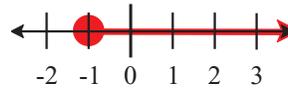
Plot the solution to the inequality $y \geq -1$ on a number line.

Solution

First place a closed circle at -1 on the number line to show that -1 is included in the solution set.



y is greater than or equal to -1 , so add a ray with an arrowhead pointing along the number line to the right of the circle.



Don't forget:

If the sign is \geq or \leq then you should use a closed circle to show that the start point is included in the solution.
If the sign is $>$ or $<$ then you should use an open circle to show that the start point isn't included in the solution.

Guided Practice

Plot the inequalities in Exercises 5–8 on the number line.

5. $h < 2$

6. $-1 < a$

7. $k \geq 1$

8. $t \leq 0$

Independent Practice

In Exercises 1–4, give a number that is part of the solution set of the inequality.

1. $m < -3$

2. $k \leq 12$

3. $w \geq 9$

4. $h > -9$

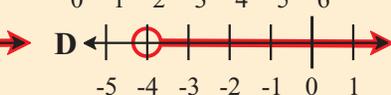
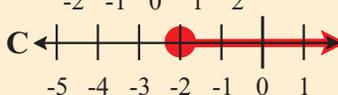
Match the inequalities 5–8 with the number line plots A–D.

5. $x > -4$



6. $x \geq 2$

7. $x < 1$



8. $x \geq -2$

9. Explain why the solution of an inequality must be graphed on a number line and not listed.

Plot the inequalities in Exercises 10–11 on number lines.

10. $-1 \leq x$

11. $x < 0.5$

Now try these:

Lesson 1.3.1 additional questions — p434

Round Up

An *inequality* is like an *unbalanced equation* — its two expressions can have *different values*. You can show all the possible solutions of an inequality by *graphing it on a number line*. Don't forget the four *symbols* — you'll need them to write and solve inequalities later.

Lesson 1.3.2

Writing Inequalities

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

What it means for you:

You'll learn how to turn a word problem into an inequality.

Key words:

- inequality
- under
- over
- minimum
- maximum

Don't forget:

The inequality $27 > 4y$ means exactly the same thing as $4y < 27$.

Don't forget:

The inequality $16 \leq h + 2$ means exactly the same thing as $h + 2 \geq 16$.

In Lesson 1.2.2 you saw how to write an equation from a word problem. To write an inequality you use exactly the same process — but this time instead of joining the two expressions with an equals sign, you use an inequality symbol.

You Need to Spot Which Symbol is Being Described

To write an inequality you write two expressions that have (or can have) different values and join them with one of the four inequality symbols. You need to be able to recognize phrases that describe the four symbols.

$>$ means “greater than” or “more than” or “over.”

$<$ means “less than” or “under.”

\geq means “greater than or equal to” or “a minimum of” or “at least.”

\leq means “less than or equal to” or “a maximum of” or “no more than.”

Example 1

Write an inequality to describe the sentence,
“Four times a number, y , is less than 27.”

Solution

The phrase “is less than” is represented by the less than symbol, $<$. It separates the expressions that make up the two sides of the inequality.

One expression is “Four times a number, y .”

This turns into the expression $4y$.

The other expression is a number, 27.

So the sentence, “Four times a number, y , is less than 27,” turns into the inequality $4y < 27$.

Example 2

Write an inequality to describe the sentence,
“A number, h , increased by two is at least 16.”

Solution

The phrase “at least” is represented by the greater than or equal to symbol, \geq .

One expression is “A number, h , increased by two.” This turns into the expression $h + 2$.

The other expression is a number, 16.

So the sentence, “A number, h , increased by two is at least 16,” turns into the inequality $h + 2 \geq 16$.

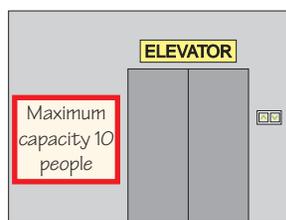
✓ Guided Practice

Write an inequality to describe each of the sentences in Exercises 1–5.

1. A number, x , increased by five is more than 12.
2. Twice a number, k , is greater than or equal to two.
3. Fifteen decreased by a number, g , is no more than six.
4. A number, p , divided by two, is under four.
5. Negative two is less than the sum of a number, m , and five.

Inequalities are Often Used in Real-Life Situations

You'll come across lots of inequality phrases in real life.



“Number of people” ≤ 10

“Height” > 4 feet



“Age” ≤ 8

In math you might be asked to write an **inequality** to represent the information given in a **word problem**.

Writing inequalities from word problems is a lot like writing equations from word problems. You need to spot key information and use it to write expressions — but you also need to work out which **inequality symbol** to use.

Example 3

Your local conservation group runs a junior award program. To get a gold award you must complete a minimum of 50 hours' conservation work. You have already done 17 hours. Write an inequality to represent the additional amount of work you need to do to gain your gold award.

Solution

First define a variable: **let the additional amount of hours you need to complete = H** .

The amount of hours you need to complete to get your award is 50. So one expression is just **50**.

The other expression describes the number of hours you have already done plus the additional number you need to spend — the variable H . So the expression is **$17 + H$** .

The phrase “**minimum**” tells you that the number of hours you complete has to be **greater than or equal to** 50. So the inequality is **$17 + H \geq 50$** .

✓ Guided Practice

Write an inequality to describe each of the situations in Exercises 6–9.

6. Lauren is three years younger than her friend Gabriela, who is k years old. Lauren is under 20.
7. Erin has visited b states. Kieran has visited two more states than Erin, and figures out that he has visited at least 28.
8. The number of boys enrolled at a university is half the number of girls, g , who are enrolled. The number of boys enrolled is more than 2000.
9. Pedro has set aside a maximum of \$100 in order to buy gifts for his family. He wants to spend the same amount, $\$x$, on each of his 3 family members.

Don't forget:

You can write all of these inequalities in the reverse direction just by changing the sign you use.

For example: $a > b$ is the same as $b < a$.

✓ Independent Practice

Write an inequality to describe each of the sentences in Exercises 1–5.

1. A number, m , decreased by seven is less than 16.
2. Nine more than a number, d , is at least 11.
3. The product of ten and a number, j , is a maximum of six.
4. Four is more than a number, y , divided by five.
5. The sum of a number, f , and six is less than negative one.
6. Explain whether the two statements “six more than a number is at least four” and “six more than a number is more than four” mean the same.

Write an inequality to describe each of the situations in Exercises 7–9.

7. Alex and Mallory both spent time cleaning the house. Alex spent y minutes cleaning. Mallory spent 15 minutes less than Alex, but over 55 minutes, cleaning.
8. Rebecca has a maximum of \$40 to spend on her cat. She buys a collar for \$17 and then spends $\$d$ on cat food.
9. Alejandra's collection of baseball cards is twice the size of Jordan's. Alejandra has collected at least 2000 cards, and Jordan has collected c baseball cards.

For each sentence in Exercises 10–12 say which inequality symbol would be used.

10. Maximum weight on this bridge is six tons.
11. The play park is for people under ten years old only.
12. This toy is for children aged three years and up.

Now try these:

Lesson 1.3.2 additional questions — p434

Round Up

When you turn a *word problem* into an *inequality*, the key thing is to figure out which of the four *inequality symbols* is being described. Then just write out the two *expressions* and join them with the correct symbol. You'll see how to *solve inequalities* like the ones you've written in Chapter 4.

Lesson 1.3.3

Two-Step Inequalities

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

What it means for you:

You'll learn how to turn a word problem into a two-step inequality.

Key words:

- inequality
- under
- over
- minimum
- maximum

Don't forget:

The four inequality symbols:
> is greater than.
< is less than.
≥ is greater than or equal to.
≤ is less than or equal to.

An inequality with two different operations in it is called a **two-step inequality**. To write a two-step inequality, follow the same steps that you learned in the last Lesson. The only difference this time will be that one of your expressions could have two operations in it.

Two-Step Inequalities Have Two Operations

A **two-step inequality** is one that involves **two different operations**.

$$4 \div x + 9 > 10$$

Diagram illustrating the operations in the inequality $4 \div x + 9 > 10$. Arrows point to the division symbol (\div) labeled "first operation", the plus sign ($+$) labeled "second operation", and the greater-than symbol ($>$) labeled "inequality symbol".

It has the same structure as a **two-step equation**, but with an **inequality symbol** instead of an equals sign.

Writing a Two-Step Inequality

Writing a **two-step inequality** is like writing a one-step inequality. You still need to write out your two expressions and join them with the correct **inequality symbol** — but now one of the expressions will contain **two operations** instead of one. That also means that you need to remember to use **PEMDAS** — the order of operations.

Example 1

Write an inequality to describe the sentence,
“Six more than the product of four and a number, h , is under 42.”

Solution

The phrase “**is under**” represents the **less than** symbol. It also separates the two **expressions** that make up the two sides of the inequality.

One expression is, “**Six more than the product of four and a number, h .**” This tells you to multiply four by h and add six to the product. It turns into the expression $4h + 6$.

The other expression is the number **42**.

So the sentence, “Six more than the product of four and a number, h , is under 42,” turns into the inequality $4h + 6 < 42$.

✓ Guided Practice

Write an inequality to describe each of the sentences in Exercises 1–6.

1. Five increased by the product of ten and a number, m , is more than 11.
2. Two plus the result of dividing a number, k , by six is at least five.
3. One subtracted from the product of a number, y , and nine is less than or equal to 33.
4. Ten subtracted from half of a number, t , is under -1 .
5. A third of a number number, r , plus nine, is no greater than -4 .
6. Double the sum of a number, x , and two is less than 20.

An Inequality Can Describe a Word Problem

To write a two-step inequality from a word problem, just follow the same rules as for a one-step inequality:

- Identify the important information you have been given.
- Spot which operation phrases are being used.
- Work out what the two expressions are.
- Join them using the correct inequality symbol.

Example 2

Hector needs to save at least \$250 to buy a new bicycle. He already has \$80, and receives \$10 each week for mowing the neighbor's lawn. Write an inequality to represent the number of weeks that Hector will need to save for.

Solution

First define a variable: **let the number of weeks Hector needs to save for = w .**

The minimum amount of money Hector needs to save is \$250. So one expression is just **250**.

The other expression describes the amount of money he will have after w weeks. This will be the \$80 he already has plus the number of weeks multiplied by the \$10 he earns each week. So the expression is **$80 + 10w$** .

The phrase “**at least**” is telling you that the amount Hector needs to save has to be **greater than or equal to** 250. So the inequality is **$80 + 10w \geq 250$** .

Don't forget:

To make sure your expression is right you can check it with a simple number using mental math.

For example: after 2 weeks Hector will have \$80 plus two times \$10. This is \$100.

Then put $w = 2$ into your expression. $80 + 10(2) = 100$. So your expression is likely to be right.

✓ Guided Practice

Write an inequality to describe each of the following situations.

7. Mrs. Clark parks by a meter that charges \$2 for the first hour and \$0.50 for each additional hour parked. She spends no more than \$10, and parks for the first hour and h additional hours.

8. Luis collects seashells. He has four boxes, each containing s shells. He gives 40 shells to Jon, and still has more than 200 shells in his collection.

9. Marcia is buying new shirts that cost \$15 each for x people in her Little League team. She has a coupon for \$12 off her order, and a maximum of \$150 to spend.

10. Daniel's teacher tells him that to be considered low-fat, a meal must contain less than three grams of fat. Daniel prepares a low-fat breakfast of yogurt topped with pumpkin seeds for a school project. A cup of yogurt contains y grams of fat. Daniel uses half a cup, and tops it with pumpkin seeds containing a total of 1 gram of fat.

✓ Independent Practice

Write an inequality to describe each of the sentences in Exercises 1–4.

1. Twenty more than twice a number, f , is less than 35.

2. Fifty subtracted from a quarter of a number, n , is at least 77.

3. Eight increased by the product of four and a number, d , is no more than 13.

4. Negative eighteen is more than a number, a , divided by 41, minus six.

5. Vanessa has ordered a meal that costs under \$12. She is having a baked potato, costing \$7, and d salads costing \$2 each. Write an inequality to describe this information.

6. Filipa and her four friends are looking for a house to rent for a vacation. The price of an airplane ticket is \$200, and they will split the cost of the house rental, $\$r$. Filipa has budgeted \$600 for the airplane ticket and house. Write an inequality to determine the maximum rental price that the house can be.

7. Tom and his two friends run a babysitting service that makes $\$p$ a month income. Each month they spend \$20 to advertise, then split the remaining money evenly. Tom wants to earn at least \$80 a month. Write an inequality to describe how much income the service must make each month for this to happen.

Now try these:

Lesson 1.3.3 additional questions — p434

Round Up

Writing a two-step inequality is just like writing a one-step inequality. You still need to look for the key information in the question, identify the operation phrases, and spot which inequality symbol is needed. But this time one of the expressions might contain two operations. You'll see how to solve two-step inequalities in Chapter 4.

Chapter 1 Investigation

Which Phone Deal is Best?

You can *write expressions* to model real-life situations. In this Investigation, you'll write expressions to represent different cell phone plans, and by *evaluating* your expressions you'll find out which is the *best value* plan for different users.

Two phone companies are offering **different family plan deals** to attract new customers.

Company A

\$30 a month for 500 minutes
\$0.02 for each additional minute

Company B

\$10 a month for 500 minutes
\$0.04 for each additional minute



Part 1:

Write **expressions** for Companies A and B that could be used to represent the **price of one month's phone bill**.

Which company offers the **better plan** for a family using **1000 minutes** a month?

Part 2:

How many minutes does a family have to talk so that Company A offers a **better deal** than Company B?

Things to think about:

- How can you **compare** the prices of both companies for different numbers of minutes?
- The **basic price** for Company B's plan is **\$20 more** than Company A's plan. Thinking just about cost, why would a family choose Company A's plan instead of Company B's?

Extensions

- 1) Write an inequality that could be used to calculate the number of minutes a family could talk with Company A if they want to spend under \$35 a month.
- 2) The Sutro family uses Company A and talks an average of 800 minutes a month. How much will they save over a year by switching to Company B?

Open-ended Extensions

- 1) Is it possible for the price of Company A's plan to be double the price of Company B's plan? Assume calls are charged to the nearest minute. Make an organized list or table to compare them.
- 2) Company C wants to charge a flat per minute fee and have their price lie between Company A's and Company B's prices when between 500 and 750 minutes are used. What per minute fees could Company C charge to accomplish this goal?

Round Up

A number that can change is called a **variable** and is represented with a letter. In the cell phone plans, the variable was the **number of minutes used**. By **evaluating expressions** with different values for the variable, you can find the prices when different numbers of minutes are used.

Chapter 2

Rational and Irrational Numbers

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Lesson 2.1.1

Section 2.1 Rational Numbers

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

Number Sense 1.4

Differentiate between rational and irrational numbers.

Number Sense 1.5

Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.

What it means for you:

You'll meet rational numbers and see which kinds of numbers that you've already met fall into this category.

Key words:

- rational number
- irrational number
- fraction
- decimal
- terminating
- repeating

Don't forget:

A fraction can be thought of as a division. So $\frac{5}{1}$ says the same thing as $5 \div 1$, and when you divide a number by 1, it doesn't change.

Check it out:

In the next Lesson you'll see how to turn terminating decimals into fractions.

*Pretty much all the numbers you've met so far are **rational** — positive and negative **integers** and **fractions** are all rational, as are most **decimals**. The only decimals that aren't rational are the ones that go on and on **forever**, without having a **repeating pattern** of digits.*

All Rational Numbers Can Be Written as Fractions

You get all sorts of numbers in the **rational set**, for example 1.05, 0.3333..., $\frac{1}{2}$, and 6 are all rational. Rational numbers have all got one thing in common — they can each be written as a **simple fraction**, with an **integer on the top and a nonzero integer on the bottom**.

In formal math:

A rational number is a number that can be written as $\frac{a}{b}$, where both a and b are integers (and b is not equal to 0).

There are basically **three types** of rational numbers.

All Integers are Rational

The **integers** are the numbers in the set $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$. Any integer can be written as a **fraction over 1**.

So integers are all **rational**. For example: $7 = \frac{7}{1}$ $-18 = \frac{-18}{1}$

Example 1

Show that 5 is a rational number.

Solution

5 can be written as $\frac{5}{1}$. This fits the above definition, so 5 must be rational.

All Terminating Decimals are Rational

Numbers like 1.2, 5.689, -3.72 , and -0.69245 are known as **terminating decimals** — they all have definite ends.

All terminating decimals can be converted to fractions of the form $\frac{a}{b}$, where a and b are both integers. So all terminating decimals are **rational**.

For example, 1.2 is equivalent to $\frac{6}{5}$, 0.125 is equivalent to $\frac{1}{8}$,

and 0.75 is equivalent to $\frac{3}{4}$.

All Repeating Decimals are Rational

Check it out:

In Lesson 2.1.3 you'll see how to turn repeating decimals into fractions.

0.09090909... is a **repeating decimal**. It will go on forever repeating the same digits (09) over and over again.

Repeating decimals can always be converted to the form $\frac{a}{b}$ where a and b are both integers — so they are always **rational**.

$$0.0909090909\dots = \frac{1}{11}$$

Other examples of repeating decimals are:

$$0.33333\dots \left(\frac{1}{3}\right), \text{ and } 0.045045045\dots \left(\frac{5}{111}\right).$$

The usual way to show that decimals are **repeating** is to put a small **bar** above them. The bar should cover one complete set of the **repeated digits**, so $0.\overline{15}$ means 0.151515..., but $0.\overline{151}$ means 0.151151151...

Never-Ending, Nonrepeating Decimals are Irrational

Check it out:

Irrational numbers are covered in more detail in Section 2.6.

A number that **cannot** be written in the form $\frac{a}{b}$ where a and b are both **integers** is an **irrational number**.

Irrational numbers are always **nonrepeating**, **nonterminating** decimals. π is an irrational number:

$$\pi = 3.141592653\dots \leftarrow \begin{array}{l} \text{goes on forever,} \\ \text{never repeats} \end{array}$$

✓ Guided Practice

Show that the numbers in Exercises 1–6 are rational.

- | | |
|---------|---------|
| 1. 2 | 2. -8 |
| 3. 0.5 | 4. 0.25 |
| 5. -0.1 | 6. 1.5 |

7. Luis does a complicated calculation and his 10-digit calculator screen shows the result 1.123456789. Can you say whether the answer of Luis's calculation is rational?

8. Is 2π rational?

Fractions Can Be Converted into Decimals by Division

All **integers**, **terminating decimals**, and **repeating decimals** are rational, so they can be written as fractions. The **opposite** is also true — **every fraction** can be converted into an integer, a terminating decimal, or a repeating decimal.

$\frac{a}{b}$ can be read as the instruction " **a divided by b** ."

If you **divide** the integer a by the integer b you'll end up with an **integer**, a **repeating decimal**, or a **terminating decimal**.

Check it out:

b can't be zero — you can't divide by zero; it's undefined.

A Remainder of Zero Means a Terminating Decimal

When you're dividing the numerator of a fraction by the denominator, you might get to a point where you have **no remainder** left — that means that it's a **terminating decimal**.

Example 2

Convert $\frac{7}{8}$ into a decimal.

Solution

Divide 7 by 8.

$$\begin{array}{r} 0.875 \\ 8 \overline{)7.0000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 00 \end{array}$$

no remainder left,
so this is a
terminating decimal \longrightarrow

So, $\frac{7}{8}$ as a decimal is **0.875**.

Don't forget:

When you add 0s to the end of a number after a decimal point, as with the 7 in Example 2, the value of the number doesn't change. So $7.0000 = 7.0 = 7$.

Guided Practice

Convert the fractions given in Exercises 9–12 into decimals without using a calculator.

9. $\frac{3}{6}$

10. $\frac{4}{5}$

11. $\frac{6}{4}$

12. $\frac{5}{32}$

A Repeated Remainder Means a Repeating Decimal

If you get a **remainder** during long division that you've **had before**, then you have a **repeating decimal**.

$$\begin{array}{r} 0.13 \\ 15 \overline{)2.000} \\ \underline{15} \\ 50 \\ \underline{45} \\ 50 \end{array}$$

repeated remainder \longrightarrow

The repeating digits are **only** the ones that you worked out since the last time you saw the same remainder.

\rightarrow In this example you've had a remainder of **50** before. Since the last time you had this remainder, you've found the digit **3**. That means **3 is the repeating part of the decimal**.

$$\begin{array}{r} 0.1\overline{3} \\ 15 \overline{)2.000} \\ \underline{15} \\ 50 \\ \underline{45} \\ 50 \end{array}$$

repeating digit \longrightarrow

repeated remainder \longrightarrow

So $\frac{2}{15} = 0.1333\dots$ with the 3 **repeating forever**.

Which you can write as $\frac{2}{15} = 0.1\overline{3}$.

Don't forget:

The quotient is what you get when you divide one number by another.

Example 3

Convert $\frac{5}{22}$ into a decimal.

Solution

Divide 5 by 22.

$$\begin{array}{r}
 \text{repeating} \\
 \text{digits} \quad \rightarrow \quad 0.\overline{227} \\
 22 \overline{)5.000} \\
 \underline{44} \\
 60 \\
 \underline{44} \\
 160 \\
 \underline{154} \\
 60
 \end{array}$$

repeated remainder

So, $\frac{5}{22}$ as a decimal is $0.\overline{227}$.

✓ Guided Practice

Convert the fractions given in Exercises 13–16 into decimals, without using a calculator.

13. $\frac{5}{27}$

14. $\frac{6}{41}$

15. $\frac{15}{7}$

16. $\frac{7}{12}$

✓ Independent Practice

1. Read statements a) and b). Only one of them is true. Which one? How do you know?

a) All integers are rational numbers.

b) All rational numbers are integers.

Show that the numbers in Exercises 2–7 are rational.

2. 4

3. 1

4. -2

5. 0.2

6. 1.25

7. $-0.\overline{3}$

Convert the fractions given in Exercises 8–13 to decimals without using a calculator. Say whether they are terminating or repeating decimals.

8. $\frac{1}{9}$

9. $\frac{8}{5}$

10. $\frac{11}{16}$

11. $\frac{5}{11}$

12. $\frac{15}{8}$

13. $\frac{5}{22}$

Now try these:

Lesson 2.1.1 additional questions — p435

Round Up

Rational numbers can all be written as fractions, where the top and bottom numbers are integers, and the bottom number isn't zero. You already know how to write integers as fractions, and you'll see how to convert terminating decimals and repeating decimals to fractions in the next two Lessons.

Lesson 2.1.2

California Standards:

Number Sense 1.5

Know that every rational number is either a terminating or a repeating decimal and **be able to convert terminating decimals into reduced fractions.**

What it means for you:

You'll see how to change terminating decimals into fractions that have the same value.

Key words:

- fraction
- decimal
- terminating

Don't forget:

You can ignore any extra 0s at the end of the decimal, because they don't change its value.

For example:

$$0.270000 = 0.27 = \frac{27}{100}.$$

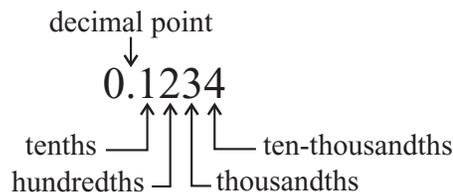
Converting Terminating Decimals to Fractions

*This Lesson is a bit like the opposite of the last Lesson — you'll be taking **decimals** and finding their **equivalent fractions**. This is how you can show that they're definitely **rational numbers**.*

Decimals Can Be Turned into Fractions

If you read **decimals** using the **place-value** system, then it's more straightforward to convert them into **fractions**. For example, a number like 0.15 is said “fifteen-hundredths,” so it turns into the fraction $\frac{15}{100}$.

You need to remember the **value** of **each position** after the **decimal point**:



Then when you are reading a decimal number, look at the **position** of the **last digit**. For example: 0.1 is one-tenth, which is the fraction $\frac{1}{10}$. 0.01 is one-hundredth, which is the fraction $\frac{1}{100}$.

Example 1

Convert 0.27 into a fraction.

Solution

0.27 is twenty-seven hundredths, so it is $\frac{27}{100}$.

Example 2

Convert 0.3497 into a fraction.

Solution

0.3497 is 3497 ten-thousandths, so it is $\frac{3497}{10,000}$.

✓ Guided Practice

Convert the decimals in Exercises 1–12 into fractions without using a calculator.

- | | | | |
|----------|------------|------------|-------------|
| 1. 0.1 | 2. 0.23 | 3. 0.17 | 4. -0.87 |
| 5. 0.7 | 6. 0.35 | 7. 0.174 | 8. -0.364 |
| 9. 0.127 | 10. 0.9827 | 11. 0.5212 | 12. -0.4454 |

Some Fractions Can Be Made Simpler

Don't forget:

Dividing the top and bottom of a fraction by the same thing doesn't change the value of the fraction.

When you convert decimals to fractions this way, you'll often get fractions that **aren't in their simplest form**. For instance, $\frac{5}{10}$ could be written more simply as $\frac{1}{2}$, and $\frac{75}{100}$ could be written more simply as $\frac{3}{4}$.

If an answer is a fraction, you should usually give it in its simplest form.

This is how to reduce a fraction to its simplest form:

- 1) Find the **biggest number** that will divide into both the **numerator** and the **denominator** without leaving any **remainder**. This number is called the **greatest common factor**, or **GCF**.
- 2) Then **divide** both the numerator and the denominator by the **GCF**.

Example 3

Convert 0.12 into a fraction.

Solution

- 0.12 is twelve-hundredths. As a fraction it is $\frac{12}{100}$.
- The factors of 12 are 1, 2, 3, 4, 6, and 12. The biggest of these that also divides into 100 leaving no remainder is 4. So the **greatest common factor** of 12 and 100 is 4.
- Divide both the numerator and denominator by 4.
 $\frac{12 \div 4}{100 \div 4} = \frac{3}{25}$. **So 0.12 as a fraction in its simplest form is $\frac{3}{25}$.**

If the greatest common factor is **1** then the fraction is already in its **simplest form**.

Example 4

Convert 0.7 into a fraction.

Solution

- 0.7 is seven-tenths so, it is $\frac{7}{10}$.
- The greatest common factor of 7 and 10 is 1, so this fraction is **already** in its simplest form.

✓ Guided Practice

Convert the decimals in Exercises 13–20 into fractions and then simplify them if possible.

13. 0.25

14. 0.65

15. -0.02

16. 0.256

17. 0.0175

18. -0.84

19. 0.267

20. 0.866

21. Priscilla measures a paper clip. She decides that it is six-eighths of an inch long. Otis measures the same paper clip with a different ruler and says it is twelve-sixteenths of an inch long. How can their different answers be explained?

Decimals Greater than 1 Become Improper Fractions

Don't forget:

A proper fraction is a fraction whose numerator is smaller than its denominator.

For example: $\frac{1}{2}$ and $\frac{9}{10}$.

An improper fraction is a fraction whose numerator is equal to or larger than its denominator.

For example: $\frac{3}{2}$ and $\frac{27}{4}$.

Don't forget:

A mixed number is a number made up of an integer and a fraction. For example:

$1\frac{1}{2}$, $2\frac{2}{3}$, and $10\frac{4}{5}$.

When you convert a **decimal** number **greater than 1** into a **fraction** it's probably easier to change it into a **mixed number** first.

Then you can change the mixed number into an **improper fraction**.

Example 5

Convert 13.7 into a fraction.

Solution

- Convert 0.7 first — this becomes $\frac{7}{10}$.
- Add on the 13. This can be written as $13\frac{7}{10}$. A mixed number.
- Now turn $13\frac{7}{10}$ into an improper fraction.

13 whole units are equivalent to $\frac{13 \cdot 10}{1 \cdot 10} = \frac{130}{10}$. So add $\frac{7}{10}$ to this:

$$\left(\frac{13 \cdot 10}{1 \cdot 10}\right) + \frac{7}{10} = \frac{130}{10} + \frac{7}{10} = \frac{137}{10} \quad \text{A quicker way of doing this is: } \frac{(13 \cdot 10) + 7}{10} = \frac{137}{10}$$

✓ Guided Practice

Convert the decimals in Exercises 22–33 into fractions without using a calculator.

- | | | | |
|------------|------------|------------|-------------|
| 22. 4.3 | 23. -1.03 | 24. 15.98 | 25. -1.7 |
| 26. 9.7 | 27. -4.5 | 28. 12.904 | 29. -13.142 |
| 30. -8.217 | 31. 0.3627 | 32. 1.8028 | 33. 4.1234 |

✓ Independent Practice

Convert the decimals given in Exercises 1–20 to fractions in their simplest form.

- | | | | |
|------------|------------|-------------|-------------|
| 1. 0.3 | 2. 0.2 | 3. 0.4 | 4. 0.30 |
| 5. 0.26 | 6. 0.18 | 7. -0.34 | 8. -1.34 |
| 9. 0.234 | 10. 2.234 | 11. 9.140 | 12. 3.655 |
| 13. -0.121 | 14. -0.655 | 15. -10.760 | 16. 5.001 |
| 17. 0.2985 | 18. 2.3222 | 19. -9.3452 | 20. -0.2400 |

Round Up

The important thing when converting a **decimal** to a **fraction** is to think about the **place value** of the last digit. Then **read** the decimal and turn it into a fraction. If the decimal is **greater than 1**, ignore the **whole number** until you get the **decimal part** figured out. Take your time, do each step carefully, and you should be OK.

Lesson 2.1.3

Converting Repeating Decimals to Fractions

California Standards:

Number Sense 1.5

Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.

What it means for you:

You'll see how to change repeating decimals into fractions that have the same value.

Key words:

- fraction
- decimal
- repeating

Don't forget:

To multiply a decimal by 10, move the decimal point one place to the right.
So $0.3333... \times 10 = 3.3333...$

Don't forget:

The greatest common factor of 3 and 9 is 3.

So:

$$\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$$

You've seen how to convert a terminating decimal into a fraction. But repeating decimals are also rational numbers, so they can be represented as fractions too. That's what this Lesson is all about — taking a repeating decimal and finding a fraction with the same value.

Repeating Decimals Can Be “Subtracted Away”

Look at the decimal $0.33333...$, or $0.\overline{3}$.

If you multiply it by 10, you get $3.33333...$, or $3.\overline{3}$.

In both these numbers, the digits after the decimal point are the same. So if you subtract one from the other, the decimal part of the number “disappears.”

Example 1

Find $3.\overline{3} - 0.\overline{3}$.

Solution

The digits after the decimal point in both these numbers are the same, since $0.\overline{3} = 0.3333...$ and $3.\overline{3} = 3.3333...$

So when you subtract the numbers, the result has no digits after the decimal point.

$$\begin{array}{r} 3.3333... \\ -0.3333... \\ \hline 3.0000... \end{array} \quad \text{or} \quad \begin{array}{r} 3.\overline{3} \\ -0.\overline{3} \\ \hline 3.0 \end{array}$$

So $3.\overline{3} - 0.\overline{3} = 3$.

This idea of getting repeating decimals to “disappear” by subtracting is used when you convert a repeating decimal to a fraction.

Example 2

If $x = 0.\overline{3}$, find: (i) $10x$, and (ii) $9x$.

Use your results to write x as a fraction in its simplest form.

Solution

(i) $10x = 10 \times 0.\overline{3} = 3.\overline{3}$.

(ii) $9x = 10x - x = 3.\overline{3} - 0.\overline{3} = 3$ (from Example 1 above).

You now know that $9x = 3$.

So you can divide both sides by 9 to find x as a fraction:

$$x = \frac{3}{9}, \text{ which can be simplified to } x = \frac{1}{3}.$$

✓ Guided Practice

In Exercises 1–3, use $x = 0.\overline{4}$.

1. Find $10x$.
2. Use your answer to Exercise 1 to find $9x$.
3. Write x as a fraction in its simplest form.

In Exercises 4–6, use $y = 1.\overline{2}$.

4. Find $10y$.
5. Use your answer to Exercise 4 to find $9y$.
6. Write y as a fraction in its simplest form.

Convert the numbers in Exercises 7–9 to fractions.

7. $2.\overline{5}$ 8. $4.\overline{1}$ 9. $-2.\overline{5}$

You May Need to Multiply by 100 or 1000 or 10,000...

If **two** digits are repeated forever, then multiply by **100** before subtracting.

Example 3

Convert $0.\overline{23}$ to a fraction.

Solution

Call the number x .

There are **two** repeating digits in x , so you need to multiply by **100** before subtracting.

$$100x = 23.\overline{23}$$

$$\text{Now **subtract**: } 100x - x = 23.\overline{23} - 0.\overline{23} = 23.$$

$$\text{So } \mathbf{99x = 23}, \text{ which means that } \mathbf{x = \frac{23}{99}}.$$

Don't forget:

This means that

$$0.\overline{23} = \frac{23}{99}$$

If **three** digits are repeated forever, then multiply by **1000**, and so on.

Example 4

Convert $1.\overline{728}$ to a fraction in its simplest form.

Solution

Call the fraction y .

There are **three** repeating digits in y , so you need to multiply by **1000** before subtracting.

$$1000y = 1728.\overline{728}$$

$$\text{Now **subtract**: } 1000y - y = 1728.\overline{728} - 1.\overline{728} = 1727.$$

$$\text{So } \mathbf{999y = 1727}, \text{ which means that } \mathbf{y = \frac{1727}{999}}.$$

Check it out:

In fact, you multiply by 10^n , where n is the number of repeating digits.

So where there's one repeating digit, you multiply by $10^1 = 10$.

Where there are two repeating digits, you multiply by $10^2 = 10 \times 10 = 100$, and so on.

✓ Guided Practice

For Exercises 10–15, write each repeating decimal as a fraction in its simplest form.

10. $0.\overline{09}$

11. $0.\overline{18}$

12. $0.\overline{909}$

13. $0.\overline{123}$

14. $2.\overline{12}$

15. $0.\overline{1234}$

The Numerator and Denominator Must Be Integers

You **won't** always get a **whole number** as the result of the subtraction. If this happens, you may need to **multiply** the numerator and denominator of the fraction to make sure they are both integers.

Example 5

Convert $3.4\overline{3}$ to a fraction.

Solution

Call the number x .

There is **one** repeating digit in x , so multiply by **10**.

$10x = 34.3\overline{3}$ (using $34.3\overline{3}$ rather than $34.\overline{3}$ makes the subtraction easier).

Subtract as usual: $10x - x = 34.3\overline{3} - 3.4\overline{3} = 30.9$.

So **$9x = 30.9$** , which means that $x = \frac{30.9}{9}$.

But the numerator here **isn't an integer**, so multiply the numerator and denominator by 10 to get an equivalent fraction of the same value.

$$x = \frac{30.9 \times 10}{9 \times 10} = \frac{309}{90}, \text{ or more simply, } x = \frac{103}{30}$$

Check it out:

Make the repeating digits line up to make the subtraction easier (and remember that the decimal points have to line up too).

So write $10x$ as $34.3\overline{3}$ rather than $34.\overline{3}$.

Then the subtraction becomes:

$$\begin{array}{r} 34.3\overline{3} \\ -3.4\overline{3} \\ \hline 30.9 \end{array}$$

Which is an easier subtraction to do than $34.\overline{3} - 3.4\overline{3}$.

✓ Guided Practice

For Exercises 16–18, write each repeating decimal as a fraction in its simplest form.

16. $1.1\overline{2}$

17. $2.33\overline{4}$

18. $0.5432\overline{1}$

✓ Independent Practice

Convert the numbers in Exercises 1–9 to fractions.

Give your answers in their simplest form.

1. $0.\overline{8}$

2. $0.\overline{7}$

3. $1.\overline{1}$

4. $0.2\overline{6}$

5. $4.8\overline{7}$

6. $0.24\overline{6}$

7. $0.1428\overline{57}$

8. $3.1428\overline{54}$

9. $10.0\overline{1}$

Now try these:

Lesson 2.1.3 additional questions — p435

Round Up

This is a really handy **3-step method** — (i) **multiply** by 10, 100, 1000, or whatever, (ii) **subtract** the original number, and (iii) **divide** to form your fraction.

Lesson 2.2.1

Section 2.2 Absolute Value

California Standards:

Number Sense 2.5

Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

What it means for you:

You'll learn how to find the absolute value of a number, and use it in calculations.

Key words:

- absolute value
- distance
- opposite

Check it out:

Because you want to know the distance from the origin but don't care about direction, the absolute value of a number is always just the number without its original sign.

Check it out:

A number and its opposite (additive inverse) always have the same absolute value.

For example:

$$|2| = |-2| = 2$$

You can think of the number “-5” as having *two parts* — a *negative sign* that tells you it's less than zero, and “5,” which tells you its size, or how far from zero it is. The *absolute value* of a number is just its size — it's not affected by whether it's greater or less than zero.

Absolute Value is Distance from Zero

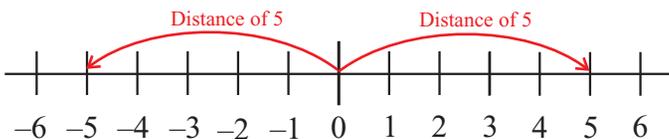
The absolute value of a number is its distance from 0 on the number line.

The **absolute value** of a number is **never negative** — that's because the absolute value describes **how far** the number is from **zero** on the number line. It doesn't matter if the number is to the **left** or to the **right** of zero — the distance **can't** be negative.

Opposites Have the Same Absolute Value

Opposites are numbers that are the **same distance from 0**, but going in **opposite directions**. Opposites have the same **absolute value**.

-5 and 5 are opposites:



So they each have an **absolute value of 5**.

A set of **bars**, $| |$, are used to represent **absolute value**.

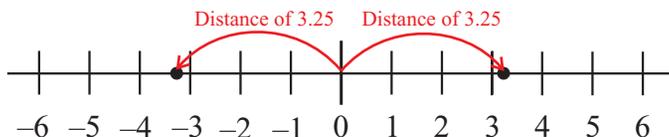
So the expression $|-10|$ means “the absolute value of negative ten.”

Example 1

What is $|3.25|$? What is $|-3.25|$?

Solution

3.25 and -3.25 are opposites. They're the same distance from 0, so they have the same absolute value.



So, $|3.25| = |-3.25| = 3.25$

✓ Guided Practice

Find the values of the expressions in Exercises 1–8.

1. $|12|$ 2. $|-9|$ 3. $|16|$ 4. $|-1|$
5. $|1.7|$ 6. $|-3.2|$ 7. $|\frac{1}{2}|$ 8. $|0|$

In Exercises 9–12, say which is bigger.

9. $|17|$ or $|16|$ 10. $|-2|$ or $|-5|$
11. $|-9|$ or $|8|$ 12. $|-1|$ or $|1|$

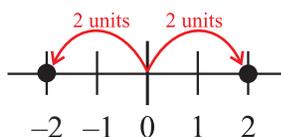
Check it out:

Find the value of each absolute value expression, then compare them.

Absolute Value Equations Often Have Two Solutions

Think about the equation $|x| = 2$. The absolute value of x is 2, so you know that x is 2 units away from 0 on the number line, but you don't know in which direction. x could be 2, but it could also be -2 .

You can show the two possibilities like this:

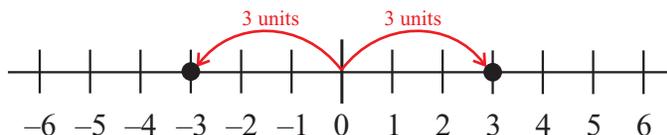


Example 2

Solve $|z| = 3$.

Solution

z can be either 3 or -3 .



✓ Guided Practice

Give the solutions to the equations in Exercises 13–16.

13. $|a| = 1$ 14. $|r| = 4$ 15. $|q| = 6$ 16. $|g| = 7$

Treat Absolute Value Signs Like Parentheses

You should treat **absolute value bars** like **parentheses** when you're deciding what **order** to do the **operations** in. Work out what's inside them first, then take the absolute value of that.

Example 3

What is the value of $|7 - 3| + |4 - 6|$?

Solution

$$\begin{aligned} &|7 - 3| + |4 - 6| \\ &= |4| + |-2| \\ &= 4 + 2 \\ &= 6 \end{aligned}$$

Simplify whatever is inside the absolute value signs
Find the absolute values
Simplify the expression

Check it out:

The absolute value bars only affect whatever's inside them. So the value of the whole expression can be negative. For example $-|a + 7|$ will be negative, unless $a = -7$.

Guided Practice

Evaluate the expressions in Exercises 17–22.

17. $|1 - 3| - |2 + 2|$

18. $|2 - 7| + |0 - 6|$

19. $-|5 - 6|$

20. $|-8| \times |2 - 3|$

21. $2 \times |4 - 6|$

22. $|7 - 2| \div |1 - 6|$

Independent Practice

Evaluate the expressions in Exercises 1–4?

1. $|-45|$

2. $|6|$

3. $|-0.6|$

4. $|\frac{5}{6}|$

5. Let x and y be two integers. The absolute value of y is larger than the absolute value of x . Which of the two integers is further from 0?

Show the solutions of the equations in Exercises 6–13 on number lines.

6. $|u| = 3$

7. $|d| = 9$

8. $|x| = 5$

9. $|w| = 15$

10. $|y| = 4$

11. $|v| = 1$

12. $|k| = 16$

13. $|z| = 7$

In Exercises 14–19, say which is bigger.

14. $|-6|$ or $|-1|$

15. $|3|$ or $|-5|$

16. $|2 - 2|$ or $|5 - 8|$

17. $|6 - 8|$ or $|2 - 1|$

18. $|3 - 2|$ or $|-5|$

19. $|11 + 1|$ or $|-2 - 8|$

Evaluate the expressions in Exercises 20–25.

20. $|3 - 5| + |2 - 5|$

21. $|0 + 5| + |0 - 5|$

22. $|5 - 10| - |0 - 2|$

23. $|-1| \times |3 - 3|$

24. $8 \times |1 - 4|$

25. $|2 - 8| \div |4 - 1|$

26. What is the sum of two different numbers that have the same absolute value? Explain your answer.

27. Is it always true that $|y| < 2y$ when y is an integer?

Now try these:

Lesson 2.2.1 additional questions — p436

Round Up

The absolute value of a number is its *distance* from zero on a number line. Absolute values are always *positive*. So if a number has a negative sign, get rid of it; if it doesn't, then leave it alone. If you see absolute value bars in an expression, work out what's between them *first* — just like parentheses.

Lesson 2.2.2

Using Absolute Value

California Standards:

Number Sense 2.5

Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

What it means for you:

You'll use absolute value to find the difference between two numbers and see how absolute values apply to real-life situations.

Key words:

- absolute value
- comparison
- difference

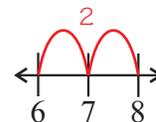
Check it out:

If $|a - b|$ is small then a and b are close to each other. If $|a - b|$ is large then a and b are far away from each other. For example, -1 and -3 are closer together than 6 and -6 , so you'd expect $|(-1) - (-3)|$ to be smaller than $|6 - (-6)|$.

You use *absolute value* a lot in real life — often without even thinking about it. For example, if the temperature falls from 3°C to -3°C you might use it to find the *overall change*. This Lesson looks at some of the ways that absolute value can apply to everyday situations.

Absolute Values Help Find Distances Between Numbers

To find **the distance between two numbers** on the number line you could count the number of spaces between them.



Using **subtraction** is a quicker way — just subtract the lesser number from the greater. $8 - 6 = 2$, so 8 and 6 are 2 units apart.

If you did the subtraction the other way around you'd get a **negative** number — and distances **can't** be negative. But if you use absolute value bars you can do the subtraction in **either order** and you'll always get a **positive value** for the distance.

$$|6 - 8| = |-2| = 2 \quad \text{and} \quad |8 - 6| = 2$$

For any numbers a and b :

The distance between a and b on the number line is $|a - b|$.

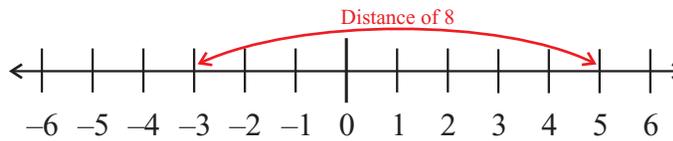
This is particularly useful when you're finding the distance between a **positive and negative number**.

Example 1

What is the distance between -3 and 5 ?

Solution

The distance between -3 and 5 is $|-3 - 5| = |-8| = 8$.



Guided Practice

Find the distance between the numbers given in each of Exercises 1–8.

- | | | | |
|------------|-------------|----------------|----------------|
| 1. 1, 5 | 2. $-3, -8$ | 3. $6, -9$ | 4. $-1, 10$ |
| 5. $3, -5$ | 6. $5, -1$ | 7. $-1.2, 2.3$ | 8. $-0.3, 2.7$ |

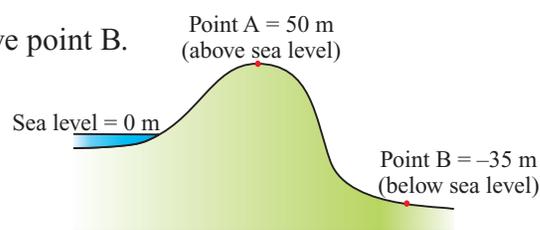
9. At 1 p.m., Amanda was 8 miles east of her home. She then traveled in a straight line west until she was 6 miles west of her home. How many miles did she travel?

Absolute Values are Used to Compare Things

You can use absolute values to **compare** numbers when it doesn't matter which side of a fixed point they are.

Example 2

Find how far point A is above point B.



Solution

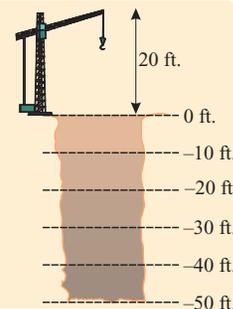
It doesn't matter that B is **below** sea level and A is **above**. It's the **distance** between them that's important.

You can find this by working out $|50 \text{ m} - (-35 \text{ m})| = |85 \text{ m}| = 85 \text{ m}$.

You'd get the same answer if you did the subtraction the other way around:
 $|-35 \text{ m} - 50 \text{ m}| = |-85 \text{ m}| = 85 \text{ m}$.

Guided Practice

10. A miner digs the shaft shown on the right. What distance was he from the top of the crane when he finished digging?



11. The top of Mount Whitney is 14,505 ft above sea level. The bottom of Death Valley is 282 ft below sea level. How much higher is the top of Mount Whitney than the bottom of Death Valley?

Absolute Values Can Describe Limits

Another use of absolute values is to describe the **acceptable limits** of a **measurement**. It might not be important whether something is above or below a set value, but **how far** above or below it is.

Example 3

The average temperature of the human body is 98.6°F , but in a healthy person it can be up to 1.4°F higher or lower. The difference between a person's temperature, x , and the average healthy temperature can be found using the expression $|98.6 - x|$.

Aaron is feeling unwell so measures his temperature. It is 100.2°F . Is Aaron's temperature within the healthy range?

Solution

The difference between Aaron's temperature and the average healthy temperature is $|98.6 - 100.2| = |-1.6| = 1.6^\circ\text{F}$.

Aaron's temperature is outside of the normal healthy range.

Check it out:

If your body temperature goes too far from the normal value in either direction it can be dangerous, so this is an important use of absolute value.

Example 4

A factory manufactures wheels that it advertises as no more than 1 inch away from 30 inches in diameter. They use the expression $|30 - d|$ to test whether wheels are within the advertised size (where d is the diameter).

Apply the expression to wheels of diameters 31, 29, and 35 inches, and say whether they meet the advertised standard.

Solution

- Wheel of diameter 31 inches: $|30 - 31| = |-1| = 1$ inch.
This wheel is within the standard.
- Wheel of diameter 29 inches: $|30 - 29| = |1| = 1$ inch.
This wheel is within the standard.
- Wheel of diameter 35 inches: $|30 - 35| = |-5| = 5$ inches.
This wheel is not within the standard.

Check it out:

Lots of manufacturers have tolerance limits for measurements. A tolerance limit is how different in size something is allowed to be from the size it should be. For example, size 6 knitting needles should have a diameter of 4 mm. Only very small differences from this size, such as 0.004 mm, will be allowed.

✓ Guided Practice

12. The height of a cupboard door should be no more than 0.05 cm away from 40 cm. The expression $|40 - h|$ is used to check whether a door of height h cm fits the size requirement. If a door measures 40.049 cm, is it within the correct range?

13. Ms. Valesquez's car needs a tire pressure, p , of 30 psi. It should be within 0.5 psi of the recommended value. She uses the expression $|30 - p|$ to test whether the pressure is acceptable. Is a pressure of 29.4 psi acceptable?

✓ Independent Practice

Find the distance between the numbers given in each of Exercises 1–4.

1. 9, -9 2. 3, -4 3. 5, 6 4. -32, -52

5. The table below shows the temperature at different times of day. How much did the temperature change by between 7 a.m. and 8 a.m.?

Time	7 a.m.	8 a.m.
Temperature	-5 °C	1 °C

6. A person stands on a pier fishing. The top of their rod is 20 feet above sea level. The line goes vertically down and hooks a fish 13 feet below sea level. How long is the line?

7. Priscilla tries to keep the balance of her checking account, b , always less than \$50 away from \$200. She uses the expression $|200 - b|$ to check that it is within these limits. Is a balance of \$242.50 acceptable?

Check it out:

It might help you to draw diagrams to go with Exercises 5 and 6.

Now try these:

Lesson 2.2.2 additional questions — p436

Round Up

Absolute values are used to find distances between numbers. They're also useful when measurements are only allowed to be a certain distance away from a set value. In these situations, it doesn't matter if the numbers are above or below the set point — it's how far away they are that's important.

Lesson 2.3.1

Section 2.3

Adding and Subtracting Integers and Decimals

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide **rational numbers** (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:

You'll practice adding and subtracting integers on a number line, and then you'll go on to working with decimals.

Key words:

- integer
- decimal

Don't forget:

Integers are all the numbers that don't involve a decimal or a fraction. They can be positive or negative.

You've had plenty of practice *adding and subtracting integers* in earlier grades — and you've probably done some *adding and subtracting of decimals* too. The same rules apply to decimals — you just have to be careful where you put the decimal point.

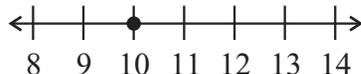
Add Integers Using a Number Line

A **number line** is a really good way to show integers in order. Adding and subtracting just involves **counting** left or right on the number line.

Example 1

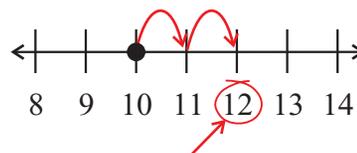
Calculate $10 + 2$ using a number line.

Solution



Find the first number on the number line.

Count the number of positions given by the second number.



Go **right** if it's a **positive** number.

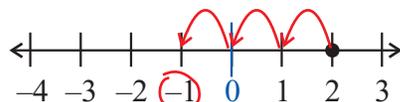
Then just **read off** the number you've ended up at. So $10 + 2 = 12$

If you're adding a **negative** number, you count to the **left**.

Example 2

Calculate $2 + (-3)$ using a number line.

Solution



So $2 + (-3) = -1$

Subtractions can be turned into additions — then you can use the methods above for subtractions.

Subtracting a **positive** number is the same as **adding** a **negative** one. For example, $5 - 6 = 5 + (-6)$

And **subtracting** a **negative** number is the same as **adding** a **positive** one. For example, $5 - (-6) = 5 + 6$

Example 3

Calculate: (i) $2 - 3$ (ii) $10 - (-2)$

Solution

(i) $2 - 3$ is the same as $2 + (-3)$. So $2 - 3 = -1$ (using Example 2).

(ii) $10 - (-2)$ equals $10 + 2$. So $10 - (-2) = 12$ (using Example 1).

Don't forget:

$$a - b \longrightarrow a + (-b)$$

$$a - (-b) \longrightarrow a + b$$

Guided Practice

Use a number line to do the calculations in Exercises 1–8.

1. $10 + 3$

2. $6 + 1$

3. $-3 + 5$

4. $-9 - 7$

5. $7 + (-6)$

6. $19 - (-5)$

7. $3 + (-2)$

8. $-1 - (-9)$

Decimal Addition Needs a Decimal Number Line

Addition with **decimals** isn't any tougher than with integers — you've just got to remember to draw a number line that includes decimals.

Example 4

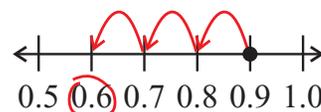
Calculate $0.9 - 0.3$ using a number line.

Solution

$0.9 - 0.3$ is a subtraction. But because **subtracting a positive number** is the same as **adding a negative one**, it can be written as:

$$0.9 - 0.3 = 0.9 + (-0.3)$$

You're dealing with **decimals**, so you need a number line that shows **decimal values**. Find 0.9, and count 0.3 to the **left** because you're **adding a negative number**.



So $0.9 - 0.3 = 0.6$

Guided Practice

Use a number line to do the calculations in Exercises 9–17.

9. $0.6 + 0.4$

10. $0.1 + 0.8$

11. $1.3 + 0.7$

12. $2.3 + (-0.4)$

13. $3.1 - 0.7$

14. $-0.7 - (-0.4)$

15. $-0.9 - 0.3$

16. $-1.2 - (-0.5)$

17. $0.3 - 0.5$

You Don't Always Have to Use a Number Line

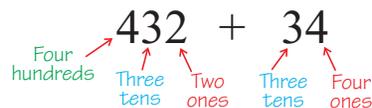
It's hard to add big numbers on a number line, so you need to be able to add and subtract **without** a number line.

Example 5

Calculate $432 + 34$ without using a number line.

Solution

You can break the calculation down into **hundreds**, **tens**, and **ones**.



You need to add the ones of each number together, then the tens, and so on. Write the numbers on **top** of each other with the ones lined up.

$$\begin{array}{r} 432 \\ +34 \\ \hline \end{array}$$

Work out the **sum** of the **ones** column first, then the **tens**, then the **hundreds**.

$$\begin{array}{r} 432 \\ +34 \\ \hline 466 \end{array}$$

So $432 + 34 = 466$

The last example was easier than some calculations because all the column sums came to less than 10. If they go **over 10** then you have to **carry** the extra numbers to the next column.

Example 6

Calculate $567 + 125$.

Solution

First you need to write the sum out **vertically** with the ones, the tens, and the hundreds lined up.

$$\begin{array}{r} 567 \\ +125 \\ \hline 1 \leftarrow 2 \end{array}$$

10 is the same as 1 ten — so carry a 1 to the tens column

The ones calculation is $7 + 5 = 12$.

12 is the same as saying **“one ten and two ones”** — so write 2 under the ones, and **carry** the 10 to the tens column.

When you add up the next column, remember to **add the 1** that you carried.

$$\begin{array}{r} 567 \\ +125 \\ \hline 692 \end{array}$$

So the tens column is now $6 + 2 + 1 = 9$

So $567 + 125 = 692$

You can use a similar method for decimals, but there are a few things to remember. The digits after a decimal point show **parts of a whole** — so 24.56 means “2 tens, 4 ones, 5 tenths, and 6 hundredths.”

Don't forget:

“Hundreds,” “tens,” “ones,” “tenths,” “hundredths,” and so on are called place values.

$$24.560$$

two tens, four units, six hundredths, five tenths, zero thousandths

You can also add **extra zeros** onto the end of a decimal and it **doesn't change** the value.

You should **always** only add digits with the same place values. So when you're adding **decimals**, line the values up by the decimal point.

Example 7

Calculate $13.93 + 5.2$.

Solution

Write the sum out **vertically** with the decimal points lined up.

$$\begin{array}{r} 13.93 \\ + 5.20 \\ \hline \end{array}$$

Make sure the decimal points are lined up
Add zeros to get the same number of decimal places

Then work out the **sum** of each **column** in turn, starting with the right-hand side. Don't forget the decimal point in the answer.

$$\begin{array}{r} 13.93 \\ + 5.20 \\ \hline 19.13 \end{array}$$

Carry numbers to the next column on the left just like before.

So $13.93 + 5.2 = 19.13$

Guided Practice

Calculate the following sums without using a number line.

18. $210 + 643$

19. $613 + 117$

20. $1264 + 527$

21. $33.7 + 12.4$

22. $14.8 + 16.2$

23. $55.82 + 34.81$

24. $75.1 + 14.31$

25. $62.4 + 31.99$

26. $2.29 + 9.92$

Some Subtractions Involve “Borrowing”

Doing column subtractions seems tough if the top number in a column is **smaller** than the number underneath — but there’s a handy method.

Example 8

Calculate $30 - 18$.

Solution

$\begin{array}{r} 30 \\ -18 \\ \hline \end{array}$ If you write this in columns, the upper number in the ones column is **smaller** than the number below.

You can **break down** 30 into different parts:

$$30 = 20 + 10 \quad \text{You could say this as “2 tens plus 1 ten” or you could say “2 tens plus 10 ones.”}$$

All you’ve done is separate one of the **tens** from the number.

You can show this in column notation: $\begin{array}{r} 2\cancel{0}^{10} \\ -18 \\ \hline 12 \end{array}$ “Borrow” 10 from the tens column, so now the column subtractions are $10 - 8$ and $2 - 1$.

So $30 - 18 = 12$

Don't forget:

If you “borrow” a 10 when you’re subtracting, remember that the number you borrowed from will now be 1 ten lower.

Example 9

Calculate $65.37 - 31.5$.

Solution

Add a zero to make them the **same** number of decimal places: $\begin{array}{r} 65.37 \\ -31.50 \\ \hline \end{array}$

If the top number in a column is smaller than the bottom one, use the same method of “**borrowing**” from the next column.

$\begin{array}{r} 65.\cancel{3}^{4}7 \\ -31.50 \\ \hline 33.87 \end{array}$ You need to borrow from the ones column. A one is ten tenths — so the 3 in the tenths column becomes **13**.

So $65.37 - 31.5 = 33.87$

✓ Guided Practice

Calculate the following sums without using a number line.

- | | | |
|---------------------|---------------------|---------------------|
| 27. $50 - 26$ | 28. $62 - 18$ | 29. $76 - 49$ |
| 30. $941 - 46$ | 31. $62.4 - 31.2$ | 32. $68.83 - 11.3$ |
| 33. $29.42 - 13.18$ | 34. $46.11 - 21.95$ | 35. $42.38 - 36.45$ |

✓ Independent Practice

Draw number lines to answer Exercises 1–4.

1. $7 + 9$ 2. $7 - (-9)$ 3. $9 + 7$ 4. $-9 - 7$

Calculate the following without using a number line.

- | | | |
|---------------------|---------------------|--------------------|
| 5. $0.99 + 0.45$ | 6. $1.86 + 3.33$ | 7. $15.64 + 3.67$ |
| 8. $45.64 + 13.88$ | 9. $164.31 + 251.3$ | 10. $32.12 - 12.1$ |
| 11. $112.13 - 38.7$ | 12. $19.4 - 5.37$ | 13. $64.11 - 44.7$ |

Now try these:

Lesson 2.3.1 additional questions — p436

Round Up

*Adding and subtracting integers and decimals — it sounds like a lot to learn, but you use the same methods over and over. You’ll need to add and subtract decimals when you’re doing **real-life math** too.*

Lesson 2.3.2

Multiplying and Dividing Integers

California Standards:

Number Sense 1.2

Add, subtract, **multiply, and divide rational numbers** (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:

You'll practice multiplying and dividing integers on a number line, and then using other methods.

Key words:

• integer

Multiplying and dividing are important skills, both in math and real life. In this Lesson you'll see how multiplication and division can be modeled.

Picture Multiplication and Division on a Number Line

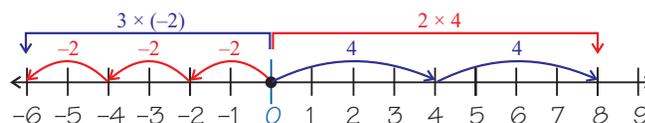
A number line is just a way of showing the **order** of numbers — so you can use it for any kind of calculation, including **multiplication** and **division**.

Multiplying by positive integers looks like a set of “hops” away from **zero**.

Example 1

Calculate using the number line: (i) 2×4 , (ii) $3 \times (-2)$

Solution



(i) Multiplying 4 by 2 means you need to move 2 groups of 4 away from zero. So $2 \times 4 = 8$

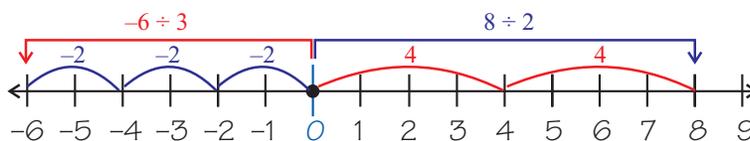
(ii) Multiplying -2 by 3 means you need to move 3 groups of -2 from zero. So $3 \times (-2) = -6$

Dividing by a positive integer looks like you're breaking a number down into **equally sized** parts.

Example 2

Calculate using the number line: (i) $8 \div 2$, (ii) $-6 \div 3$

Solution



(i) Dividing 8 by 2 means finding 8 on the number line, and then splitting the distance between 8 and 0 into 2 equal parts. So $8 \div 2 = 4$

(ii) Dividing -6 by 3 means you find -6 on the number line, and then split the distance between -6 and 0 into 3 equal parts. So $-6 \div 3 = -2$

Don't forget:

You learned about the rules for multiplying by positive and negative numbers in grade 6.

Don't forget:

The “sign” of a number just means whether it is positive or negative.

Sometimes it's hard to tell whether your answer will be **positive or negative**. The table below shows what the sign of the answer will be:

positive	\times/\div	positive	=	positive
positive	\times/\div	negative	=	negative
negative	\times/\div	positive	=	negative
negative	\times/\div	negative	=	positive

So for example, $8 \div (-2) = -4$
while $-2 \times (-4) = 8$

✓ Guided Practice

Use a number line to work out the problems given in Exercises 1–12.

1. 4×3

2. $2 \times (-5)$

3. -4×6

4. $-4 \times (-3)$

5. -6×4

6. $-3 \times (-3)$

7. $20 \div 5$

8. $18 \div 6$

9. $22 \div 2$

10. $-15 \div 3$

11. $16 \div (-4)$

12. $-14 \div (-7)$

You Must Know How to Multiply Without a Calculator

You can only really use a number line for simple multiplications. Here are two good ways of multiplying any two numbers together:

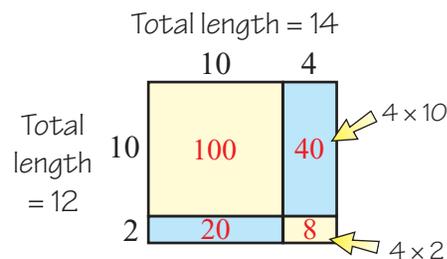
Example 3

Calculate 12×14 .

Solution

Picture each number as the length of a side of a rectangle — but break each number down into tens and ones. Then you can do the multiplication “part by part.”

The total area of the rectangle is 12×14 , and you can see this equals $100 + 40 + 20 + 8 = 168$



Another method is to write the numbers on **top** of each other.

$$\begin{array}{r} 14 \\ \times 12 \\ \hline 28 \\ + 140 \\ \hline 168 \end{array}$$

First write the numbers as a vertical calculation. Multiply the top number by the **ones** digit of the bottom number. $2 \times 14 = 28$

Then multiply the top number by the **tens** of the bottom number: $10 \times 14 = 140$

Then add them together, just like in the “rectangle” method.

This is known as **long multiplication**.

Notice how the above two methods are **very similar**.

The first line of work in the long multiplication is the same as the area of the bottom part of the rectangle, and the second line of work in the long multiplication is the same as the area of the top part of the rectangle.

In both methods you then **add these together** to get the overall result.

✓ Guided Practice

Use the methods in Example 3 for Exercises 13–20.

13. 12×13

14. 22×16

15. 32×18

16. -14×37

17. -46×42

18. 25×58

19. 52×67

20. 85×95

Work Through Long Division from Left to Right

Long division is a good way of writing down and solving tricky division problems. It involves dividing big numbers bit by bit, by breaking them into collections of smaller numbers.

Check it out:

Long division means breaking a division down into small problems like figuring out that 9×5 goes into 46 (with 1 left over).

But, in Example 4, remember that the 4 is in the hundreds column and the 6 is in the tens column, so you're really working out that 90×5 goes into 460 (with 10 left over). Long division is just a way of breaking up complicated division problems.

Example 4

Calculate $467 \div 5$.

Solution

You need to divide the whole of 467 by 5, but you can work through it bit by bit.

The number you're dividing by goes on the left. Work from the left to find numbers that divide by 5.

Keep going until you've divided the whole number.

The answer to this division is

93 with remainder 2.

The standard way of writing this is $467 = (93 \times 5) + 2$.

$$\begin{array}{r}
 9 \leftarrow \text{5 doesn't divide into 4, but it divides into 46 (this is actually 460). The maximum number of times 5 divides into 46 is 9, so put 9 in the tens column. You now know that the answer is in the 90s.} \\
 5 \overline{)467} \\
 \underline{-45} \downarrow \leftarrow 9 \times 5 = 45, \text{ so put 45 underneath, lined up with 46.} \\
 17 \leftarrow \text{Subtract 45 from 46, and bring the 7 down to find the amount left.} \\
 \\
 93 \leftarrow \text{There's 17 remaining. Now divide 17 by 5. 5 goes into 17 a maximum of 3 times. Write this in the units column.} \\
 5 \overline{)467} \\
 \underline{-450} \\
 17 \\
 \underline{-15} \leftarrow 3 \times 5 = 15, \text{ so put 15 underneath, lined up the units.} \\
 2 \leftarrow \text{There's just 2 left. 5 won't go into 2, so it's a remainder.} \\
 \\
 93 \text{ R}2 \\
 5 \overline{)467}
 \end{array}$$

Guided Practice

Write out and solve these calculations using long division.

21. $72 \div 6$

22. $104 \div 4$

23. $105 \div 7$

24. $274 \div 13$

25. $1955 \div 8$

26. $5366 \div 13$

Independent Practice

Show the multiplications in Exercises 1–3 on a number line.

1. 3×1

2. 2×5

3. 4×-2

Use the area method to find the products in Exercises 4–6.

4. 13×15

5. 12×65

6. 33×56

Find the products in Exercises 7–12.

7. 11×18

8. 13×22

9. 25×21

10. -33×12

11. 16×-15

12. -23×-51

Find the quotients in Exercises 13–18.

13. $710 \div 5$

14. $138 \div 6$

15. $1248 \div -8$

16. $190 \div 3$

17. $274 \div 4$

18. $172 \div 5$

Check it out:

A quotient is what you get when you divide one number by another.

A product is what you get when you multiply numbers.

Now try these:

Lesson 2.3.2 additional questions — p437

Round Up

Multiplying and dividing large numbers without a calculator seems like quite a task. But if you break the numbers up into ones, tens, and hundreds, then they're much simpler to handle.

Lesson 2.3.3

Multiplying Fractions

California Standards:

Number Sense 1.2

Add, subtract, **multiply**, and divide **rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:

You'll practice multiplying fractions, and you'll extend this to multiplying fractions by integers and mixed numbers.

Key words:

- area model
- mixed numbers

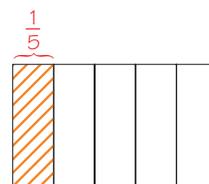
You've *multiplied fractions* in other grades — but it's still not an easy topic. In this Lesson you'll get plenty more practice at it.

Area Models Show Fraction Multiplication

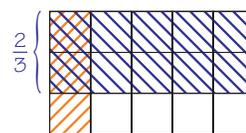
Multiplying a fraction by another fraction means working out **parts of a part**. For example, $\frac{1}{5} \times \frac{2}{3}$ means “**one-fifth of two-thirds**.”

You can show this graphically — it's called an **area model**. You need to start by drawing a rectangle.

- Shade in $\frac{1}{5}$ of the rectangle in one direction:



- Then shade in $\frac{2}{3}$ of it in the other direction, using a different color:



The part showing $\frac{1}{5} \times \frac{2}{3}$ is the part that represents **one-fifth of two-thirds** — this is the part that's shaded in both colors.

There are 2 squares shaded out of a total of 15, so $\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$.

Example 1

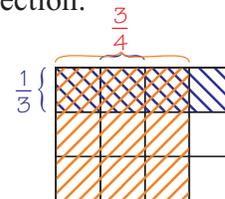
Calculate $\frac{3}{4} \times \frac{1}{3}$ using the area model method.

Solution

You need to work out **three-quarters of one-third** — so shade in $\frac{3}{4}$ of the rectangle in one direction, and $\frac{1}{3}$ in the other direction:

There are 3 out of 12 squares shaded

in both colors, so $\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$.



Check it out:

The answer here is correct, although it could be simplified. See the next page (or Section 2.1) for more information.

Guided Practice

Calculate these fraction multiplications by drawing area models:

1. $\frac{1}{3} \times \frac{2}{5}$

2. $\frac{3}{4} \times \frac{1}{5}$

3. $\frac{3}{4} \times \frac{1}{6}$

You Can Multiply Fractions Without Drawing Diagrams

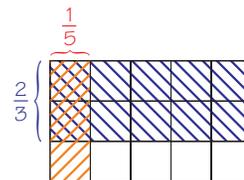
Don't forget:

The top number in a fraction is called the numerator, and the bottom number is called the denominator.

$\frac{2}{5}$
 \leftarrow numerator
 \leftarrow denominator

When you draw an area model, the **total number of squares** is always the same as the **product of the denominators** of the fractions you're multiplying.

You've already seen the area model for $\frac{1}{5} \times \frac{2}{3}$:



Multiply the denominators:

- The total number of squares is $5 \times 3 = 15$.

Also, the **number of squares shaded in both colors** is always the same as the **product of the numerators**.

Multiply the numerators:

- The total number of squares shaded in both colors is $1 \times 2 = 2$.

That means you can work the product out without drawing the area model.

Example 2

Calculate $\frac{3}{4} \times \frac{1}{3}$ without drawing a diagram.

Solution

Multiply the numerators: $3 \times 1 = 3$

Multiply the denominators: $4 \times 3 = 12$

Now write this as a fraction: $\frac{3}{12}$
 \leftarrow numerator
 \leftarrow denominator

So $\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$.

The solution to Example 2 could be simplified a bit more.

Simplifying just means writing the solution using **smaller numbers**, but so that the fraction still means the same thing.

The numerator and denominator can both be divided by 3... $\frac{3}{12} = \frac{1}{4}$...so $\frac{1}{4}$ represents the same amount, but simplified.

That means you could write $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$.

Don't forget:

You can simplify fractions using the greatest common factor (GCF) — there's a lot more about this in Section 2.1.

Don't forget:

A fraction is in its simplest form if 1 is the only number that divides exactly into both the numerator and denominator.

Guided Practice

Calculate these fraction multiplications without drawing area models. Simplify your answer where possible.

4. $\frac{5}{6} \times \frac{1}{2}$

5. $\frac{2}{3} \times \frac{1}{3}$

6. $\frac{1}{3} \times \frac{6}{7}$

7. $\frac{3}{8} \times \frac{1}{4}$

8. $\frac{5}{7} \times \frac{3}{5}$

9. $\frac{11}{12} \times \frac{6}{7}$

First Convert Whole or Mixed Numbers to Fractions

Don't forget:

A mixed number contains an integer and a fraction.

For example, $3\frac{1}{2}$ is a mixed number.

Don't forget:

The whole number 3 is

equivalent to $3 \times \frac{2}{2} = \frac{6}{2}$.

$\frac{2}{2}$ is the same as one whole.

To multiply **fractions** by **mixed numbers**, you can just write out the mixed numbers as a **single fraction** and carry on multiplying as normal.

The same is true if you need to multiply a fraction by an **integer** — you can write the integer as a fraction and use the multiplication method from before.

Example 3

Calculate: (i) $3\frac{1}{2} \times \frac{1}{4}$, (ii) $\frac{4}{5} \times 8$

Solution

(i) Convert $3\frac{1}{2}$ to a fraction: $3\frac{1}{2} = \frac{(3 \times 2) + 1}{2} = \frac{7}{2}$

Then just multiply out the fractions as normal:

$$3\frac{1}{2} \times \frac{1}{4} = \frac{7}{2} \times \frac{1}{4} = \frac{7}{8}$$

(ii) The integer 8 can be written as $\frac{8}{1}$.

$$\text{So you can multiply as normal: } \frac{4}{5} \times 8 = \frac{4}{5} \times \frac{8}{1} = \frac{32}{5}$$

Guided Practice

Calculate the following, and simplify your solutions where possible.

10. $1\frac{1}{3} \times \frac{1}{5}$

11. $\frac{1}{3} \times 2\frac{1}{3}$

12. $1\frac{2}{3} \times \frac{2}{3}$

13. $3 \times \frac{2}{7}$

14. $1\frac{1}{4} \times \frac{1}{5}$

15. $1\frac{5}{7} \times 1\frac{1}{2}$

Don't forget:

If you're multiplying negative numbers, remember the rules on p75 for working out the sign of the answer.

Don't forget:

You should usually write answers as mixed numbers instead of improper fractions — this keeps the numbers simpler.

Now try these:

Lesson 2.3.3 additional questions — p437

Independent Practice

Find the product and simplify each calculation in Exercises 1–3.

1. $\frac{2}{3} \times \frac{7}{10}$

2. $-\frac{4}{9} \times \frac{3}{15}$

3. $-\frac{5}{12} \times 2\frac{1}{3}$

4. A positive whole number is multiplied by a positive fraction smaller than one. Explain how the size of the product compares to the original whole number.

5. A rectangular patio measures $8\frac{1}{4}$ feet wide and $12\frac{1}{2}$ feet long. What is the area of the patio?

6. A recipe for 12 muffins calls for $3\frac{1}{4}$ cups of flour. How many cups of flour are needed to make 42 muffins?

Round Up

Multiplying fractions is OK because you *don't* need to put each fraction over the same denominator. If you need to multiply by *integers* or *mixed numbers*, just turn them into fractions too.

Lesson 2.3.4

Dividing Fractions

California Standard:

Number Sense 1.2

Add, subtract, multiply, and **divide rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:

You'll learn about the reciprocal of a fraction and how to use it when dividing by fractions.

Key words:

- reciprocal
- divisor

Don't forget:

The reciprocal (or multiplicative inverse) of a number is what you have to multiply the number by to get 1. There's more about reciprocals in Lesson 1.1.4.

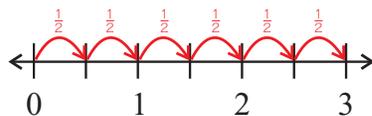
Don't forget:

Also, since the reciprocal of $\frac{1}{2}$ is 2, dividing something by 2 is the same as multiplying by $\frac{1}{2}$. There's more about this on the next page.

*Dividing fractions is hard to grasp — but once you learn a couple of useful techniques it'll all seem a lot easier. The most important thing to learn is how to use **reciprocals**.*

You Can Show Fraction Division on a Number Line

A problem like $3 \div \frac{1}{2}$ can be hard to imagine. In words, it means “work out how many halves are in three.” You can show it on a number line.



You can see that $\frac{1}{2}$ fits into 3 six times.

$$\text{So } 3 \div \frac{1}{2} = 6.$$

You can use a number line to show that the number of halves in any number will always be double that number.

For example, there are 8 halves in 4, so $4 \div \frac{1}{2} = 4 \times 2 = 8$.

Dividing is the Same as Multiplying by the Reciprocal

Multiplication and division are really closely linked. In fact, you can write any **division** problem as a **multiplication** problem using a **reciprocal**.

Dividing by a number is the same as multiplying by its reciprocal.

The **product** of a number and its **reciprocal** is 1.

For example, the reciprocal of 2 is $\frac{1}{2}$. This means that dividing something by $\frac{1}{2}$ is the **same** as multiplying by 2.

Example 1

Calculate $3 \div \frac{1}{4}$.

Solution

The reciprocal of $\frac{1}{4}$ is 4.

So you can rewrite this as a multiplication: $3 \div \frac{1}{4} = 3 \times 4 = 12$.

So $3 \div \frac{1}{4} = 12$.

Guided Practice

Calculate these by converting each division problem into a multiplication problem. Give your solutions in their simplest form.

1. $3 \div \frac{1}{3}$

2. $8 \div \frac{1}{4}$

3. $8 \div \frac{2}{3}$

4. $3 \div \frac{3}{5}$

5. $\frac{5}{6} \div 5$

6. $\frac{11}{12} \div 11$

Solve Fraction ÷ Fraction Using Reciprocals Too

You can turn **any** division into a **multiplication** using the **reciprocal** of the divisor (the thing you're dividing by).

It makes dividing fractions by fractions much easier than it seems at first.

Example 2

Calculate $\frac{2}{3} \div \frac{1}{9}$.

Solution

Dividing by a number is the same as multiplying by its reciprocal.

The reciprocal of $\frac{1}{9}$ is 9, or $\frac{9}{1}$.

Fraction: Reciprocal:

$$\frac{1}{9} \quad \frac{9}{1}$$

So you need to work out $\frac{2}{3} \times \frac{9}{1}$.

$$\text{This is } \frac{2}{3} \times \frac{9}{1} = \frac{2 \times 9}{3 \times 1} = \frac{18}{3} = 6$$

Don't forget:

$\frac{1}{9}$ and $\frac{9}{1}$ are reciprocals.

They multiply to give 1.

$$\frac{1}{9} \times \frac{9}{1} = \frac{1 \times 9}{9 \times 1} = \frac{9}{9} = 1$$

Don't forget:

To multiply two fractions, you first multiply the numerators to get the numerator of your answer. You then multiply the two denominators, and this becomes the denominator of your answer.

See the previous Lesson for more information.

To convince yourself that $\frac{2}{3} \div \frac{1}{9}$ really does equal 6, remember that in words, the problem means, “how many ninths are in two-thirds?”

Look at the square on the right.

Two-thirds of it has been colored in.

The square has then been divided into ninths — there are **six ninths** in the colored two-thirds.

In other words, $\frac{2}{3} \div \frac{1}{9} = 6$.

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Guided Practice

Find the reciprocal of the following fractions:

7. $\frac{1}{5}$

8. $\frac{2}{3}$

9. $\frac{5}{7}$

Calculate Exercises 10–15 by converting each division problem into a multiplication problem. Give your solutions in their simplest form.

10. $\frac{1}{3} \div \frac{1}{5}$

11. $\frac{2}{5} \div \frac{1}{6}$

12. $\frac{1}{5} \div \frac{3}{4}$

13. $\frac{1}{2} \div \frac{3}{8}$

14. $\frac{1}{3} \div \frac{9}{1}$

15. $\frac{6}{7} \div \frac{1}{4}$

Convert Mixed Numbers into Fractions

If you have a division problem involving mixed numbers, you need to write them out as **fractions** before you start to divide.

Example 3

Calculate $-4\frac{6}{7} \div 2\frac{1}{6}$.

Solution

First convert the mixed numbers to fractions.

$$-4\frac{6}{7} = -\frac{(4 \times 7) + 6}{7} = -\frac{34}{7} \quad \text{and} \quad 2\frac{1}{6} = \frac{(2 \times 6) + 1}{6} = \frac{13}{6}$$

Write the division as a multiplication using the reciprocal of the **divisor**. The divisor is the number that you're **dividing by**.

$$-4\frac{6}{7} \div 2\frac{1}{6} = -\frac{34}{7} \div \frac{13}{6} = -\frac{34}{7} \times \frac{6}{13}$$

Do the multiplication in the normal way.

$$-\frac{34}{7} \times \frac{6}{13} = -\frac{34 \times 6}{7 \times 13} = -\frac{204}{91}, \text{ or } -2\frac{22}{91}$$

Don't forget:

1 can be written as $\frac{7}{7}$.

So 4 is equal to $4 \times \frac{7}{7}$.

Guided Practice

Calculate the answers to the following divisions.

Write each solution as a mixed number or an integer.

16. $4 \div 2\frac{1}{2}$

17. $5\frac{1}{3} \div \frac{4}{9}$

18. $-9\frac{1}{2} \div -3\frac{1}{8}$

Independent Practice

Calculate the following.

1. $\frac{3}{4} \div 5$

2. $\frac{2}{3} \div 7$

3. $\frac{9}{13} \div 2$

4. $\frac{1}{2} \div \frac{1}{3}$

5. $\frac{2}{5} \div \frac{3}{4}$

6. $\frac{7}{9} \div \frac{2}{18}$

7. $2\frac{1}{3} \div \frac{1}{2}$

8. $4 \div \frac{5}{6}$

9. $8\frac{9}{10} \div 7\frac{1}{6}$

10. A one-pound bag of sugar is equal to $2\frac{2}{5}$ cups. A recipe for cornbread requires $\frac{2}{3}$ of a cup of sugar. How many batches of cornbread can be made with the bag of sugar?

11. The area of a rectangular floor is $62\frac{3}{4}$ square feet. The length of the room is $10\frac{1}{4}$ feet. What is the width of the room?

Now try these:

Lesson 2.3.4 additional questions — p437

Round Up

The most important thing to remember here is that you can convert any fraction division into a multiplication using the reciprocal. Remember that little fact, and you'll make your life a lot easier.

Lesson 2.3.5

Common Denominators

California Standards:

Number Sense 1.1

Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), **compare rational numbers in general.**

Number Sense 1.2

Add, subtract, multiply, and divide **rational numbers** (integers, **fractions,** and terminating decimals) and take positive rational numbers to whole-number powers.

Number Sense 2.2

Add and subtract fractions by using factoring to find common denominators.

What it means for you:

You'll learn the first step of adding and subtracting fractions — putting fractions over a common denominator. You'll also see how this allows you to compare fractions to find which is greater.

Key words:

- prime
- factorization
- least common multiple
- common denominator

Don't forget:

A number that has no factors except itself and 1 is called a prime number.

Don't forget:

Factors of a number are any whole numbers that divide exactly into it.

*Adding and subtracting fractions isn't always straightforward. Before you can add or subtract fractions, they need to be over a **common denominator**, and one way to find a common denominator is to find the **least common multiple (LCM)** of the denominators.*

Prime Factorization — the First Step to Finding an LCM

Finding the **prime factorization** of a number involves writing it as a **product of prime factors** (prime numbers multiplied together).

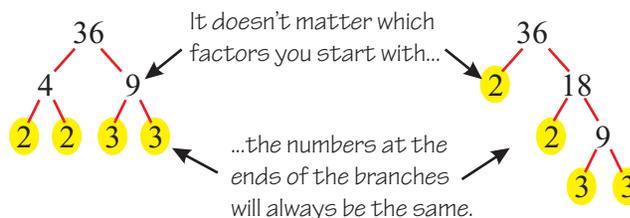
Example 1

Write 36 as a product of prime factors.

Solution

One way to do this is to look for **factors of 36**, then look for **factors of those factors**, and then look for **factors of those factors**, and so on.

You can arrange your factors in a “tree.” Break down 36 into two factors and write them underneath. Then look for factors of your factors.



Each branch ends in a prime number — these can't be broken down further. So the prime factorization of 36 is $36 = 2 \times 2 \times 3 \times 3$

Example 2

Write the following as products of prime factors: a) 35, b) 37

Solution

a) $35 = 5 \times 7$

5 and 7 are both prime, so the tree stops here. The prime factorization of 35 is $35 = 5 \times 7$

b) 37

37 is prime, so the tree has no branches. 37 **doesn't factor**.

✓ Guided Practice

Write the following as products of prime factors.

1. 15

2. 30

3. 17

4. 16

5. 24

6. 50

Don't forget:

Multiples are what you get in multiplication tables. For example, the multiples of 3 are 3, 6, 9, 12, 15, 18... and so on.

Check it out:

The least common multiple is useful when finding a common denominator for two fractions.

The LCM is the Least Common Multiple

You can use prime factorizations to find the **least common multiple** of two numbers, which is the **smallest** number that **both** will **divide into evenly**.

Example 3

Find the least common multiple of 8 and 14.

Solution

There are two ways of doing this.

1. You can **start listing all the multiples** of each number, and the **first one** that they have **in common** is the least common multiple:

- **Multiples of 8:** 8, 16, 24, 32, 40, 48, **56**, 64, 72...
- **Multiples of 14:** 14, 28, 42, **56**..

So **the least common multiple is 56**.

This method is fine, but it can get a bit tedious when the LCM is high.

2. You can first write each number as **the product of its prime factors**.

$$8 = 2 \times 2 \times 2 \text{ and } 14 = 2 \times 7.$$

Then make **a table** showing these factorizations. Wherever possible, put each factor of the second number in a column with the same factor. Start a **new column** if the number's not already there — so in this example, start a new column for the 7.

8	2	2	2	
14	2			7

Now make a new row — **the LCM row**.

For each box in the LCM row, write down the number in the boxes above.

8	2	2	2	
14	2			7
LCM	2	2	2	7

Finally, **multiply together** all the numbers in the LCM row to find the least common multiple.

So the least common multiple of 8 and 14 is $2 \times 2 \times 2 \times 7 = 56$.

Example 4

Find the least common multiple of 10 and 15.

Solution

- Prime factorizations: $10 = 2 \times 5$ and $15 = 3 \times 5$.
- Make a table:
- Multiply the numbers in the LCM row.

So the least common multiple of 10 and 15 is $2 \times 5 \times 3 = 30$.

10	2	5	
15		5	3
LCM	2	5	3

✓ Guided Practice

Find the least common multiple of each pair of numbers in Exercises 7–15.

7. 6 and 15

8. 10 and 12

9. 8 and 9

10. 4 and 10

11. 8 and 16

12. 20 and 30

13. 13 and 7

14. 9 and 81

15. 42 and 30

Compare Fractions Using a Common Denominator

Least common multiples can be used as **common denominators**.

Check it out:

You don't have to use the least common multiple here — any common multiple would be fine. For example, you could use $8 \times 14 = 112$. However, using the least common multiple will mean all the numbers will be smaller, which can be very useful.

Don't forget:

When you multiply both the numerator and denominator of a fraction by the same number, you don't change its value. This is because you are really just multiplying the fraction by 1, since $\frac{7}{7} = \frac{4}{4} = 1$.

Example 5

By using a common denominator, find which is greater, $\frac{5}{8}$ or $\frac{9}{14}$.

Solution

It's not easy to say which is the larger fraction when they have different denominators. You need to find fractions **equivalent** to each of these that have a **common denominator**.

You can use **any common multiple** of 8 and 14 as the common denominator. Here, we'll use the **least common multiple** (which is 56 — see Example 3). You need to decide how many 8s and how many 14s make 56 — and then **multiply the top and bottom** of each fraction by the right number:

$$\frac{5}{8} = \frac{5 \times 7}{8 \times 7} = \frac{35}{56} \qquad \frac{9}{14} = \frac{9 \times 4}{14 \times 4} = \frac{36}{56}$$

You can now see that $\frac{9}{14}$ is greater than $\frac{5}{8}$, since $\frac{36}{56}$ is greater than $\frac{35}{56}$.

✓ Guided Practice

In Exercises 16–21, put each pair of fractions over a common denominator to find which is the greater in each pair.

16. $\frac{1}{4}$ and $\frac{2}{10}$

17. $\frac{1}{3}$ and $\frac{2}{7}$

18. $\frac{3}{10}$ and $\frac{2}{7}$

19. $\frac{4}{9}$ and $\frac{5}{11}$

20. $\frac{11}{15}$ and $\frac{12}{17}$

21. $\frac{6}{9}$ and $\frac{2}{3}$

✓ Independent Practice

Write the numbers in Exercises 1–6 as products of prime factors.

1. 21

2. 100

3. 32

4. 50

5. 49

6. 132

Find the least common multiple of each pair in Exercises 7–12.

7. 6 and 8

8. 10 and 25

9. 48 and 21

10. 32 and 50

11. 100 and 49

12. 49 and 132

Find the greater fraction in each pair in Exercises 13–15.

13. $\frac{10}{21}$ and $\frac{51}{100}$

14. $\frac{50}{132}$ and $\frac{13}{49}$

15. $\frac{33}{50}$ and $\frac{32}{49}$

16. Order these fractions from least to greatest: $\frac{2}{3}, \frac{14}{17}, \frac{13}{16}, \frac{3}{4}, \frac{16}{21}$

Now try these:

Lesson 2.3.5 additional questions — p438

Round Up

Finding common denominators is something you should get real comfortable with doing, because you need to do it a lot in math — in the next few Lessons, for example.

Lesson 2.3.6

Adding and Subtracting Fractions

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide **rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

Number Sense 2.2

Add and subtract fractions by using factoring to find common denominators.

What it means for you:

You'll see how to add and subtract fractions.

Key words:

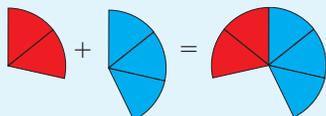
- denominator
- numerator
- common denominator

Don't forget:

The numerator is the top line of a fraction.
The denominator is the bottom line of a fraction.

Don't forget:

You can think of adding fractions as adding parts of whole shapes.



$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

*Adding and subtracting fractions can be **quick**, or it can be quite a **long process** — it all depends on whether the fractions **already have a common denominator**, or whether you have to **find it first**.*

You Can Add Fractions with a Common Denominator

If fractions have a **common denominator** (their denominators are the **same**), adding them is fairly straightforward.

To find the **numerator of the sum**, you **add the numerators** of the individual fractions. The **denominator** stays the same.

Example 1

Find $\frac{2}{7} + \frac{3}{7}$.

Solution

These two fractions have a common denominator, 7.
So **7** will also be the **denominator of the sum**.

The **numerator** of the sum will be $2 + 3 = 5$.

$$\text{So } \frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}.$$

You **subtract** fractions with a common denominator in exactly the same way.

Example 2

Find $\frac{7}{9} - \frac{2}{9}$.

Solution

The **denominator** of the result will be **9** (the fractions' common denominator). The **numerator** of the result will be $7 - 2 = 5$.

$$\text{So } \frac{7}{9} - \frac{2}{9} = \frac{7-2}{9} = \frac{5}{9}.$$

✓ Guided Practice

Find the sums and differences in Exercises 1–8.

1. $\frac{1}{5} + \frac{2}{5}$

2. $\frac{3}{11} + \frac{4}{11}$

3. $\frac{4}{5} - \frac{1}{5}$

4. $\frac{10}{21} - \frac{2}{21}$

5. $\frac{7}{15} + \frac{4}{15}$

6. $\frac{23}{50} + \frac{19}{50}$

7. $\frac{9}{25} - \frac{7}{25}$

8. $\frac{5}{17} + \frac{1}{17}$

You May Need to Find a Common Denominator First

Fractions with **unlike** denominators **cannot** be directly added or subtracted. You must first find **equivalent** fractions with a **common denominator**.

Example 3

Find $\frac{7}{9} + \frac{5}{6}$.

Solution

The denominators are different here. This means you need to find two fractions equivalent to them, but with a **common denominator**.

The common denominator can be **any** common multiple of 9 and 6.

- You could use $9 \times 6 = 54$ as your common denominator. Then the equivalent fractions will be:

$$\frac{7}{9} \times \frac{6}{6} = \frac{7 \times 6}{9 \times 6} = \frac{42}{54} \qquad \frac{5}{6} \times \frac{9}{9} = \frac{5 \times 9}{6 \times 9} = \frac{45}{54}$$

Now you can add these fractions: $\frac{42}{54} + \frac{45}{54} = \frac{87}{54}$, which you can simplify to $\frac{29}{18}$ by dividing the numerator and denominator by 3.

- Or you could find the **LCM** (least common multiple) using prime factorizations. Since $9 = 3^2$ and $6 = 2 \times 3$, the LCM is $2 \times 3 \times 3 = 18$.

This time, the equivalent fractions are:

$$\frac{7}{9} \times \frac{2}{2} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18} \quad \text{and} \quad \frac{5}{6} \times \frac{3}{3} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$$

Now you can add these to get $\frac{14}{18} + \frac{15}{18} = \frac{29}{18}$.

You can use the **LCM** or **any other common multiple** as your common denominator — you'll end up with the **same answer**. But using the LCM means that the numbers in your fractions are smaller and easier to use.

Example 4

By putting both fractions over a common denominator, find $\frac{11}{15} - \frac{3}{20}$.

Solution

Use a table to find the LCM of 15 and 20 — this is $5 \times 3 \times 2 \times 2 = 60$.

Find equivalent fractions with denominator 60:

$$\frac{11}{15} = \frac{11 \times 4}{15 \times 4} = \frac{44}{60} \quad \text{and} \quad \frac{3}{20} = \frac{3 \times 3}{20 \times 3} = \frac{9}{60}$$

So rewriting the subtraction gives: $\frac{11}{15} - \frac{3}{20} = \frac{44}{60} - \frac{9}{60} = \frac{35}{60}$

This can be simplified to $\frac{35}{60} = \frac{7}{12}$.

15	5	3		
20	5		2	2
LCM	5	3	2	2

Don't forget:

A fraction whose numerator and denominator are the same is equal to 1 — so multiplying another fraction by, say, $\frac{6}{6}$ or $\frac{9}{9}$ doesn't change its value. This is why you can multiply the numerator and denominator by the same amount, and leave the value of the fraction unchanged.

Don't forget:

To find an LCM, you can use a table like this one. (See Lesson 2.3.5 for more information.)

9		3	3
6	2	3	
LCM	2	3	3

Check it out:

Again, any common multiple of 15 and 20 can be used as a common denominator — including their product $15 \times 20 = 300$.

But this will mean the numbers in the calculation will be bigger than necessary. For example:

$$\begin{aligned} \frac{11}{15} - \frac{3}{20} &= \frac{220}{300} - \frac{45}{300} \\ &= \frac{175}{300} \\ &= \frac{7}{12} \end{aligned}$$

Check it out:

Even if you use the LCM as your common denominator, you may still be able to simplify the answer at the end.

✓ Guided Practice

Calculate the answers in Exercises 9–11.

9. $\frac{14}{15} - \frac{2}{5}$

10. $\frac{7}{10} + \frac{2}{3}$

11. $-\frac{2}{3} + \frac{4}{7}$

Be Extra Careful if There are Negative Signs

As always in math, if there are **negative numbers** around, you have to be extra careful.

Don't forget:

Remember... subtracting a negative number is exactly the same as adding a positive number.

Don't forget:

You could also swap the terms around to form a subtraction:

$$\frac{-3}{5} + \frac{2}{7} = \frac{2}{7} - \frac{3}{5}$$

Don't forget:

$\frac{-11}{35}$, $-\frac{11}{35}$, and $\frac{11}{-35}$ are all equal.

Example 5

Find $-\frac{3}{5} - \left(-\frac{2}{7}\right)$.

Solution

This looks tricky because of all the negative signs. So take things slowly and carefully.

This sum can be rewritten as $\frac{-3}{5} + \frac{2}{7}$ — it means exactly the same.

The LCM of 5 and 7 is $5 \times 7 = 35$.

So put both these fractions over a common denominator of 35.

$$\frac{-3}{5} = \frac{-3 \times 7}{5 \times 7} = \frac{-21}{35} \qquad \frac{2}{7} = \frac{2 \times 5}{7 \times 5} = \frac{10}{35}$$

Now you can add the two fractions in the same way as before.

$$\frac{-3}{5} - \left(-\frac{2}{7}\right) = \frac{-21}{35} + \frac{10}{35} = \frac{-21+10}{35} = \frac{-11}{35} \text{ or } -\frac{11}{35}$$

✓ Guided Practice

Calculate the answers in Exercises 12–14. Simplify your answers.

12. $-\frac{2}{3} - \left(-\frac{3}{8}\right)$

13. $\frac{7}{6} - \left(-\frac{4}{9}\right)$

14. $-\frac{25}{48} - \left(-\frac{7}{16}\right)$

✓ Independent Practice

Calculate the following. Give all your answers in their simplest form.

1. $\frac{2}{7} + \frac{8}{7}$

2. $\frac{8}{5} - \frac{2}{5}$

3. $\frac{7}{9} - \frac{2}{9}$

4. $\frac{7}{16} - \frac{3}{8}$

5. $\frac{19}{30} + \frac{7}{20}$

6. $\frac{5}{7} + \frac{5}{8}$

7. $-\frac{3}{16} - \frac{5}{17}$

8. $\frac{19}{20} + \left(-\frac{20}{21}\right)$

Round Up

Keep practicing this until it becomes routine... (i) find a common denominator, (ii) put both fractions over this denominator, (iii) do the addition or subtraction, (iv) simplify your answer if possible. You'll see this again next Lesson.

Lesson 2.3.7

Adding and Subtracting Mixed Numbers

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide **rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

Number Sense 2.2

Add and subtract fractions by using factoring to find common denominators.

What it means for you:

You'll learn to add and subtract numbers like $2\frac{1}{2}$ and $3\frac{5}{16}$.

Key words:

- mixed number
- common denominator

Don't forget:

You can convert $2\frac{2}{3}$ to a fraction by remembering that:

(i) $2\frac{2}{3}$ means $(2 \times 1) + \frac{2}{3}$.

(ii) 1 can be written as $\frac{3}{3}$.

$$\begin{aligned} \text{So } 2\frac{2}{3} &= \left(2 \times \frac{3}{3}\right) + \frac{2}{3} \\ &= \frac{2 \times 3}{3} + \frac{2}{3} \\ &= \frac{(2 \times 3) + 2}{3} \end{aligned}$$

This Lesson covers nothing new — it just combines things that you've learned before — converting mixed numbers to fractions, and adding and subtracting fractions. So if they seemed tricky the first time around, this is a good chance to get a bit more practice.

Adding Mixed Numbers — Convert to Fractions First

Mixed numbers are things like $4\frac{1}{2}$ and $-6\frac{7}{8}$, with both an integer and a fraction part.

It's easiest to add the numbers together if you **convert them to fractions first**. Once they're written as **improper fractions** you can add them by finding the **least common multiple** as before.

Example 1

Find $1\frac{1}{2} + 2\frac{2}{3}$.

Solution

Convert both the numbers to fractions.

$$1\frac{1}{2} = \frac{(1 \times 2) + 1}{2} = \frac{3}{2} \qquad 2\frac{2}{3} = \frac{(2 \times 3) + 2}{3} = \frac{8}{3}$$

Put both fractions over a common denominator of $2 \times 3 = 6$.

$$\frac{3}{2} = \frac{3 \times 3}{2 \times 3} = \frac{9}{6} \qquad \frac{8}{3} = \frac{8 \times 2}{3 \times 2} = \frac{16}{6}$$

Do the addition: $1\frac{1}{2} + 2\frac{2}{3} = \frac{9}{6} + \frac{16}{6} = \frac{25}{6}$

Example 2

Find $4\frac{1}{2} - 6\frac{7}{8}$.

Solution

Convert both the numbers to fractions.

$$4\frac{1}{2} = \frac{(4 \times 2) + 1}{2} = \frac{9}{2} \qquad 6\frac{7}{8} = \frac{(6 \times 8) + 7}{8} = \frac{55}{8}$$

Put both fractions over a common denominator of 8.

$$\frac{9}{2} = \frac{9 \times 4}{2 \times 4} = \frac{36}{8}, \text{ and } \frac{55}{8} \text{ is already over a denominator of 8.}$$

Do the subtraction: $4\frac{1}{2} - 6\frac{7}{8} = \frac{36}{8} - \frac{55}{8} = \frac{36 - 55}{8} = -\frac{19}{8}$

✓ Guided Practice

For Exercises 1–6, give your answers as fractions.

1. $3\frac{3}{4} + 2\frac{3}{4}$

2. $8\frac{1}{2} + 1\frac{2}{3}$

3. $6\frac{2}{3} - \frac{1}{2}$

4. $2\frac{3}{4} - 4\frac{3}{5}$

5. $1\frac{1}{7} - 2\frac{5}{8}$

6. $4\frac{4}{5} - 1\frac{1}{6}$

Take Extra Care With Negative Signs

If there are a lot of **negative signs**, you should take extra care.

Example 3

Find $-3\frac{2}{3} - (-2\frac{6}{7})$.

Solution

Convert the mixed numbers to fractions.

$$-3\frac{2}{3} = -\left(\frac{(3 \times 3) + 2}{3}\right) = -\frac{11}{3} \quad \text{and} \quad -2\frac{6}{7} = -\left(\frac{(2 \times 7) + 6}{7}\right) = -\frac{20}{7}$$

Now find a common denominator — you can use $3 \times 7 = 21$.

$$-\frac{11}{3} = -\frac{11 \times 7}{3 \times 7} = -\frac{77}{21} \quad \text{and} \quad -\frac{20}{7} = -\frac{20 \times 3}{7 \times 3} = -\frac{60}{21}$$

So the calculation becomes

$$-\frac{77}{21} - \left(-\frac{60}{21}\right) = -\frac{77}{21} + \frac{60}{21} = \frac{-77 + 60}{21} = -\frac{17}{21}$$

Don't forget:

Subtracting a negative number is the same as adding a positive number.

Also take extra care if the calculation has more than two terms — you have to make sure that all the fractions have the same denominator.

Example 4

Find $\frac{5}{3} - \frac{3}{5} + 2\frac{2}{3}$.

Solution

Convert the mixed number to a fraction. $2\frac{2}{3} = \frac{(2 \times 3) + 2}{3} = \frac{8}{3}$

Now find a common denominator — you can use $5 \times 3 = 15$.

$$\frac{5}{3} = \frac{5 \times 5}{3 \times 5} = \frac{25}{15} \quad \frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15} \quad \frac{8}{3} = \frac{8 \times 5}{3 \times 5} = \frac{40}{15}$$

So the calculation becomes $\frac{25}{15} - \frac{9}{15} + \frac{40}{15} = \frac{25 - 9 + 40}{15} = \frac{56}{15}$

Don't forget:

Always do additions and subtractions from left to right.

✓ Guided Practice

Do each of the calculations in Exercises 7–9.

Give your answers as fractions.

7. $-4\frac{5}{6} - 2\frac{2}{3}$

8. $-5\frac{5}{6} - \left(-4\frac{11}{12}\right)$

9. $2\frac{1}{2} - \frac{5}{6} + 1\frac{1}{12}$

Simplify Your Answers if Possible

It's usually a good idea to **simplify** your answers if possible.

Example 5

Find $1\frac{2}{3} + 2\frac{5}{6}$.

Solution

Convert the mixed numbers to fractions.

$$1\frac{2}{3} = \frac{(1 \times 3) + 2}{3} = \frac{5}{3} \quad \text{and} \quad 2\frac{5}{6} = \frac{(2 \times 6) + 5}{6} = \frac{17}{6}$$

You can use **6** as a common denominator.

Find a fraction equivalent to $\frac{5}{3}$ with a denominator of 6: $\frac{5}{3} = \frac{5 \times 2}{3 \times 2} = \frac{10}{6}$.

Do the addition: $1\frac{2}{3} + 2\frac{5}{6} = \frac{10}{6} + \frac{17}{6} = \frac{10+17}{6} = \frac{27}{6}$

Simplify this to get your final answer: $\frac{27}{6} = \frac{9}{2}$ **Divide the numerator and denominator by 3**

Don't forget:

To simplify a fraction you divide the numerator and denominator by the same number (a common factor).

✓ Guided Practice

For Exercises 10–12, give your answers in their simplest form.

10. $2\frac{3}{8} + 3\frac{1}{8}$

11. $3\frac{5}{16} - 1\frac{1}{16}$

12. $1\frac{3}{8} + 2\frac{5}{10}$

✓ Independent Practice

Calculate the following. Give all your answers in their simplest form.

1. $2\frac{2}{3} + 3\frac{1}{3}$

2. $3\frac{1}{4} - 1\frac{1}{2}$

3. $2\frac{2}{7} - 1\frac{4}{5}$

4. $2\frac{3}{5} - 9\frac{4}{15}$

5. $5\frac{7}{8} - 12\frac{3}{16}$

6. $8\frac{1}{4} - 1\frac{7}{11}$

7. $2\frac{3}{4} + 3\frac{2}{3} - 5\frac{3}{8}$

Round Up

You can hopefully see how you can use the same old routine over and over...

Convert mixed numbers to **fractions**, find a **common denominator**, do the **calculation**.

Lesson 2.4.1

Section 2.4

Further Operations with Fractions

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

Number Sense 2.2

Add and subtract fractions by using factoring to find common denominators.

What it means for you:

You'll see how doing complex calculations involving fractions uses exactly the same ideas you've seen earlier in this Section.

Key words:

- order of operations
- common denominator

Don't forget:

LCM means least common multiple — see Lesson 2.3.5.

Don't forget:

You can use PEMDAS to help you remember the correct order of operations. Parentheses, Exponents, Multiplication and Division, Addition, and Subtraction.

Don't forget:

Dividing by a fraction is the same as multiplying by its reciprocal.

Don't forget:

A raised dot \cdot means multiplication.
So $2\frac{4}{5} \cdot 2 = 2\frac{4}{5} \times 2$

The problems start to get more and more *complicated* now. There are all kinds of calculations in this Lesson. But you just have to remember what you've learned before, and use it carefully.

Order of Operations: Multiplication Before Addition

Math problems with fractions can involve **combinations** of operations — for example, you might need to do a multiplication and addition.

Example 1

Calculate $\frac{2}{5} + \frac{5}{2} \times \frac{7}{3}$.

Solution

Remember — if there are no parentheses, you do multiplication **before** addition. So this calculation becomes

$$\frac{5 \times 7}{2 \times 3} + \frac{2}{5} = \frac{35}{6} + \frac{2}{5}$$

The LCM of 6 and 5 is $6 \times 5 = 30$. Use this as the common denominator.

$$\frac{35}{6} = \frac{35 \times 5}{6 \times 5} = \frac{175}{30} \quad \text{and} \quad \frac{2}{5} = \frac{2 \times 6}{5 \times 6} = \frac{12}{30}$$

Find the result.

$$\frac{175}{30} + \frac{12}{30} = \frac{175 + 12}{30} = \frac{187}{30}$$

Example 2

Find $\frac{30}{45} \div \frac{2}{3} - 1\frac{3}{4}$.

Solution

Do the division first. $\frac{30}{45} \div \frac{2}{3} = \frac{30}{45} \times \frac{3}{2} = \frac{30 \times 3}{45 \times 2} = \frac{90}{90} = 1$

Then the subtraction. $1 - 1\frac{3}{4} = \frac{4}{4} - \frac{7}{4} = \frac{4 - 7}{4} = -\frac{3}{4}$

Guided Practice

Calculate the following. Simplify your answers where possible.

1. $\frac{1}{2} + \frac{3}{2} \times \frac{5}{6}$

2. $4\frac{3}{4} \div 2 - \frac{1}{8}$

3. $1\frac{7}{8} \cdot 2 + 2\frac{4}{5} \cdot 2$

Remember PEMDAS with Really Complex Expressions

With calculations that **look complicated**, take things **slowly**.

Example 3

Calculate $\left(4\frac{2}{3} + \frac{1}{2}\right) \times \frac{1}{6}$.

Solution

First convert the mixed number to a fraction so that you can do the **calculation** more easily.

$$4\frac{2}{3} = \frac{(4 \times 3) + 2}{3} = \frac{14}{3}$$

Then calculate the **sum inside the parentheses** using **6 as the common denominator**.

$$\begin{aligned}\left(\frac{14}{3} + \frac{1}{2}\right) \times \frac{1}{6} &= \left(\frac{28}{6} + \frac{3}{6}\right) \times \frac{1}{6} \\ &= \left(\frac{31}{6}\right) \times \frac{1}{6}\end{aligned}$$

Finally, do the **multiplication**.

$$\frac{31}{6} \times \frac{1}{6} = \frac{31}{36}$$

Example 4

Calculate $\left(3\frac{3}{4} + \frac{1}{4}\right) - 3 \cdot \frac{1}{4}$

Solution

First turn the mixed number into a fraction. $3\frac{3}{4} = \frac{(3 \times 4) + 3}{4} = \frac{15}{4}$

Evaluate the expression in the **parentheses**. $3\frac{3}{4} + \frac{1}{4} = \frac{15}{4} + \frac{1}{4} = \frac{16}{4} = 4$

Do the **multiplication**. $3 \cdot \frac{1}{4} = \frac{3}{4}$

Now you have: $\left(3\frac{3}{4} + \frac{1}{4}\right) - 3 \cdot \frac{1}{4} = 4 - \frac{3}{4}$

Finally, do the **subtraction**. $= \frac{16}{4} - \frac{3}{4} = \frac{13}{4}$

Don't forget:

It's a good idea to simplify fractions whenever you can — don't wait until you've got your final answer.

This way, you're making the numbers smaller and easier to use in the rest of the calculation.

Example 5

Calculate $\frac{1}{2} \times \left(1 + 4\frac{1}{3}\right)$

Solution

Parentheses first:

To add the numbers, put them over a common denominator of 3.

$$1 + 4\frac{1}{3} = \frac{3}{3} + \frac{(4 \times 3) + 1}{3} = \frac{3}{3} + \frac{13}{3} = \frac{16}{3}$$

The **division** is next, but you'll need to convert $3\frac{1}{3}$ to $\frac{10}{3}$ first.

$$\frac{1}{2} = \frac{1}{2} \div \frac{10}{3} = \frac{1}{2} \times \frac{3}{10} = \frac{1 \times 3}{2 \times 10} = \frac{3}{20}$$

You can now do the **multiplication**.

$$\frac{1}{2} \times \left(1 + 4\frac{1}{3}\right) = \frac{3}{20} \times \frac{16}{3} = \frac{3 \times 16}{20 \times 3} = \frac{48}{60}$$

The final step is to simplify the answer: $\frac{48}{60} = \frac{4}{5}$

Don't forget:

Do divisions and multiplications from left to right.

Don't forget:

$$3\frac{1}{3} = \frac{(3 \times 3) + 1}{3} = \frac{10}{3}$$

✓ Guided Practice

Calculate the expressions in Exercises 4–5.

Give your answers in their simplest form.

4. $\frac{1}{6} \cdot \frac{2}{7} - \frac{5}{7} \cdot \frac{1}{2} + \frac{7}{12} \cdot \frac{8}{7}$

5. $3\frac{1}{8} + \frac{5}{8}$

✓ Independent Practice

Calculate the following. Give all your answers in their simplest form.

1. $\frac{3}{4} \cdot \frac{1}{6} + \frac{5}{12}$

2. $\frac{8}{15} - \frac{3}{5} \cdot \frac{1}{3} + \frac{27}{30}$

3. $\frac{5}{12} \cdot \frac{9}{2} + \frac{3}{4} \cdot \frac{5}{3} - \frac{5}{8} \cdot \frac{1}{3}$

4. $\frac{3}{5} + \frac{2}{15} + 2\frac{2}{3} - \frac{1}{2}$

Now try these:

Lesson 2.4.1 additional questions — p439

Round Up

Some of the expressions in this Lesson were really complicated. When you have a tricky expression to evaluate, you just need to *take your time*, remember **PEMDAS**, and work through it very *carefully*.

Lesson 2.4.2

Multiplying and Dividing Decimals

California Standards:

Number Sense 1.2

Add, subtract, **multiply, and divide rational numbers** (integers, fractions, and **terminating decimals**) and take positive rational numbers to whole-number powers.

What it means for you:

You'll practice multiplying and dividing decimals.

Key words:

- integer
- decimal

Check it out:

This is really similar to the fraction multiplication area model in Lesson 2.3.3. You would have shaded exactly the same areas if you were multiplying $\frac{1}{5}$ and $\frac{3}{5}$. These fractions are equivalent to the decimals 0.2 and 0.6.

Check it out:

In Example 1, multiplying two decimals each with 1 decimal place gives you a solution with 2 decimal places. In fact, you always count the decimal places in both factors to find out how many decimal places the solution has.

The trickiest thing about multiplying and dividing decimals is working out where the decimal point should go. Once you understand the method, it's a lot more straightforward though.

Modeling Decimal Multiplication

You can use an **area model** to show what happens when you multiply two decimal numbers.

Example 1

Calculate 0.2×0.6 .

Solution

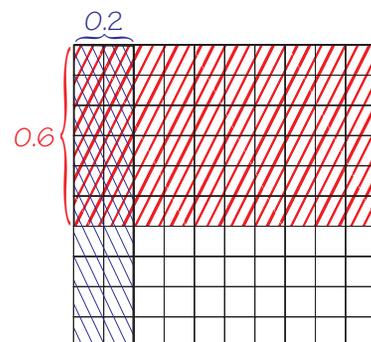
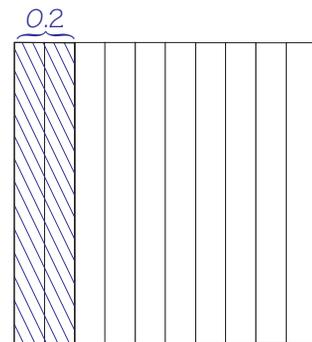
0.2 is two-tenths — that's how much of the square on the right is shaded.

0.6 is six-tenths — this is shaded in the other direction on the diagram below.

The part that's been **shaded twice** is 0.2×0.6 . The square's now divided into **100**, so each small square is $1 \div 100 = 0.01$.

There are **12 squares** that have been shaded twice and each represents 0.01.

So $0.2 \times 0.6 = 0.12$



Guided Practice

Use an area model to show the following multiplications.

1. 0.4×0.3

2. 0.8×0.2

3. 0.1×0.9

Multiplying Decimals

Multiplying decimals is just like multiplying integers — only you have to make sure you put the **decimal point** in the correct place.

You can **rewrite** a decimal multiplication as a **fraction calculation**.

Example 2

Calculate: 2×1.6

Solution

$$2 \times 1.6 = 2 \times \frac{16}{10} \quad \leftarrow \text{Write the decimal as a fraction...}$$

$$= \frac{2 \times 16}{10} \quad \leftarrow \text{...which can be rewritten like this.}$$

$$= \frac{32}{10} = 32 \div 10 = \mathbf{3.2} \quad \leftarrow \text{Now divide by the 10 to get the decimal answer.}$$

This works if **both** numbers are decimals too:

Example 3

Calculate: 2.1×0.04

Solution

$$2.1 \times 0.04 = \frac{21}{10} \times \frac{4}{100} \quad \leftarrow \text{Write both decimals as fractions.}$$

$$= \frac{21 \times 4}{10 \times 100} \quad \leftarrow \text{Multiply them.}$$

$$= \frac{84}{1000} = 84 \div 1000 = \mathbf{0.084} \quad \leftarrow \text{Now divide by the 1000 to get the decimal answer.}$$

Check it out:

Another way of multiplying decimals is to just ignore the decimal point, do an integer multiplication, then figure out where the decimal point goes.

For example:

$$2.1 \times 0.04$$

Ignore the decimal points and do $21 \times 4 = 84$.

The factors had a total of 3 decimal places:

$$2.1 \text{ (1 decimal place)} \times \\ 0.04 \text{ (2 decimal places)}$$

So you know the product must have 3 decimal places, so move the decimal point 3 places to the left:

$$\curvearrowright 84. \Rightarrow \mathbf{0.084}$$

Don't forget:

Multiplying a positive by a negative gives a negative — see page 75 for more.

Always Check the Position of Your Decimal Point

You can check you have the **correct number of decimal places** in the product by making sure it's the same as the **total** number of decimal places **in the factors**.

For example, $2.\underline{1} \times 3.\underline{67} = 7.\underline{707}$
 $1 \text{ digit} + 2 \text{ digits} = 3 \text{ digits}$

Watch out for calculations that give decimals with **zeros at the end** — you have to count the final zeros. For example, multiplying 1.5 by 1.2 gives **1.80**. You must count up the decimal places before rewriting this as 1.8.

Guided Practice

Write out and solve these calculations.

4. $1.2 \times (-1.1)$

5. 4.5×5.9

6. $1.6 \times (-8.2)$

7. 6.31×6.4

8. $(-2.77) \times (-7.3)$

9. 9.1×2.44

Dividing Decimals — Take Care with the Decimal Point

You can think about **decimal division** in a similar way.

Example 4

Calculate: $46.5 \div 0.05$

Solution

$$\begin{aligned}
 46.5 \div 0.05 &= \frac{465}{10} \div \frac{5}{100} && \text{Write the decimals as fractions} \\
 &= \frac{465}{10} \times \frac{100}{5} && \text{To divide by a fraction, multiply by its reciprocal} \\
 &= \frac{4650}{5} = 4650 \div 5 = \mathbf{930}
 \end{aligned}$$

Don't forget:

You can do integer division using long division. Check back to Lesson 2.3.2.

Check it out:

You could also do the division $46.5 \div 0.05$ in Example 4 by doing integer division, then working out the decimal places: $465 \div 5 = 93$. There's **1 decimal place** in 46.5, and **2 decimal places** in 0.05. So move **1 decimal place** to the left, and **2 to the right**. So the decimal point moves one place right overall: $46.5 \div 0.05 = 930$

Check it out:

0.5 is 10 times smaller than 5. This means that the result of dividing something by 0.5 will be 10 times bigger than the result of dividing the same thing by 5. To make a result 10 times bigger, you move the decimal point one place to the right.

Now try these:

Lesson 2.4.2 additional questions — p439

Another way is to ignore the decimal points and just use **integer division**. Then you have to find the correct position for the **decimal point**:

- Count the number of digits after the decimal point in the **first** number and move the decimal point this many places to the **left**.
- Count the number of digits after the decimal point in the **second** number and move the decimal point this many places to the **right**.

Example 5

If $5 \div 25 = 0.2$, calculate $0.005 \div 2.5$.

Solution

0.005 is the first number — it has **3 decimal places**.

2.5 is the second number — it has **1 decimal place**.

So you have to move **3 places to the left**, and then **1 to the right**.

$$0.0000.2 \Rightarrow 0.0002 \Rightarrow 0.005 \div 2.5 = \mathbf{0.002}$$

✓ Guided Practice

Solve these divisions.

10. $273 \div 13$

11. $195.2 \div 8$

12. $53.56 \div 1.3$

13. $1.56 \div 3$

14. $1.44 \div 3$

15. $6.55 \div 5$

16. $3.84 \div 1.2$

17. $4.64 \div 1.6$

18. $33.58 \div 2.3$

✓ Independent Practice

In Exercises 1–6, find the product of each pair of numbers.

1. 1.03×0.5

2. 4.781×-6.0

3. $-3.11 \times (-9.14)$

4. 7.2×0.6

5. 1.04×4

6. 10.5×0.07

Find the quotient in Exercises 7–9.

7. $-7.25 \div 0.25$

8. $-16.8 \div (-0.04)$

9. $12.48 \div -1.2$

Round Up

Wow — that was a lot of information about multiplying and dividing. But don't panic — when you're multiplying and dividing with decimals, you just need to take it slow, working bit by bit.

Lesson 2.4.3

Operations with Fractions and Decimals

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:

You'll practice adding, subtracting, multiplying, and dividing fractions and decimals together.

Key words:

- fraction
- decimal
- multiply
- divide

You've probably now seen nearly all the ideas, techniques, and tricks you'll ever need for almost *any kind* of number problem. Now you're going to put them all into action...

Calculations Can Involve Both Fractions and Decimals

To do a calculation involving **a fraction and a decimal**, you either have to make them **both fractions** or **both decimals**.

For example, if you needed to find $0.3 \times \frac{3}{5}$, you could...

• Convert 0.3 to a fraction $\left(\frac{3}{10}\right)$ and find $\frac{3}{10} \times \frac{3}{5} = \frac{9}{50}$.

• Convert $\frac{3}{5}$ to a decimal (0.6) and find $0.3 \times 0.6 = \mathbf{0.18}$.

Both methods are correct, and they both give the same answer (you can check by calculating $9 \div 50$, which gives **0.18**).

Example 1

Calculate $0.25 \times \frac{4}{5}$ by:

a) converting 0.25 to a fraction,

b) converting $\frac{4}{5}$ to a decimal.

Solution

a) 0.25 is $\frac{25}{100} = \frac{1}{4}$. So $0.25 \times \frac{4}{5} = \frac{1}{4} \times \frac{4}{5} = \frac{4}{20} = \frac{1}{5}$

b) $\frac{4}{5}$ is $4 \div 5 = 0.8$. So $0.25 \times \frac{4}{5} = 0.25 \times 0.8 = \mathbf{0.2}$

The best idea is to choose the method that makes the calculation **easiest**.

Example 2

Calculate $\frac{1}{3} - 0.5$.

Solution

If you convert $\frac{1}{3}$ to a decimal, you get 0.33333333..., which is difficult to subtract 0.5 from. So it's best to use **fractions** here.

$$\text{So } \frac{1}{3} - 0.5 = \frac{1}{3} - \frac{1}{2} = \frac{2}{6} - \frac{3}{6} = -\frac{1}{6}$$

Don't forget:

0.3333... can also be written as $0.\overline{3}$. (But that doesn't make it any easier to subtract 0.5 from.)

✓ Guided Practice

For Exercises 1–6, find the answers to the calculations by:

(i) converting all the numbers to fractions,

(ii) converting all numbers to decimals.

Check that both your answers for each exercise are equivalent.

1. $\frac{1}{2} + 0.2$

2. $\frac{1}{4} - 0.2$

3. $\frac{1}{4} + 0.4$

4. $\frac{1}{3} \times 0.3$

5. $\frac{1}{6} \times 1.2$

6. $\frac{1}{10} \times 2.5$

For Exercises 7–9, use the easiest method to do each calculation.

7. $\frac{1}{7} + \frac{3}{98}$

8. $0.15 \div \frac{1}{2}$

9. $0.3 - \left(\frac{1}{3} \times 4.5\right)$

You Can Think of Fraction Multiplication Another Way

There's another useful way to think about multiplying by fractions.

When you multiply by the fraction $\frac{3}{5}$, it is the same as either:

(i) **multiplying by 3** and then **dividing by 5**, or

(ii) **dividing by 5** and then **multiplying by 3**.

Check it out:

Multiplying by $\frac{1}{5}$ is the same as dividing by 5.

Example 3

Calculate $6.93 \times \frac{2}{3}$.

Solution

This would be difficult to do using decimals ($6.93 \times 0.6666\dots$).

And 6.93 doesn't convert to an "easy" fraction $\left(\frac{693}{100}\right)$.

But multiplying by $\frac{2}{3}$ means **dividing by 3** and then **multiplying by 2**.

This is much quicker.

Dividing by 3 gives: $6.93 \div 3 = \mathbf{2.31}$

Then multiplying by 2 gives: $2.31 \times 2 = \mathbf{4.62}$

So, $6.93 \times \frac{2}{3} = \mathbf{4.62}$

Example 4

Calculate $\frac{2}{7} \times 1.5$.

Solution

You're multiplying 1.5 by $\frac{2}{7}$, so **multiply it by 2**, then **divide by 7**.

So $1.5 \times 2 = \mathbf{3}$, and then $3 \div 7 = \frac{\mathbf{3}}{\mathbf{7}}$. So, $\frac{2}{7} \times 1.5 = \frac{\mathbf{3}}{\mathbf{7}}$

Check it out:

You can do the multiplication and division in either order — so choose whichever will be easier.

Example 5

Calculate $\frac{1}{3}(6.35 - 3.02)$.

Solution

As always, work out the parentheses first.

$$6.35 - 3.02 = \mathbf{3.33}.$$

Then you can divide by 3.

$$3.33 \div 3 = \mathbf{1.11}$$

Don't forget:

Multiplying by $\frac{1}{3}$ is the same as dividing by 3.

✓ Guided Practice

Calculate the value of the expressions in Exercises 10–14.

10. $\frac{3}{4} \times 0.24$

11. $\frac{7}{15} \times 1.5$

12. $0.9 \times \frac{8}{9}$

13. $3.5 \times \left(\frac{3}{5} - \frac{1}{5}\right)$

14. $\frac{5}{9} \times (8.03 + 0.97)$

✓ Independent Practice

Calculate the following.

1. $\frac{1}{2} \times 0.8$

2. $\frac{1}{3} \times 1.2$

3. $\frac{1}{2} + 0.25$

4. $1.25 + \frac{7}{4}$

5. $0.8 - \frac{1}{5}$

6. $-1.3 - \frac{3}{10}$

7. $2.5 \times \frac{2}{5}$

8. $1.3 \times \frac{10}{13}$

9. $2.7 \div \frac{27}{13}$

10. $(4.57 + 3.53) \times \frac{1}{9}$

11. $(-5.36 - 1.64) \times \frac{2}{7}$

12. $(2.89 - 18.89) \div \frac{8}{5}$

13. $\frac{(5.67 + 12.53)}{\frac{2}{3}}$

Don't forget:

Dividing by $\frac{27}{13}$ is the same as multiplying by its reciprocal, $\frac{13}{27}$.

Now try these:

Lesson 2.4.3 additional questions — p439

Round Up

The theme that's been running through the last few Lessons is this — although questions might look complicated, you just need to *take things real slow*, and use all those things you learned earlier in the Section. *Don't* try to hurry — that makes you more likely to make a *mistake*.

Lesson 2.4.4

Problems Involving Fractions and Decimals

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

Mathematical Reasoning 2.2

Apply strategies and results from simpler problems to more complex problems.

What it means for you:

You'll use ideas from previous Lessons in this Section to solve some real-life problems.

Key words:

- word problem
- units

Check it out:

Very often, writing your answer in a sensible way just means adding units. You should also always check that it's realistic — see Chapter 1 for more information.

This is the last Lesson in this Section. And again, it's all about using the skills you've already learned. But this time before you can do the math, you have to write the problem down in math language using a description of a real-life situation.

First Write Real-Life Problems as Math

Example 1

The length of a rectangular room is $54\frac{1}{3}$ feet, while its width is 33.3 feet. What is the area of the room?

Solution

You find the area of a rectangle by multiplying its length by its width.

So the area of this room is given by $54\frac{1}{3} \times 33.3$.

Now you need to work through all the steps from the previous Lessons.

$$54\frac{1}{3} \times 33.3 = \frac{163}{3} \times 33.3$$

Convert mixed number to a fraction

$$= (33.3 \div 3) \times 163$$

Rewrite as “ \div then \times ”

$$= 11.1 \times 163$$

Do the division

You could use a calculator here, but you don't really need to.

To do this multiplication without a calculator, you can rewrite it using the ideas in Section 2.3.2.

$$\begin{aligned} 11.1 \times 163 &= (10 \times 163) + (1 \times 163) + (0.1 \times 163) \\ &= 1630 + 163 + 16.3 \\ &= \mathbf{1809.3} \end{aligned}$$

With real-life problems you must always think about what your answer means, and then write it in a sensible way. Here, you need to add **units**.

So the area of the room is **1809.3 square feet**.

✓ Guided Practice

1. A rectangular dance floor is 28.6 feet wide and $15\frac{1}{2}$ feet long.

What is the area of the dance floor?

2. A rectangular playing field is $16\frac{2}{3}$ yards wide and 30.9 yards long.

What is the area of the playing field?

Drawing a Diagram Can Make Things Clearer

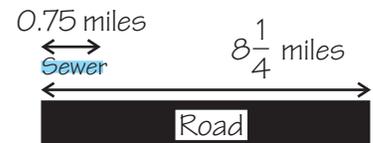
Sometimes it's not **doing** the math that's the hardest thing in a problem. It's working out **what math to do** in the first place.

Example 2

Some public sewer lines are being installed along $8\frac{1}{4}$ miles of road. The supervisor says they will be able to complete 0.75 of a mile a day. How long will the project take?

Solution

If you can't see how to answer a question, draw a picture.



You need to find out how many times 0.75 goes into $8\frac{1}{4}$.

In math language, this is a division — so you need to solve $8\frac{1}{4} \div 0.75$.

Now that the problem is written in math language, you can use the techniques from previous Lessons.

$$\begin{aligned} 8\frac{1}{4} \div 0.75 &= 8\frac{1}{4} \div \frac{3}{4} && \text{Convert the decimal to a fraction} \\ &= \frac{33}{4} \div \frac{3}{4} && \text{Convert the mixed number to a fraction} \\ &= \frac{33}{4} \times \frac{4}{3} && \text{Rewrite the division as a multiplication} \\ &= \frac{33 \times 4}{4 \times 3} && \text{Multiply the fractions} \\ &= \frac{33}{3} = 11 && \text{Simplify} \end{aligned}$$

Don't forget: about **units**...

The project will take **11 days**.

Don't forget:

Your diagram doesn't have to be accurate — just good enough to get an idea of what you need to do.

Check it out:

You could work out the fraction here as $\frac{132}{12}$, and then simplify it. But here, the factor of 4 in the numerator and the factor of 4 in the denominator have been canceled straightaway.

$$\frac{33 \times \cancel{4}}{\cancel{4} \times 3}$$

Guided Practice

3. A piece of rope 17.25 feet long must be divided up into smaller lengths of $\frac{3}{4}$ of a foot long.

How many lengths can be made from this piece of rope?

4. A birthday cake needs $2\frac{1}{2}$ cups of raisins. However, because a big party is planned, several cakes are made by increasing the amount of all the ingredients used. In fact, $8\frac{3}{4}$ cups of raisins are used.

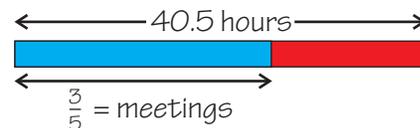
How many cakes is this enough raisins for?

Example 3

Aisha worked 40.5 hours in one particular week. Three-fifths of these hours she was in meetings, while the rest of the time was spent traveling. How many hours did Aisha spend traveling during the week?

Solution

Again, a diagram might help.



You need to work out how many hours the red part of the bar represents.

If the blue part of the bar is $\frac{3}{5}$ of the total hours, then the red part must

$$\text{represent: } 1 - \frac{3}{5} = \frac{2}{5}$$

So you need to work out $\frac{2}{5}$ of 40.5 hours

— this can be written as a multiplication: $40.5 \times \frac{2}{5}$

Now you can do the math.

$$\begin{aligned} 40.5 \times \frac{2}{5} &= (40.5 \div 5) \times 2 \\ &= 8.1 \times 2 \\ &= \mathbf{16.2} \end{aligned}$$

Rewrite the fraction multiplication as a division, followed by a multiplication

Do the division

Do the multiplication

Now say what your answer means.

Aisha spent **16.2 hours** traveling during the week.

Check it out:

Here, you could do the math as fractions...

$$\begin{aligned} 40.5 \times \frac{2}{5} &= \frac{81}{2} \times \frac{2}{5} \\ &= \frac{81}{5} \\ &= 16\frac{1}{5} \end{aligned}$$

...or decimals...

$$40.5 \times 0.4 = 16.2$$

You get the same answer whatever method you use.

Guided Practice

5. A small town has a sports field with a total area of 1533.75 square yards. One third of this area is used only by parents with small children, while the rest can be used by anyone.

How many square yards are for anyone's use?

Independent Practice

1. The floor area of a room is $99\frac{15}{16}$ square feet.

The length of the room is $10\frac{1}{4}$ feet. What is the width of the room?

2. Approximately $\frac{1}{5}$ of the students in a high school take an advanced placement class. Of those students it is thought 50% will receive a scholarship for college. What fraction of the students in the high school are expected to receive a scholarship?

Now try these:

Lesson 2.4.4 additional questions — p440

Round Up

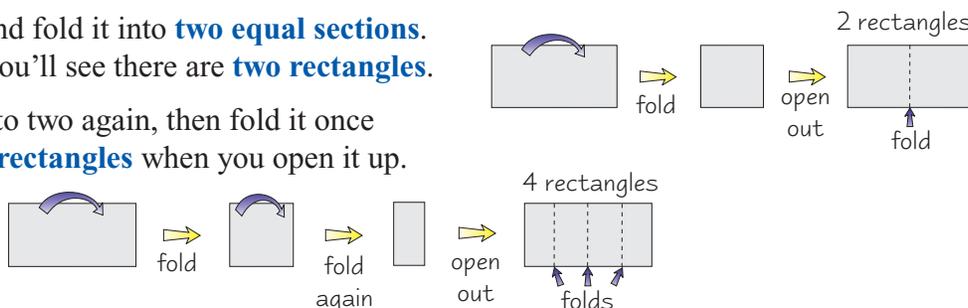
*That's the end of a Section containing a lot of little bits of information. An important thing to remember from it all is that you must work through complex problems **slowly and carefully**.*

Section 2.5 introduction — an exploration into: Basic Powers

You can find the *powers of numbers* by repeatedly folding a piece of paper. Each extra fold you make produces more rectangles. You'll see how the *numbers of rectangles* produced are powers.

Take a sheet of paper and fold it into **two equal sections**.
When you open it up, you'll see there are **two rectangles**.

If you fold the paper into two again, then fold it once more, you'll have **four rectangles** when you open it up.



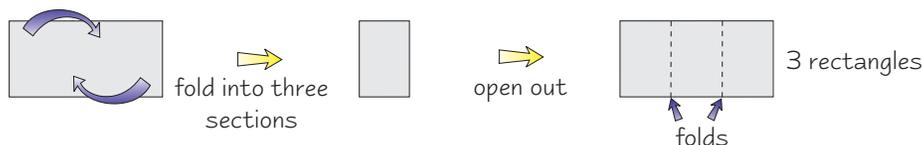
The number of rectangles produced by each fold represents the powers of 2.

$$2^0 = 1 \quad 2^1 = 2 \quad 2^2 = 4$$

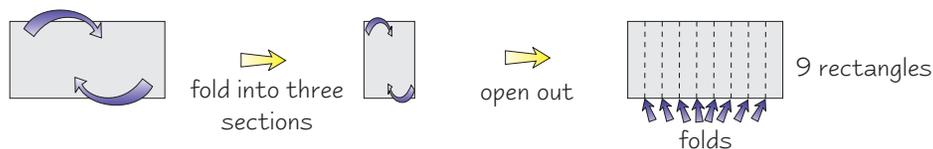
✓ Exercises

- Continue to fold the paper into two equal sections each time.
Write the total number of rectangles produced after the given number of folds.
 - 3
 - 4
 - 5
- Write your answers to **1a**, **b**, and **c** as powers of 2.

Now take a sheet of paper and make **two folds**, so that it forms **three equal sections**.



Repeat this by folding the paper into **three equal sections each time**.



✓ Exercises

- Fold a piece of paper repeatedly into three, in the way described above. A long piece of paper makes this part easier.
What is the total number of rectangles produced from the:
 - second set of folds?
 - third set of folds?
 - fourth set of folds?
- Write your answers to **3a**, **b**, and **c** as powers of 3.
- With a new piece of paper, experiment with this process to make 25 rectangles.
- Is it possible to make 10 rectangles of equal size by using the process above?

Round Up

When you repeatedly fold a piece of paper into **two**, you *repeatedly multiply* the number of rectangles by **two**. And when you repeatedly fold a piece of paper into **three**, you *repeatedly multiply* the number of rectangles by **three**. This gives you the *powers of 2* and the *powers of 3* — because powers are produced by *repeated multiplication*.

Lesson 2.5.1

Section 2.5 Powers of Integers

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and **take positive rational numbers to whole-number powers.**

Algebra and Functions 2.1

Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

What it means for you:

You'll learn how to write repeated multiplications in a shorter form.

Key words:

- power
- base
- exponent
- factor

Check it out:

Raising something to the second power is called squaring it. Raising something to the third power is called cubing it. There's more on this in Lesson 2.5.3.

Don't forget:

A factor is one of the terms in a multiplication expression.

Check it out:

You can describe a number as being "raised to" or "taken to" a power. So 2^4 could be read as, "two to the fourth power," or "two raised to the fourth power," or "two taken to the fourth power."

A **power** is just the **product** that you get when you **repeatedly multiply** a number by itself, like $2 \cdot 2$, or $3 \cdot 3 \cdot 3$. **Repeated multiplication expressions** can be very long. So there's a special system you can use for writing out **powers** in a **shorter way** — and that's what this Lesson is about.

A Power is a Repeated Multiplication

A **power** is a **product** that results from **repeatedly multiplying** a number by itself. For example:

$$2 \cdot 2 = 4, \text{ or "two to the second power."}$$

$$2 \cdot 2 \cdot 2 = 8, \text{ or "two to the third power."}$$

$$2 \cdot 2 \cdot 2 \cdot 2 = 16, \text{ or "two to the fourth power."}$$

So **4**, **8**, and **16** are all **powers of 2**.

You Can Write a Power as a Base and an Exponent

If every time you used a **repeated multiplication** you wrote it out in full, it would make your work very complicated. So there's a **shorter way** to write them. For example:

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$

2 is the **base** — it's **the number that's being multiplied**

This is the **exponent** — it tells you **how many times** the base number is a **factor** in the multiplication expression.

For example, the expression $10 \cdot 10$ can be written in this form — **the base is 10** and since 10 occurs twice, **the exponent is 2**.

$$\text{base} \rightarrow 10^2 \leftarrow \text{exponent}$$

You can rewrite any repeated multiplication in this form.

So any number, x , to the **n th power** can be written as:

$$\text{base} \rightarrow x^n \leftarrow \text{exponent}$$

Example 1

Write the expression $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ in base and exponent form.

Solution

The number that is being multiplied is 3. So the base is 3.

3 occurs as a factor five times in the multiplication expression. So the exponent is 5.

So $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$.

If a number has an **exponent of 1** then it occurs only once in the expanded multiplication expression. So any number to the **power 1** is just the **number itself**. For example:

$$\begin{aligned} 5^1 &= 5, \\ 137^1 &= 137, \\ x^1 &= x. \end{aligned}$$

 **Guided Practice**

Write each of the expressions in Exercises 1–8 as a power in base and exponent form.

1. $8 \cdot 8$

2. $2 \cdot 2 \cdot 2$

3. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

4. 5

5. $9 \cdot 9 \cdot 9 \cdot 9$

6. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

7. $-5 \cdot -5$

8. $-8 \cdot -8 \cdot -8 \cdot -8 \cdot -8$

Check it out:

When you write a negative number raised to a power, you need to put parentheses around the number. For example, $(-2)^4$ tells you to raise negative two to the fourth power — it has a value of 16.

Evaluate a Power by Doing the Multiplication

Evaluating a power means working out its value. Just write it out in its **expanded form** — then treat it as any other multiplication calculation.

Example 2

Evaluate 5^4 .

Solution

5^4 means “four copies of the number five multiplied together.”

$$5^4 = 5 \cdot 5 \cdot 5 \cdot 5$$

$$5^4 = 625.$$

Check it out:

A negative number raised to an odd power always gives a negative answer. For example:
 $(-2)^3 = -2 \cdot -2 \cdot -2 = -8$

A negative number raised to an even power always gives a positive answer. For example:
 $(-2)^4 = -2 \cdot -2 \cdot -2 \cdot -2$
 $= (-2 \cdot -2) \cdot (-2 \cdot -2)$
 $= 4 \cdot 4 = 16$

Example 3

Evaluate $(-2)^2$.

Solution

$(-2)^2$ means “two copies of the number negative two multiplied together.”

$$(-2)^2 = -2 \cdot -2$$

$$(-2)^2 = 4.$$

✓ Guided Practice

Evaluate the exponential expressions in Exercises 9–16.

9. 10^2

10. 5^3

11. 7^1

12. 3^6

13. 47^1

14. $(-15)^1$

15. $(-3)^2$

16. $(-4)^3$

✓ Independent Practice

Write each of the expressions in Exercises 1–6 in base and exponent form.

1. $4 \cdot 4 \cdot 4$

2. $9 \cdot 9$

3. 8

4. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

5. $-4 \cdot -4 \cdot -4$

6. $-3 \cdot -3 \cdot -3 \cdot -3$

7. Kiera and 11 of her friends are handing out fliers for a school fund-raiser. Each person hands out fliers to 12 people. How many people receive a flier?

Evaluate the exponential expressions in Exercises 8–13.

8. 15^2

9. 4^3

10. 8^1

11. 1^8

12. $(-5)^3$

13. $(-5)^4$

14. A single yeast cell is placed on a nutrient medium. This cell will divide into two cells after one hour. These two cells will then divide to form four cells after another hour. The process continues indefinitely.

a) How many yeast cells will be present after 1 hour, 2 hours, and 6 hours?

b) Write exponential expressions with two as the base to describe the number of yeast cells that will be present after 1 hour, 2 hours, and 6 hours.

c) How many hours will it take for the yeast population to reach 256?

Now try these:

Lesson 2.5.1 additional questions — p440

Round Up

If you need to use a *repeated multiplication*, it's useful to have a *shorter* way of writing it. That's why *bases* and *exponents* come in really handy when you're writing out *powers* of numbers. You'll see lots of powers used in *expressions*, *equations*, and *formulas*. For example, the formula for the area of a circle is πr^2 where r is the radius. So it's important you know what they mean.

Lesson 2.5.2

Powers of Rational Numbers

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and **take positive rational numbers to whole-number powers**.

Algebra and Functions 2.1

Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

What it means for you:

You'll learn how to take fractions and decimals to powers.

Key words:

- power
- exponent
- base
- decimal
- fraction

Check it out:

If you are raising a negative fraction to a power, just keep the minus sign with the numerator all the way through your work.
For example:

$$\left(-\frac{3}{4}\right)^4 = \frac{(-3)^4}{4^4}$$

In just the same way that you can raise whole numbers to powers, you can also raise fractions and decimals to powers.

You Can Raise a Fraction to a Power

A **fraction raised to a power** means exactly the same as a whole number raised to a power — repeated multiplication. But now the **complete fraction** is the **base**.

$$\text{base} \rightarrow \left(\frac{2}{3}\right)^2 \leftarrow \text{exponent}$$

This expression means $\frac{2}{3} \cdot \frac{2}{3}$.

When you raise a fraction to a power, you are raising the **numerator** and the **denominator separately** to the **same power**. For example:

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$$

This makes **evaluating** the fraction easier. You can **evaluate** the **numerator** and the **denominator separately**.

Example 1

Evaluate $\left(\frac{1}{4}\right)^3$.

Solution

$$\left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3}$$

$1^3 = 1 \cdot 1 \cdot 1 = 1$
 $4^3 = 4 \cdot 4 \cdot 4 = 64$

Raise both the numerator and the denominator to the third power.

$$\frac{1^3}{4^3} = \frac{1}{64}$$

Example 2

Evaluate $\left(-\frac{2}{5}\right)^2$.

Solution

$$\left(-\frac{2}{5}\right)^2 = \frac{(-2)^2}{5^2}$$

$(-2)^2 = -2 \cdot -2 = 4$
 $5^2 = 5 \cdot 5 = 25$

Raise both the numerator and the denominator to the second power.

$$\frac{(-2)^2}{5^2} = \frac{4}{25}$$

✓ Guided Practice

Evaluate the exponential expressions in Exercises 1–6.

1. $\left(\frac{1}{2}\right)^2$

2. $\left(\frac{1}{2}\right)^4$

3. $\left(\frac{5}{3}\right)^1$

4. $\left(\frac{9}{7}\right)^3$

5. $\left(-\frac{3}{10}\right)^2$

6. $\left(-\frac{3}{10}\right)^3$

You Can Raise a Decimal to a Power

A **decimal** raised to a power means exactly the same as a whole number raised to a power — it's a repeated multiplication. The decimal is the **base**.

$$\text{base} \rightarrow 0.24^2 \leftarrow \text{exponent}$$

This expression is the same as saying $0.24 \cdot 0.24$.

When you evaluate a decimal raised to a power, you **multiply the decimal by itself** the specified number of times. The tricky thing when you're multiplying decimals is to get the decimal point in the right place — you saw how to do this in Section 2.4.

Example 3

Evaluate $(0.3)^3$.

Solution

The multiplication you are doing here is $(0.3)^3 = 0.3 \cdot 0.3 \cdot 0.3$.

$$0.3 \times 0.3 \times 0.3 = \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \quad \text{write the decimals as fractions}$$

$$= \frac{3 \times 3 \times 3}{10 \times 10 \times 10} \quad \text{multiply the fractions}$$

$$= \frac{27}{1000} = 27 \div 1000 = \mathbf{0.027}$$

So, $(0.3)^3 = \mathbf{0.027}$

Check it out:

To check the number of decimal places your answer should have, just count the number of decimal places in all of the factors and add them together. For example, in the expression $0.3 \cdot 0.3 \cdot 0.3$, you have three factors, and each contains one decimal place. So your answer should contain three decimal places, which 0.027 does.

Check it out:

When you put the decimal point back into your answer, put in 0s to fill up any places between the decimal point and the numerical part of the answer.

For example, here the numerical part of your answer is 529, but you need four decimal places in your answer. So use a 0 before the numerical part — so the answer is 0.0529.

Example 4

Evaluate $(0.23)^2$.

Solution

$$0.23 \times 0.23 = \frac{23}{100} \times \frac{23}{100}$$

write the decimals as fractions

$$= \frac{529}{10,000} = 529 \div 10,000 = \mathbf{0.0529} \quad \text{multiply}$$

Guided Practice

Evaluate the exponential expressions in Exercises 7–12.

7. $(0.5)^2$

8. $(0.2)^3$

9. $(0.78)^1$

10. $(0.12)^2$

11. $(0.15)^3$

12. $(0.08)^2$

Independent Practice

Write each of the expressions in Exercises 1–4 in base and exponent form.

1. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

2. $0.25 \cdot 0.25$

3. $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7}$

4. $-\frac{1}{4} \cdot -\frac{1}{4} \cdot -\frac{1}{4}$

Evaluate the exponential expressions in Exercises 5–10.

5. $\left(\frac{1}{9}\right)^2$

6. $\left(\frac{1}{4}\right)^3$

7. $\left(\frac{2}{3}\right)^3$

8. $\left(\frac{9}{8}\right)^1$

9. $\left(\frac{1}{5}\right)^5$

10. $\left(-\frac{2}{5}\right)^3$

11. Mark is feeding chickens. He divides 135 g of corn into thirds. Each portion is then divided into thirds again to give small portions. What fraction of the original amount is in each small portion? How much does each small portion weigh?

Evaluate the exponential expressions in Exercises 12–17.

12. $(0.4)^2$

13. $(0.1)^4$

14. $(0.21)^2$

15. $(0.97)^1$

16. $(0.02)^2$

17. $(0.25)^3$

Now try these:

Lesson 2.5.2 additional questions — p440

Round Up

To raise a *fraction* to a *power*, you raise the *numerator* and the *denominator* separately to the *same power*. To raise a *decimal* to a *power*, you use the decimal as the *base* and raise it to a power as you would a whole number — just by *multiplying it by itself* the correct number of times.

Lesson 2.5.3

Uses of Powers

California Standards:

Number Sense 1.1

Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:

You'll see how you can use exponents to work out areas of squares and volumes of cubes, and learn about a shorter way to write very large numbers.

Key words:

- squared
- cubed
- scientific notation

Check it out:

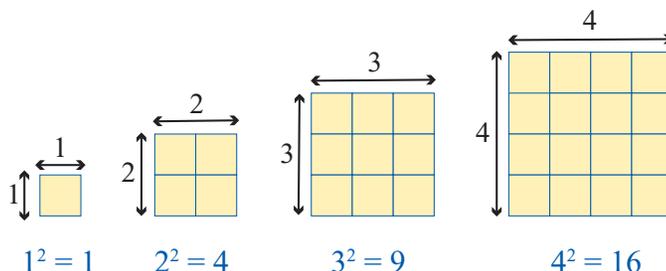
The areas of all shapes, not just squares, are measured in square units. These could be cm^2 , m^2 , inches², feet², or even miles². You'll learn more about this in Chapter 3.

You'll come across powers a lot both in math and real-life situations. That's because you use them to work out *areas* and *volumes*. They're also handy when you need to write out a very *big number* — you can use powers to write these numbers in a *shorter form*.

Exponents are Used in Some Formulas

Exponents are used in the **formulas** for the **areas** of **squares** and **circles**. In this Lesson you'll see how **exponents** are used in finding the **area** of a **square**. In the next Chapter you'll use a formula to find the area of a circle.

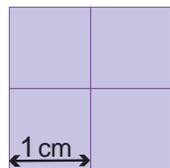
The formula for the area of a square is **Area** = $s \cdot s = s^2$, where s represents the **side length** of the **square**.



When you find the area of a square, the side length is used as a **factor twice** in the multiplication. So raising a number to the **second power** is called **squaring** it.

Example 1

Find the area of the square shown below.



Solution

Each small square is **1 cm wide**. So the side length of the whole square is **2 cm**. The **area** of the whole square is **2 cm • 2 cm = 4 cm²**.

You can see that this is true, because it is made up of **four** smaller **1 cm² squares**.

Example 2

A square has a side length of 11 inches. Find its area.

Solution

$$\text{Area} = (\text{side length})^2$$

$$\text{Area} = 11^2 = 11 \cdot 11 = 121$$

$$\text{Units: inches} \cdot \text{inches} = \text{in}^2$$

$$\text{Area} = 121 \text{ in}^2$$

Don't forget:

When you're working out the units that go with a calculation, use the unit analysis method you saw in Chapter 1 (p42). Just apply the same operations to the units as you did to the numbers.

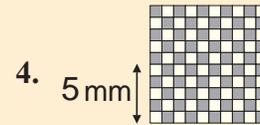
Guided Practice

Find the areas of the squares in Exercises 1–4.



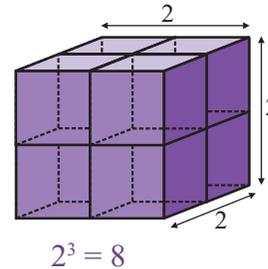
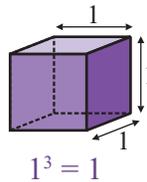
2. Square of side length 6 feet.

3. Square of side length 3.5 m.

**Exponents are Used to Find Volumes of Some Solids**

Exponents are also used in **formulas** to work out **volumes** of **solids**, like cubes, spheres, and prisms.

The formula for the volume of a cube is **Volume** = $s \cdot s \cdot s = s^3$, where s represents the **side length** of the **cube**.



When you find the volume of a cube, the side length is used as a **factor three times** in the multiplication. So raising a number to the **third power** is called **cubing** it.

Example 3

A cube has a side length of 5 cm. Find its volume.

Solution

$$\text{Volume} = (\text{side length})^3$$

$$\text{Volume} = 5 \cdot 5 \cdot 5 = 5^3 = 125$$

$$\text{Units: cm} \cdot \text{cm} \cdot \text{cm} = \text{cm}^3$$

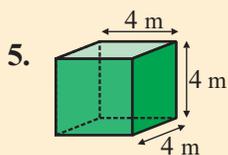
$$\text{Volume} = 125 \text{ cm}^3.$$

Check it out:

The volumes of all solids, not just cubes, are measured in cubed units. For example, the units could be cm^3 , m^3 , inches^3 , feet^3 , or even miles^3 . You'll learn more about this in Chapter 7.

✓ Guided Practice

Find the volumes of the cubes in Exercises 5–8.



6. Cube of side length 5 feet.

7. Cube of side length 7.5 mm.



Use Scientific Notation to Write Big Numbers

Sometimes in math and science you'll need to deal with numbers that are **very big**, like 570,000,000. To avoid having to write numbers like this out in full every time, you can rewrite them as a **product of two factors**.

For instance: $570,000,000 = 5.7 \times 100,000,000$

The second factor is a **power of ten**. You can write it in **base and exponent** form.

$$5.7 \times 100,000,000 = 5.7 \times 10^8.$$

So 5.7×10^8 is 570,000,000 written in **scientific notation**.

Scientific Notation

To write a number in scientific notation turn it into two factors:

→ the first factor must be a number that's at least one but less than ten.

→ the second factor must be a power of 10 written in exponent form.

Check it out:

To work out what power of ten the second factor is, just count the zeros in it.

For example, 10 is 10^1 , 1000 is 10^3 , and 10,000,000 is 10^7 .

Example 4

Write the number 128,000,000,000 in scientific notation.

Solution

$$128,000,000,000 = 1.28 \times 100,000,000,000$$

Split the number into a decimal between 1 and 10 and a power of ten.

$$= 1.28 \times 10^{11}$$

Write the number as a product of the two factors.

Check it out:

If the number you were putting into scientific notation was 51,473,582, then you would probably round it before putting it into scientific notation. You'll see more about how to round numbers in Chapter 8.

Example 5

The number 5.1×10^7 is written in scientific notation. Write it out in full.

Solution

$$\begin{aligned} 5.1 \times 10^7 &= 5.1 \times 10,000,000 \\ &= \mathbf{51,000,000} \end{aligned}$$

Write out the power of ten as a factor in full.

Multiply the two together: move the decimal point as many places to the right as there are zeros in the power of ten.

Guided Practice

Write the numbers in Exercises 9–12 in scientific notation.

9. 6,700,000

10. 32,800

11. -270,000

12. 1,040,000,000

Write out the numbers in Exercises 13–16 in full.

13. 3.1×10^3

14. 8.14×10^6

15. -5.05×10^7

16. 9.091×10^9

Independent Practice

Find the areas of the squares in Exercises 1–4.

1. Square of side length 2 cm.

2. Square of side length 8 yd.

3. Square of side length 13 m.

4. Square of side length 5.5 ft.

5. Maria is painting a wall that is 8 feet high and 8 feet wide.

She has to apply two coats of paint. Each paint can will cover 32 feet². How many cans of paint does she need?

Find the volumes of the cubes in Exercises 6–9.

6. Cube of side length 3 ft.

7. Cube of side length 6 yd.

8. Cube of side length 9 cm.

9. Cube of side length 1.5 in.

10. Tyreese is tidying up his little sister's toys. Her building blocks are small cubes, each with a side length of 3 cm. They completely fill a storage box that is a cube with a side length of 15 cm. How many blocks does Tyreese's sister have?

Write the numbers in Exercises 11–14 in scientific notation.

11. 21,000

12. -51,900,000

13. 42,820,000

14. 31,420,000,000,000

Write out the numbers in Exercises 15–18 in full.

15. 8.4×10^5

16. 2.05×10^8

17. -9.1×10^4

18. 3.0146×10^{10}

19. In 2006 the population of the USA was approximately 299,000,000. Of those 152,000,000 were female.

How many were male? Write your answer in scientific notation.

Now try these:

Lesson 2.5.3 additional questions — p441

Round Up

When you're finding the *area* of a *square* or the *volume* of a *cube*, your calculation will always involve *powers*. That's because the formulas for both the *area* of a *square* and the *volume* of a *cube* involve repeated multiplication of the *side length*. Powers also come in useful for writing very *large* numbers in a *shorter form* — that's what *scientific notation* is for.

Lesson 2.5.4

More on the Order of Operations

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:

You'll learn how to use the PEMDAS rules with expressions that have decimals, fractions, and exponents.

Key words:

- PEMDAS
- operations
- exponent

Check it out:

If you see parentheses with an exponent, the exponent applies to the whole expression inside them. So $(1 + 2)^2$ is $(1 + 2) \cdot (1 + 2)$. Everything in the parentheses is the repeated factor in the multiplication. So to follow the PEMDAS rules you need to simplify the contents of the parentheses first, and then apply the exponent to the result. So $(1 + 2)^2 = 3^2$

In Chapter One you saw how the *order of operations rules* help you to figure out which *operation* you need to do *first* in a calculation. This Lesson will review what the order is, and give you practice at applying it to expressions with *exponents* in them.

PEMDAS Tells You What Order to Follow

When you come across an **expression** that contains **multiple operations**, the **PEMDAS rule** will help you to work out which one to do **first**. For example:

Parentheses $4 + 6 \cdot (2 + 4)^2 - 10 \div 2$

Exponents $= 4 + 6 \cdot (6)^2 - 10 \div 2$

Multiplication and Division $= 4 + 6 \cdot 36 - 10 \div 2$  **Multiplication and division have equal priority** in PEMDAS. You work them out from **left to right**.

Addition and Subtraction $= 4 + 216 - 5$  **Addition and subtraction have equal priority** too — work them out from **left to right**.
 $= 215$

Example 1

Evaluate the expression $5^2 - 16 \div 2^3 \cdot (3 + 2)$.

Solution

$$5^2 - 16 \div 2^3 \cdot (3 + 2)$$

$$= 5^2 - 16 \div 2^3 \cdot 5$$

$$= 25 - 16 \div 8 \cdot 5$$

$$= 25 - 2 \cdot 5$$

$$= 25 - 10$$

$$= 15$$

Do the addition in the parentheses

Then evaluate the two exponents

Next it's multiplication and division — do the division first, as it comes first, then do the multiplication

Finally do the subtraction

Guided Practice

Evaluate the expressions in Exercises 1–6.

1. $6 - 10 \cdot 3^2$

2. $(5 - 3)^3 + 4^3 \div 8$

3. $2^4 + (3 \cdot 2 - 10)^2$

4. $5 + 6^4 \div (6 - 2)^1$

5. $(36 \div 12 - 2^4)^2$

6. $(10 \cdot 2 - 5)^2 - (4 \div 2)^3 \cdot 3$

Check it out:

- When you multiply a negative number by a negative number, the result is positive.
- When you multiply a positive number and a negative number, the result is negative.
- So if you raise a negative number to an even power, the result will be positive.
- But if you raise a negative number to an odd power, the result will be negative.
- For example:
 $(-2)^3 = -2 \cdot -2 \cdot -2$
 $= 4 \cdot -2 = -8$
 $(-2)^4 = -2 \cdot -2 \cdot -2 \cdot -2$
 $= 4 \cdot 4 = 16$

Check it out:

If you have parentheses inside parentheses, for example, $(3 + (4 + 2))$, you should start with the innermost parentheses and work outward.

Take Care with Expressions That Have Negative Signs

When an expression contains a combination of negative numbers and exponents, you need to think carefully about what it means. For example:

$$-(2^2) = -(2 \cdot 2) = -4$$

$$(-2)^2 = -2 \cdot -2 = 4$$

Example 2

Evaluate the expression $(-3^2) \cdot 5 + (-3)^2$.

Solution

$$\begin{aligned} & (-3^2) \cdot 5 + (-3)^2 \\ &= (-9 \cdot 5) + (-3)^2 \\ &= -45 + (-3)^2 \\ &= -45 + 9 \\ &= -36 \end{aligned}$$

Evaluate the exponent in the inner parentheses
Do the multiplication in the parentheses
Evaluate the exponent
Finally do the addition

Guided Practice

Evaluate the expressions in Exercises 7–12.

7. $-(4^2)$

8. $(-4)^2$

9. $-(2^2) \cdot 5 + 1$

10. $(-4)^2 \div 2 - 4$

11. $10 + (2 \cdot -(5^2)) + (-7)^2$

12. $12 + (-2^2) + (-2)^2 \div 2$

The Order Applies to Decimals and Fractions Too

When you're working out a problem involving decimals or fractions you follow the same order of operations.

Example 3

Evaluate the expression $\left(\frac{1}{2}\right)^4 + \frac{1}{16} \cdot (10 - 7)^2$.

Solution

$$\begin{aligned} & \left(\frac{1}{2}\right)^4 + \frac{1}{16} \cdot (10 - 7)^2 \\ &= \left(\frac{1}{2}\right)^4 + \frac{1}{16} \cdot 3^2 && \text{Do the subtraction in the parentheses} \\ &= \frac{1}{16} + \frac{1}{16} \cdot 9 && \text{Then evaluate the two exponents} \\ &= \frac{1}{16} + \frac{9}{16} && \text{Perform the multiplication} \\ &= \frac{10}{16} = \frac{5}{8} && \text{Finally do the addition and simplify} \end{aligned}$$

Example 4

Evaluate the expression $0.25 + 7.75 \div 3.1 - (0.3)^4$.

Solution

$$\begin{aligned} & 0.25 + 7.75 \div 3.1 - (0.3)^4 \\ &= 0.25 + 7.75 \div 3.1 - 0.0081 \\ &= 0.25 + 2.5 - 0.0081 \\ &= 2.75 - 0.0081 \\ &= \mathbf{2.7419} \end{aligned}$$

Evaluate the exponent
Then perform the division
Do the addition first, as it comes first
Finally do the subtraction

Don't forget:

If your calculation involves a mixture of fractions and decimals, convert everything to either fractions or decimals first.

For a reminder of how to do this, see Section 2.1.

Guided Practice

Evaluate the expressions in Exercises 13–20.

13. $\left(\frac{1}{2} + \frac{3}{2}\right)^3 - \frac{1}{8}$

14. $\left(\frac{1}{4}\right)^2 \cdot 3 + 4 \div \frac{1}{2} - 2$

15. $0.1 + (0.25)^2 - 0.2 \div 2$

16. $(0.72 + 0.08) \div 16 + (0.4)^2$

17. $\left(\frac{3}{4} \div \frac{1}{2}\right)^2 + \left(\frac{2}{3} \cdot \frac{1}{2}\right)^3$

18. $0.5 \cdot (1 + 0.25)^2 + 1.2$

19. $2 \cdot \left(\frac{1}{2}\right)^2 + (5 \div 10)^2 \cdot 4$

20. $(5 \cdot 0.1 + 0.2) \cdot \left(\frac{1}{5}\right)^2$

Independent Practice

Evaluate the expressions in Exercises 1–6.

1. $\frac{12+2^3}{5}$

2. $(4^2 - 2^3) \div 2^2 + 8^1$

3. $(10 + 2^4 \cdot 3) + (5^2 - 15)^2$

4. $-3^3 \cdot 2^2 + 9$

5. $(-6)^3 \cdot 3 - 12^2$

6. $(4^3 - 3^4)^2 \div (17)^2$

7. In the expression $(x - y^2 \cdot z)^6$, x , y , and z stand for whole numbers. If you evaluate it, will the expression have a positive or a negative value? (The expression is not equal to zero.) Explain your answer.

Evaluate the expressions in Exercises 8–13.

8. $\left(\frac{1}{3}\right)^2 + 2 \cdot \frac{2}{27}$

9. $(0.5)^2 + 0.8 \div (0.1)^3$

10. $\left(\frac{2}{3} \div \frac{4}{5}\right)^2 + \left(\frac{1}{6}\right)^2 \cdot 4$

11. $(0.5 + 1.8)^2 \cdot 1.5 + 0.065$

12. $\left(0.5 \cdot \frac{6}{8}\right)^2 - \left(\frac{1}{4}\right)^3$

13. $(0.2 \cdot 4 - 0.3)^2 + \left(\frac{1}{2}\right)^3 \cdot 2$

14. Lakesha is making bread. She has $\frac{5}{4}$ lb of flour, which she splits into two equal piles. Then she splits each of these in half again. She adds three of the resulting piles to her mixture. How much flour has she added to her mixture? Give your answer as a fraction.

Now try these:

Lesson 2.5.4 additional questions — p441

Round Up

When you have an expression containing *exponents*, you must follow the *order of operations* to evaluate it. You use the same order with expressions that contain *fractions* and *decimals* too.

Section 2.6 introduction — an exploration into: The Side of a Square

A *perfect square* has sides whose lengths are whole numbers. You'll be given square tiles and be asked to construct larger squares with particular *areas* — you'll be able to produce some of the larger squares, but not others. The lengths of the sides of the squares are the *square roots* of the areas. You'll see that the areas of some squares have *whole number square roots*, but others don't.

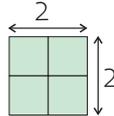
Each small square has an area of **1 square unit**.

Example

Make a square with an area of 4 square units. Then write down the square root of 4.

Solution

With 4 square tiles:



This is a **perfect square** — it's got an area of **4 square units** and sides of **2 units**.

So, **2 is the square root of 4**.

Exercises

- Use the tiles to make squares with the given areas.
When you have made a square, write the lengths of the sides.
 - 9 square units
 - 16 square units
 - 25 square units
 - 36 square units
- What are the square root of the following? Use your answers to Exercise 1 to help you.
 - 9
 - 16
 - 25
 - 36

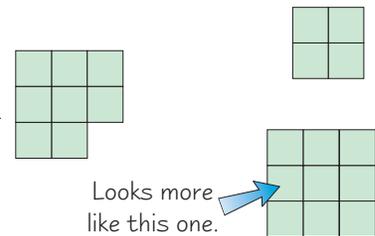
You can use the tiles to **estimate** the square root of a number that is **not** perfect square.

Example

Use tiles to estimate the square root of 8.

Solution

This is the **closest shape** you can make to a square using 8 tiles — It's bigger than a 2 by 2 square, but smaller than a 3 by 3 square. As it's closer to a 3 by 3 square, you can estimate that the square root of 8 is **about 3**.



Exercises

- Construct a figure that is as close to a square as possible. Use this to estimate the square roots of these numbers.
 - 5
 - 14
 - 22

Round Up

Some numbers are *perfect squares* — like 4, 9, 16, 25. These numbers have square roots that are *whole numbers*. If you make a square with a perfect square area, its sides will be whole numbers.

Lesson 2.6.1

Section 2.6

Perfect Squares and Their Roots

California Standards:

Number Sense 2.4

Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

What it means for you:

You'll see what a square root is and how to find the square root of a square number.

Key words:

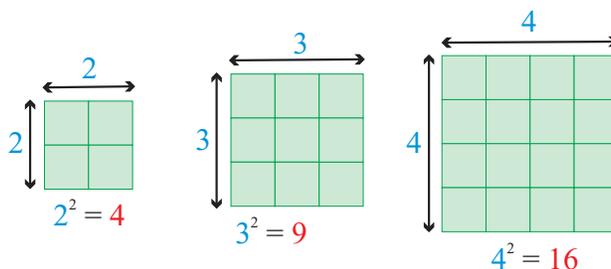
- perfect square
- square root
- positive root
- negative root

If you multiply the *side length* of a square by itself you get the area of the square. You can do the *opposite* too — find the *side length* of the square from the area. That's called finding the *square root*.

The Square of an Integer is a Perfect Square Number

Raising a number to the power two is called **squaring** it. That's because you find the area of a square by multiplying the side length by itself.

So, the **area of a square** = $s \cdot s = s^2$, where s is the side length.



All the numbers in **red** are the squares of the numbers in **blue**.

The square of an **integer** is called a **perfect square**.

Perfect squares are always integers too.

$1 \cdot 1 = 1$] — These numbers are perfect squares
$2 \cdot 2 = 4$	
$4 \cdot 4 = 16$	
$3.5 \cdot 3.5 = 12.25$] — These numbers aren't perfect squares
$5.1 \cdot 5.1 = 26.01$	

Example 1

Is the number 81 a perfect square?

Solution

$$9 \cdot 9 = 81$$

As 9 is an integer, **81 is a perfect square**.

Guided Practice

Give the square of each of the numbers in Exercises 1–6.

1. 4

2. 7

3. 12

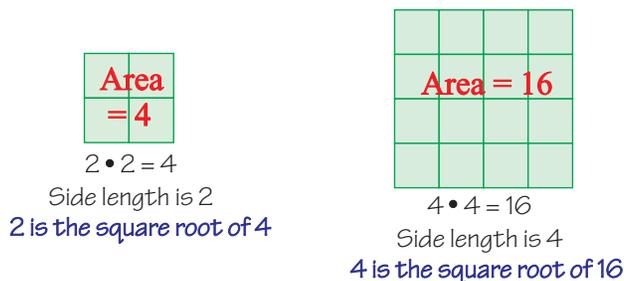
4. 1

5. -2

6. -12

The Opposite of Squaring is Finding the Square Root

You might **know the area** of a square and **want to know the side length**. You find the side length of a square by finding the **square root**.



So, to find the square root of a square number, you have to **find the number that multiplied by itself gives the square number**.

Check it out:

Your calculator should have a square root button on it that looks like this: $\sqrt{\square}$.

To find the square root of 2 on your calculator press:

$$\sqrt{\square} \quad 2 \quad =$$

On some types of calculators you have to press: $2 \quad \sqrt{\square}$

Check it out:

You can't find the square root of a negative number. To square a number you have to multiply it by itself.

- If you multiply a positive number by a positive number the result is always positive.
- If you multiply a negative number by a negative number the result is also positive.

So if you multiply any number by itself the answer is always positive.

Check it out:

These numbers are all perfect square numbers. So their square roots will all be whole numbers.

For example: $5 \cdot 5 = 25$. 25 is a square number. 5 is a **square root** of 25.

The symbol $\sqrt{\quad}$ is used to show a square root.

So you can say that $\sqrt{25} = 5$.

Unless the number you're finding the square root of is a perfect square, the square root will be a **decimal** — and may well be **irrational**. (There's more on this in Lesson 2.6.2.)

All Positive Numbers Have Two Square Roots

Every positive number has **one positive**, and **one negative**, square root. This is because $4 \cdot 4 = 16$ and $-4 \cdot -4 = 16$ — so the square root of 16 could be **either** 4 or -4 .

A square root symbol, $\sqrt{\quad}$, by itself means just “**the positive square root**.”

$$\sqrt{16} = 4 \text{ — this is the positive square root of 16.}$$

$$-\sqrt{16} = -4 \text{ — this is the negative square root of 16.}$$

✓ Guided Practice

Evaluate the square roots in Exercises 7–14.

7. $\sqrt{36}$

8. $-\sqrt{64}$

9. $\sqrt{100}$

10. $-\sqrt{144}$

11. $\sqrt{121}$

12. $-\sqrt{169}$

13. $\sqrt{1}$

14. $\sqrt{400}$

Writing Square Roots with Fractional Exponents

Check it out:

If you multiply a square root by itself you get back to the original number.

$$\sqrt{x} \cdot \sqrt{x} = x.$$

Don't forget:

$-100^{\frac{1}{2}}$ means $-(100^{\frac{1}{2}})$. It can't be $(-100)^{\frac{1}{2}}$ because that would be the square root of a negative number.

Now try these:

Lesson 2.6.1 additional questions — p441

In Section 2.4 you saw that an **exponent** means a **repeated multiplication**. When you **square** a number it is repeated as a factor **two times** in the multiplication expression. So you can write $x \cdot x = x^2$.

Taking the **square root** of a number is the **reverse process** to squaring it. Because it **undoes** squaring we can also write \sqrt{x} as $x^{\frac{1}{2}}$.

So, $\sqrt{9} = 9^{\frac{1}{2}}$ and $\sqrt{16} = 16^{\frac{1}{2}}$.

✓ Guided Practice

Evaluate the expressions in Exercises 15–18.

15. $4^{\frac{1}{2}}$

16. $-25^{\frac{1}{2}}$

17. $49^{\frac{1}{2}}$

18. $-100^{\frac{1}{2}}$

✓ Independent Practice

Give the square of each of the numbers in Exercises 1–6.

1. 6

2. 11

3. 16

4. -10

5. -13

6. -15

7. Marissa is making patterns with small square mosaic tiles. She has 50 tiles. Can she arrange them to make one larger square, using all the tiles? Explain your answer.

8. This year's senior class will have 225 students graduating. The faculty wants the chairs to be arranged in the form of a square. How many chairs should be put in each row?

Evaluate the square roots in Exercises 9–16.

9. $\sqrt{25}$

10. $-\sqrt{25}$

11. $\sqrt{64}$

12. $-\sqrt{49}$

13. $\sqrt{9}$

14. $\sqrt{289}$

15. $361^{\frac{1}{2}}$

16. $-49^{\frac{1}{2}}$

17. Give the square roots of 16.

18. Give the square roots of 81.

19. A square deck has an area of 81 feet². Paul is planning to enlarge the deck by increasing the length of each side by 2 feet. How much will the area of the deck increase by?

Round Up

If you multiply any integer by itself you will get a **perfect square number**. The factor that you multiply by itself to get a square number is called its **square root**. Every square number has one **positive** and one **negative** square root. And don't forget — **negative numbers don't have square roots**.

Lesson 2.6.2

Irrational Numbers

California Standards:

Number Sense 1.4

Differentiate between rational and irrational numbers.

What it means for you:

You'll learn what irrational numbers are, and how they're different from rational numbers.

Key words:

- rational
- irrational
- integer
- terminating decimal
- repeating decimal

Check it out:

You could never write an irrational number out in full because it would go on forever.

Check it out:

In Chapter 3, you'll use π to calculate the circumference of a circle. You can never calculate the circumference exactly, because π goes on forever.

If you find the *square root of 2* on your calculator, you get a number that fills the display, and none of the digits repeat. In this Lesson you'll learn what makes numbers like that special.

Rational Numbers Can Be Written as Fractions

In Section 2.1 you saw that any number that can be written in the form $\frac{a}{b}$ where a and b are both integers, and b isn't 0, is called a **rational number**.

For example:

$$\begin{array}{l} 2 \xrightarrow{\text{can be written as}} \frac{2}{1} \\ 3.7 \xrightarrow{\text{can be written as}} \frac{37}{10} \\ 0.\overline{81} \xrightarrow{\text{can be written as}} \frac{9}{11} \end{array}$$

You can write all of these as fractions as described above, so they are all rational numbers.

All **fractions**, **integers**, **terminating decimals**, and **repeating decimals** are **rational numbers**. You can add **square roots of perfect squares** to that list too, because they are always **integers**.

✓ Guided Practice

Prove that the numbers in Exercises 1–4 are rational by writing each one as a fraction in its simplest form.

1. 6

2. 0.8

3. $0.\overline{3}$

4. $\sqrt{16}$

Irrational Numbers Can't Be Written as Fractions

- Any number that **can't** be written as a ratio of two integers is called an **irrational number**.
- Irrational numbers are **nonterminating, nonrepeating decimals**.

0.123456789101112131415161718192021...

5.121121112111121111121111121111112...

Neither of these decimals terminate or have repeating patterns of digits. They're both irrational numbers.

The most famous irrational number is π — you can't write π as a fraction.

π starts **3.1415926535897932384626433832795...**

The value of π has been calculated to over a million decimal places so far — it **never ends** and **never repeats**.

The square roots of **perfect squares** are **integers** — so they're **rational**.

The square root of **any integer** other than a perfect square is **irrational**.

$$\sqrt{1} = 1, \sqrt{2}, \sqrt{3}, \sqrt{4} = 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9} = 3, \sqrt{10} \dots$$

These numbers are rational
These numbers are irrational

✓ Guided Practice

Classify the numbers in Exercises 5–10 as rational or irrational.

5. $\frac{7}{9}$

6. π

7. 5

8. $\sqrt{100}$

9. $1.25\overline{43}$

10. $\sqrt{14}$

Check it out:

The “repeat period” of any decimal is the length of the bit of it that repeats. So the repeat period of $0.\overline{3}$ is one digit — the digit 3, the repeat period of $2.\overline{945}$ is three digits — the group of digits 945, and the repeat period of $5.27\overline{316}$ is two digits — the pair of digits, 16.

Check it out:

You know that $1 \div 7$ must be a rational number because it can be expressed as $\frac{1}{7}$.

Some Decimals Have a Long Repeat Period

Sometimes it might not be obvious straightaway whether a number is **rational** or **irrational**. Some decimals that have a **large repeat period** may look as if they are irrational but are actually rational.

For example, when you **divide 1 by 7** on your calculator you get a decimal number. From the number on your calculator display, you can't tell if that decimal ever **ends or repeats**.

$$1 \div 7 = 0.1428571\dots$$

To show that the decimal does in fact **repeat**, work out $1 \div 7$ using **long division**:

$$\begin{array}{r}
 0.14285714\dots \\
 7 \overline{) 1.00000000\dots} \\
 \underline{7} \\
 30 \\
 \underline{28} \\
 20 \\
 \underline{14} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{35} \\
 50 \\
 \underline{49} \\
 10 \\
 \underline{07} \\
 30 \\
 \underline{28}
 \end{array}$$

The repeat period starts here. So you can write this decimal as $0.\overline{142857}$

now the same cycle begins again — this is a repeating decimal →

You can see that the same long division cycle **begins again**. This means the decimal does **repeat**.

It's really hard to prove a number's **irrational** — because it could just be a decimal with a **really really long** repeat pattern.

The only way you can know that a number is irrational is if it has a pattern that you know will never repeat — there's an endless set of irrational numbers like this to choose from:

→ They might have a pattern of digits that you could generate with a formula. $0.\overline{1248163264128256512\dots}$

This pattern of digits represents the powers of 2: $2^0, 2^1, 2^2, 2^3, 2^4$, etc. It goes on forever without repeating.

→ They could have any pattern of digits where each time the pattern is repeated the number of copies of each digit increases.

$0.\overline{12112211122211112222\dots}$

In this pattern the numbers 1 and 2 alternate, but each time they are repeated the number of copies of the digits increases by 1.

Check it out:

A bar above some digits in a decimal number shows that these digits are repeated over and over again.

✓ Guided Practice

Classify the numbers in Exercises 11–15 as rational or irrational.

11. $0.369121518\dots$ where the pattern of digits are the multiples of 3
12. $3.1224\overline{68}$
13. 0.2647931834
14. $0.141144111444\overline{14}$
15. $0.123112233111222\dots$

✓ Independent Practice

Prove that the numbers in Exercises 1–4 are rational by expressing them as fractions in their simplest form.

1. 14
2. $\sqrt{121}$
3. 2.6
4. $1.\overline{6}$
5. Read statements a) and b). Only one of them is true. Which one? How do you know?
 - a) All fractions can be written as decimals.
 - b) All decimals can be written as fractions.

Classify the numbers in Exercises 6–13 as rational or irrational.

6. 10
7. $\frac{14}{5}$
8. 0.497623
9. 3π
10. $9.12958725364\overline{8}$
11. $\sqrt{27}$
12. $\sqrt{225}$
13. $22.343344333444\dots$
14. Write any four irrational numbers between zero and five.

Now try these:

Lesson 2.6.2 additional questions — p442

Round Up

*Irrational numbers can't be written as fractions where the numerators and denominators are both integers. Irrational numbers are always **nonterminating** and **nonrepeating** decimals.*

Lesson 2.6.3

Estimating Irrational Roots

California Standard:

Number Sense 2.4

Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; **for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.**

Mathematical Reasoning 2.7

Indicate the relative advantages of exact and approximate solutions to problems and **give answers to a specified degree of accuracy.**

What it means for you:

You'll learn how to find the approximate square root of any number without using a calculator.

Key words:

- irrational
- perfect square
- square root

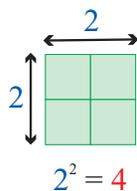
Don't forget:

Rounding a number makes it shorter and easier to work with. How much you round a number depends on how accurate you need to be. For example, you might round 99.26 up to 100, down to 99, or up to 99.3 — depending on how accurate you want to be. There are rules for rounding — they're explained in Chapter 8.

You saw in the last Lesson that all square roots of integers that aren't perfect squares are *irrational numbers*. That means that you could never write their exact decimal values, because the numbers would go on *forever*. But you can use an *approximate* value instead.

Square Roots of Nonperfect Squares are Irrational

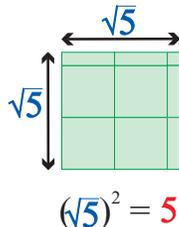
Perfect square numbers have square roots that are **integers**.



The area of this square is 4 units.
So its side length must be $\sqrt{4}$ units = 2 units — which is **rational**.

Numbers that are **not perfect squares** still have square roots.

Square roots of integers that are **not perfect squares** are always **irrational numbers**.



The area of this square is 5 units.
So its side length must be $\sqrt{5}$ units — which is **irrational**.

✓ Guided Practice

Say whether each number in Exercises 1–6 is rational or irrational.

1. $\sqrt{9}$

2. $\sqrt{2}$

3. $\sqrt{12}$

4. $\sqrt{16}$

5. $\sqrt{169}$

6. $\sqrt{140}$

You Can Approximate Irrational Square Roots

If you are asked to give a **decimal** value for $\sqrt{2}$, or for any other irrational number, you would have to give an **approximation**. You could never give an exact answer because the exact answer goes on forever.

Example 1

Find the approximate value of $\sqrt{2}$ using a calculator.

Solution

You should have a button on your calculator that has the square root symbol on it. It will look like this: $\sqrt{\square}$.

To find the square root of 2, press the square root button, then the number 2, and then the equals button, like this: $\sqrt{\square}$ \square 2 \square =.

You will get an answer on the screen that looks something like this:



Even though the answer on your screen stops, it's not the exact answer. It's just an approximation based on how many digits can fit on the screen.

So you should write your answer like this:

$\sqrt{2} = 1.414213562$ (to 9 decimal places) State the accuracy you've given the approximate answer to.

Or like this: $\sqrt{2} \approx 1.41$

Then you've shown that you know it's an approximate answer.

Check it out:

Some calculators need you to press \square \square

Check it out:

The symbol " \approx " means "is approximately equal to." You can use it in situations where you can't write the exact value of a number because it's irrational.

Check it out:

When you round a decimal answer you will usually be told how many decimal places to round it to. That's just how many digits there are after the decimal point.

If you're rounding a number to six decimal places, you have to look at the digit in the seventh decimal place. If it's 5 or more, then round the sixth digit up. If it's less than 5, then round the sixth digit down.

For example,

$\sqrt{28} = 5.291502622\dots$

seventh digit

The seventh digit is more than 5, so the sixth digit is rounded up to 3.

So, $\sqrt{28} = 5.291503$ to 6 d.p.

There's more on how and when to round numbers in Section 8.3.

Guided Practice

Use your calculator to approximate the square roots in Exercises 7–12. Give the values to six decimal places.

7. $\sqrt{5}$

8. $\sqrt{6}$

9. $\sqrt{10}$

10. $\sqrt{29}$

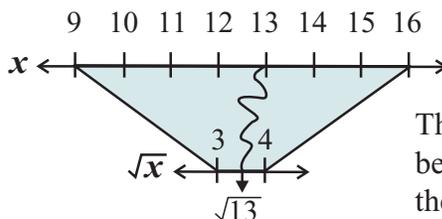
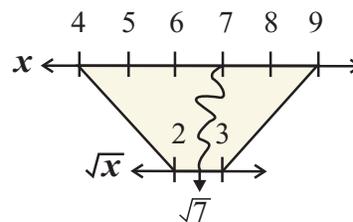
11. $\sqrt{47}$

12. $\sqrt{160}$

Estimate Square Roots Using a Number Line

Estimating the square root of a number without using a calculator involves working out which two **perfect square numbers** it lies between.

For example, the square roots of all the numbers between **4 and 9** lie between **2 and 3** on the number line.

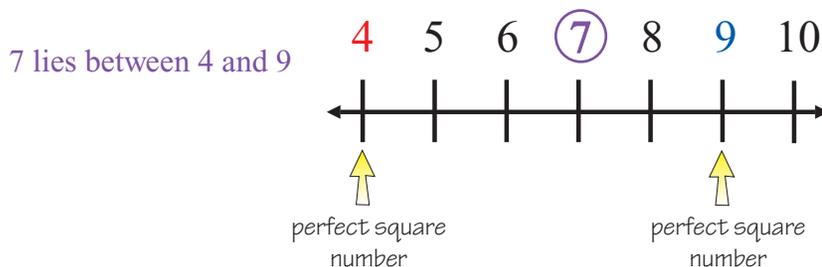


The square roots of all the numbers between **9 and 16** lie between **3 and 4** on the number line.

There are two steps to finding an approximation of the square root of a number.

For example: find the two numbers that $\sqrt{7}$ lies between on the number line.

- 1) First find the two **perfect square numbers** that 7 lies **between** on the number line.



- 2) Find the **square roots of these two perfect square numbers**. The square root of 7 must be **between** these two square roots.



So $\sqrt{7}$ must lie between **2 and 3**.

Example 2

Find the two numbers that $\sqrt{14}$ lies between on the number line.

Solution

First find the two **perfect squares** that 14 lies between on the number line. 14 lies between **9 and 16**.

$$\sqrt{9} = 3 \text{ and } \sqrt{16} = 4.$$

So $\sqrt{14}$ lies between **3 and 4** on the number line.

Example 3

Find the numbers that $\sqrt{18}$ lies between on the number line.

Solution

First find the two perfect squares that 18 lies between on the number line. 18 lies between 16 and 25.

$$\sqrt{16} = 4 \text{ and } \sqrt{25} = 5.$$

So $\sqrt{18}$ lies between **4 and 5** on the number line.

✓ Guided Practice

In Exercises 13–20 find the whole numbers that the root lies between.

13. $\sqrt{5}$

14. $\sqrt{15}$

15. $\sqrt{24}$

16. $\sqrt{46}$

17. $\sqrt{68}$

18. $\sqrt{98}$

19. $\sqrt{125}$

20. $\sqrt{150}$

✓ Independent Practice

Use your calculator to approximate the square roots in Exercises 1–4 to four decimal places.

1. $\sqrt{17}$

2. $\sqrt{28}$

3. $\sqrt{73}$

4. $\sqrt{155}$

In Exercises 5–10 say which two perfect square numbers the number lies between.

5. 3

6. 29

7. 50

8. 95

9. 125

10. 200

In Exercises 11–18 find the whole numbers that the root lies between.

11. $\sqrt{3}$

12. $\sqrt{13}$

13. $\sqrt{22}$

14. $\sqrt{33}$

15. $\sqrt{58}$

16. $\sqrt{93}$

17. $\sqrt{160}$

18. $\sqrt{216}$

19. A square has an area of 85 inches². What whole-inch measurements does the side length lie between?

20. If $\sqrt{a} \approx 2.4$ then which two perfect squares does a lie between?

21. Latoya has a new office. It is a square room, with a floor area of 230 feet². She wants to fit a 15 ft desk area along one wall — will this fit along one of the sides? Explain your reasoning.

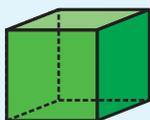
22. A math class is shown a cube made of card. Pupils are told that the total surface area of the cube is 90 cm². They are asked to guess the length of each side of the cube in centimeters. Peter guesses 10 centimeters, and John guesses 4 centimeters. Whose guess is the closer?

Now try these:

Lesson 2.6.3 additional questions — p442

Don't forget:

A cube has 6 faces that are all identical squares.



Round Up

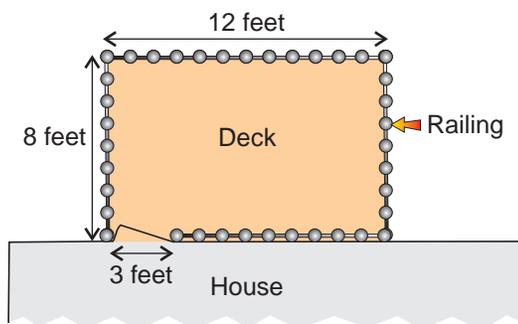
You can never write out the *exact* value of the square root of a nonperfect square number — but you can use an *approximation*. To figure out which two integers a number's square root lies between, it's just a case of knowing which two *perfect squares* the number *lies between*, and finding their square roots.

Chapter 2 Investigation

Designing a Deck

You have to add, subtract, multiply, and divide numbers to solve lots of *real-life problems*. Being able to use *powers* and find *square roots* can sometimes come in useful too.

The Dedona family has a deck on the back of their house that they want to **make bigger**. The current deck is a **rectangle** with a length of 12 feet and width of 8 feet. They want the new deck to be **225 square feet** in area.



Part 1:

What would the dimensions of the enlarged deck be if it were in the shape of a square?

Part 2:

Make four different designs for additions to the deck that satisfy the Dedonas' area requirement.

Things to think about:

- The new deck **must** contain the original, rectangular deck. However, the new, enlarged deck **doesn't** have to be rectangular itself.

Extensions

The cost of railing is \$18.95 for a 3-foot section. Railing is only sold in 3-foot sections.

- 1) What design for the new deck provides the required area with the least amount of railing?
- 2) How many sections of railing do they need to buy for the new deck?
What is the total cost for the railing?

Open-ended Extensions

- 1) Propose a design that would satisfy the Dedonas' area requirements and cost the most money for railings. The narrowest any section of the deck can ever be is 1 foot.
How would your solution change if the existing deck was torn down?
How would it change if the 3-foot railing sections could not be cut?
- 2) Would a square design be the least expensive if there was no railing against the house?

Round Up

When you're solving real-life problems, you often have to *combine* all the operations. You use *multiplication* to find areas and *addition* to find the *total length* around the edge. And if you have a square with a set area, you can find its *side length* by finding the square root of the area.

Chapter 3

Two-Dimensional Figures

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Section 3.1 introduction — an exploration into: Area and Perimeter Patterns

You can draw shapes which have the *same area*, but *different perimeters*.
In this Exploration, you'll look at how to *maximize the perimeter* for a given area.
You'll also look at shapes that have the *same perimeters*, but *different areas*.

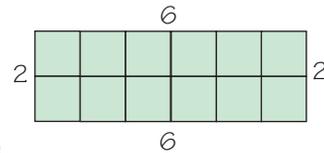
You can find the **area** of a shape by counting up the number of **unit squares**.
The **perimeter** is calculated by finding the **sum of all the side lengths**.

Example

Find the area and the perimeter of this shape.

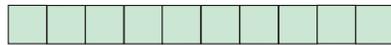
Solution

Area = **12 square units** Perimeter = $6 + 2 + 6 + 2 =$ **16 units**



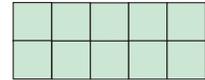
When given a **set area**, you can draw shapes with **different perimeters** — like these:

These shapes both have an area of 10 square units.



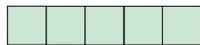
This one has a perimeter of 22 units...

...but this one has a perimeter of 14 units.



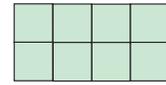
And when given a **set perimeter**, you can draw shapes with **different areas** — like these:

These shapes both have a perimeter of 12 units.



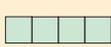
This one has an area of 5 square units...

...but this one has an area of 8 square units.



Exercises

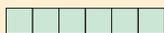
1. Find the area and perimeter of each shape.



A



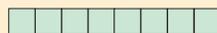
B



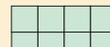
C



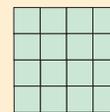
D



E



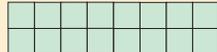
F



G



H



I

2. Look at the areas and perimeters of the following sets of shapes.

What do you notice about them? Which type of shape maximizes the perimeter in each set?

A and B C and D E and F G, H and I

3. Draw three different rectangles with areas of 12 square units that all have different perimeters. What are the dimensions of the rectangle that has the largest perimeter?

Use only whole-number dimensions for Exercises 3–6.

4. Draw the rectangle with an area of 20 square units that has the largest perimeter possible.

5. Draw two rectangles that have different areas but both have perimeters of 14 units.

6. Draw a rectangle that has a perimeter of 20 units and has the largest area possible.

Round Up

A rectangle with a *big difference* between its length and width measurement will have a *large perimeter* for its area. It works the other way for *maximizing the area* of a rectangle with a *fixed perimeter* — the closer the shape is to a *square*, the *bigger* the area will be.

Lesson 3.1.1

Section 3.1 Polygons and Perimeter

California Standards:

Measurement and Geometry 2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures, and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

What it means for you:

You'll learn about the names of different shapes and use formulas for finding the perimeters of some shapes.

Key words:

- polygon
- perimeter
- regular polygon
- irregular polygon
- quadrilateral
- parallelogram
- trapezoid

Don't forget:

Dashes are used to show that certain lengths are equal. You might see single and double dashes. Sides with double dashes are the same length as each other, but not the same as those with single dashes.

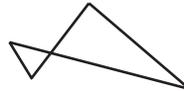
You're probably pretty familiar with a lot of shapes — this Lesson gives you a chance to brush up on their names, and shows you how you can use formulas to find the distance around the outside of some shapes.

Polygons Have Straight Sides

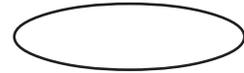
Polygons are flat shapes. They're made from straight line segments that never cross. The line segments are joined end to end.



This **is** a polygon



This **isn't** a polygon (the lines cross)



This **isn't** a polygon (it's not made from straight lines)

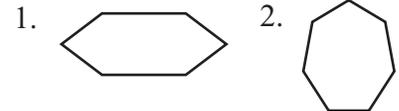
The name of a polygon depends on how many sides it has.

3 sides	Triangle
4 sides	Quadrilateral
5 sides	Pentagon
6 sides	Hexagon

7 sides	Heptagon
8 sides	Octagon
9 sides	Nonagon
10 sides	Decagon

Example 1

Identify each of the following shapes.



Solution

1. This shape has **6 sides**, so it's a **hexagon**.
2. This shape has **7 sides**, so it's a **heptagon**.

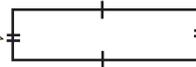
A Quadrilateral is a Polygon with Four Sides

A **quadrilateral** is any shape that has **four sides**.

You need to be able to name a few of them.

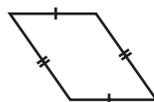
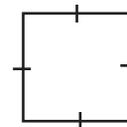
Rectangle

A rectangle has opposite sides of equal length. It also has four right angles.



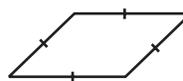
Square

This is a special kind of rectangle. It has four sides all of equal length.



Parallelogram

Its opposite sides are parallel.



Rhombus

This is a parallelogram with all sides of the same length.



Trapezoid

This has only one pair of parallel sides.

✓ Guided Practice

Identify each of the following polygons:

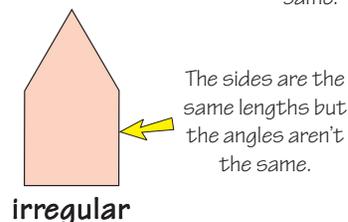
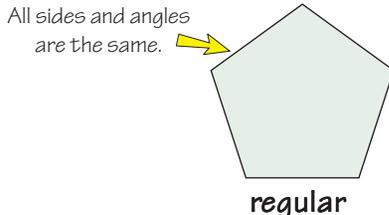
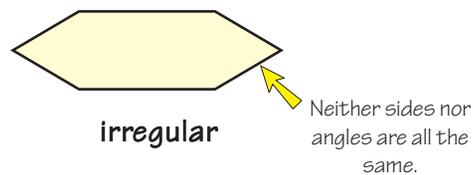
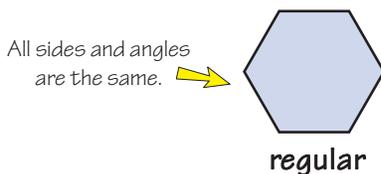


Check it out:

A rhombus has all sides of equal length, but the angles aren't all the same. So a rhombus is **always irregular**. A square, however, has all sides and all angles equal, so a square is **always regular**.

Regular Polygons Have Equal Sides and Angles

Regular polygons have **equal angles**, and sides of **equal length**. **Irregular polygons don't** have all sides and angles equal.



Example 2

Decide whether this polygon is regular or irregular.

Solution

The shape has **all angles equal**.



But the lengths of the sides are **not** the same, so it is an **irregular polygon**.

Check it out:

Polygons occur a lot in everyday life — for example:



The Pentagon



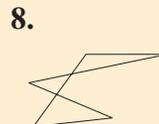
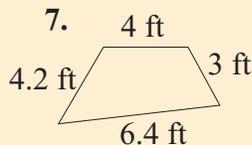
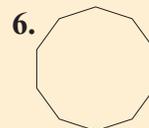
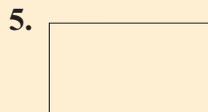
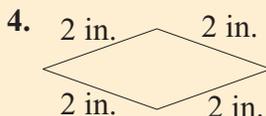
octagon



triangle

✓ Guided Practice

Decide whether each of the following shapes is a regular polygon, an irregular polygon, or not a polygon at all.



Perimeter is the Distance Around a Polygon

The **perimeter** is the **distance around the edge** of a shape.

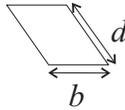
You can find the perimeter by **adding** up the lengths of the sides of a polygon, but some **polygons** have a **formula** you can use to find the perimeter more quickly.

Don't forget:

The “*d*” in the formula for the perimeter of a parallelogram stands for the **length of the diagonal**. Don't get this confused with the vertical height, which you'll use when you work out the area in the next Lesson.

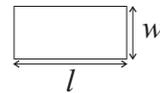
Parallelogram:

$$P = 2(b + d)$$



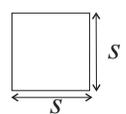
Rectangle:

$$P = 2(l + w)$$



Square:

$$P = 4s$$



Example 3

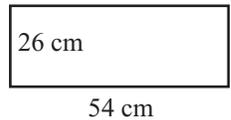
Find the perimeter of a rectangle of length 54 cm and width 26 cm.

Solution

Draw a diagram, and use the formula $P = 2(l + w)$.

Substitute the values for *l* and *w*, and **evaluate**.

$$P = 2 \times (54 \text{ cm} + 26 \text{ cm}) = 2 \times 80 \text{ cm} = \mathbf{160 \text{ cm}}$$



Don't forget:

Perimeter is a distance and needs a unit. Check what unit the question contains and make sure your answer has the correct unit.

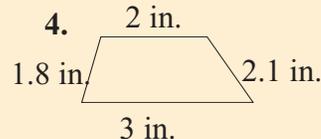
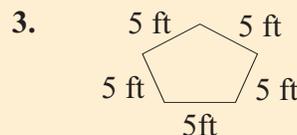
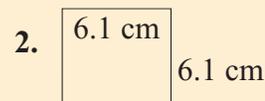
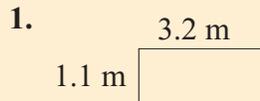
Guided Practice

9. Find the length of the diagonal of a parallelogram that has a base of 4.6 in. and a perimeter of 14 in.

10. Find the perimeter of a square of side 8.3 m.

Independent Practice

Find the perimeter of the figures in Exercises 1–4.



5. Brandy wants to know how many pieces of wood she needs to mark out the boundary of her new house. How many pieces of wood will she need if each piece of wood is 50 in. long and her boundary is a square of side 650 in.?

Now try these:

Lesson 3.1.1 additional questions — p443

Round Up

There are a few **formulas** here that make it **much quicker** to do perimeter calculations. If you can't apply one of the formulas, remember that you can always just **add up the lengths of the sides**.

Lesson 3.1.2

Areas of Polygons

California Standards:

Measurement and
Geometry 2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures, and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

What it means for you:

You'll use formulas to find the areas of regular shapes.

Key words:

- area
- triangle
- parallelogram
- trapezoid
- formula
- substitution

Check it out:

The area of a parallelogram is exactly the same as the area of a rectangle of the same base length and vertical height.

Check it out:

Remember — the height you use to calculate the area of a parallelogram is the vertical height and not the length of the side.

Don't forget:

Use dimensional analysis to make sure your answer has the correct units. You're multiplying a length by a length — for example, meters \times meters. So areas should always be a square unit — such as square meters (m^2).

Area is the *amount of space inside a shape*. Like for perimeter, there are *formulas* for working out the areas of some polygons. You'll practice using some of them in this Lesson.

Area is the Amount of Space Inside a Shape

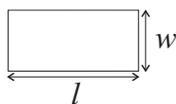
Area is the amount of **surface covered by a shape**.

Parallelograms, rectangles, and squares all have useful formulas for finding their **areas**.

Triangles and other shapes can be a little more difficult, but there are formulas for those too — which we'll come to next.

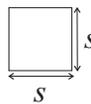
Rectangle:

$$A = lw$$



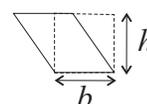
Square:

$$A = s^2$$



Parallelogram:

$$A = bh$$



Example 1

Use a formula to evaluate the area of this shape.

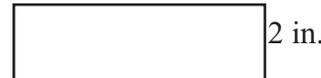
7 in.

Solution

Use the formula for the area of a rectangle.

Substitute in the values given in the question to evaluate the area.

$$A = lw = 7 \text{ in.} \times 2 \text{ in.} = 14 \text{ in}^2$$



You can also **rearrange** the formulas to find a **missing length**:

Example 2

Find the height of a parallelogram of area 42 cm^2 and base length 7 cm.

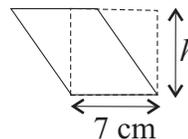
Solution

Rearrange the formula for the area of a parallelogram, and substitute.

$$A = bh$$

$$\frac{A}{b} = \frac{b}{b}h = h$$

$$h = \frac{42}{7} = 6 \text{ cm}$$

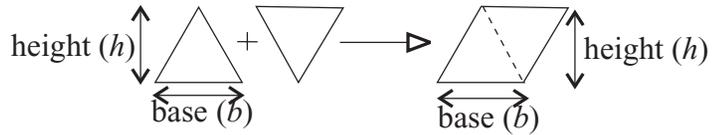


Guided Practice

1. Find the area of a square of side 2.4 m.
2. Find the length of a rectangle if it has area 30 in^2 , and width 5 in.

The Area of a Triangle is Half that of a Parallelogram

The area of a **triangle** is **half** the area of a **parallelogram** that has the **same base length** and **vertical height**.



$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \cdot \text{area of parallelogram} \\ &= \frac{1}{2} (\text{base} \times \text{height}) \\ &= \frac{1}{2} bh \end{aligned}$$

$$A = \frac{1}{2}bh$$

Don't forget:

The height you use for working out the area of a triangle is the **vertical height** and **not** the length of a side. This vertical height is often called the **altitude**.

Don't forget:

Multiplying by $\frac{1}{2}$ is the same as dividing by 2.

Example 3

Find the base length of the triangle opposite if it has an area of 20 in^2 and a height of 8 in.

Solution

Rearrange the formula for the area to give an expression for the base length of the triangle.

$$A = \frac{1}{2}bh$$

$$2A = bh$$

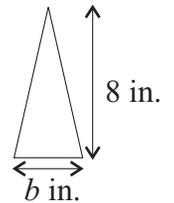
$$\frac{2A}{h} = \frac{h}{h}b = b. \text{ So } b = \frac{2A}{h}$$

Multiply both sides by 2

Divide both sides by the height (h)

Now **substitute** in the values and **evaluate** to give the base length.

$$b = (2 \times 20) \div 8 = \mathbf{5 \text{ in.}}$$

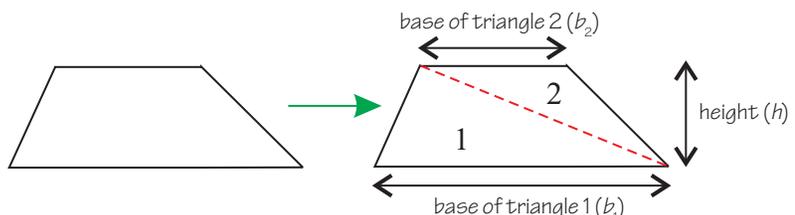


Guided Practice

- Find the area of a triangle of base length 3 ft and height 4.5 ft.
- Find the base length of a triangle with height 50 m and area 400 m^2 .

Break a Trapezoid into Parts to Find its Area

The most straightforward way to find the area of a trapezoid is to **split it up** into **two triangles**. You then have to work out the **area of both triangles** and **add** them together to find the **total area**.



Notice that both triangles have the **same height** but **different bases**.

Check it out:

Splitting the trapezoid into two triangles isn't the only way — it's just the easiest. You will get the same answer if you split it into two triangles and a rectangle, for example.

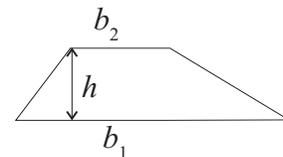
So, the area of the trapezoid is the **sum of the areas of each triangle**.

Area of trapezoid = area of Triangle 1 + area of Triangle 2

$$\text{Area of trapezoid} = \frac{1}{2}b_1h + \frac{1}{2}b_2h$$

Take out the **common factor** of $\frac{1}{2}h$ to give:

$$\text{Area of trapezoid} = \frac{1}{2}h(b_1 + b_2)$$



$$A = \frac{1}{2}h(b_1 + b_2)$$

Check it out:

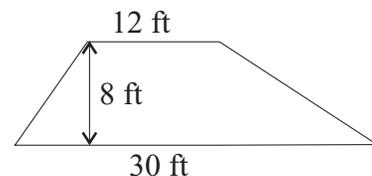
Taking out the common factor uses the distributive property that you learned about in Chapter 1 (see p8).

Example 4

Find the area of the trapezoid shown.

Solution

$$\text{Area of trapezoid} = \frac{1}{2}h(b_1 + b_2)$$



Substitute in the values given in the question and **evaluate**.

$$\text{Area of trapezoid} = \frac{1}{2} \times 8 \text{ ft} \times (12 \text{ ft} + 30 \text{ ft}) = \frac{1}{2} \times 8 \text{ ft} \times 42 \text{ ft} = \mathbf{168 \text{ ft}^2}.$$

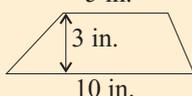
Don't forget:

It's always best to draw a diagram before attempting to solve an area question. It's easy to make mistakes otherwise.

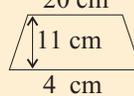
Guided Practice

Find the areas of the trapezoids in Exercises 5–8, using the formula.

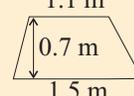
5. 5 in.



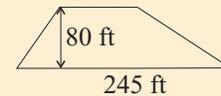
6. 20 cm



7. 1.1 m

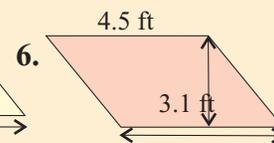
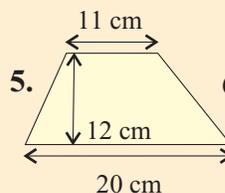
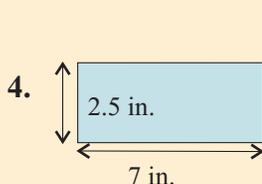
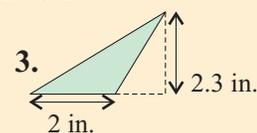
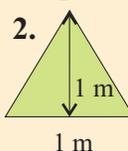
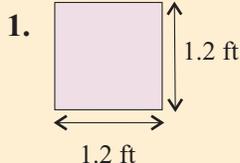


8. 105 ft

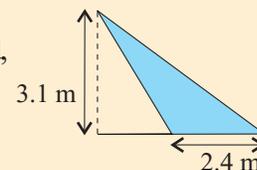


Independent Practice

Find the area of each of the shapes in Exercises 1–6.



7. Miguel wants to know the area of his flower bed, shown opposite. Find the area using the correct formula.



Now try these:

Lesson 3.1.2 additional questions — p443

Round Up

Later you'll use these formulas to find the areas of **irregular shapes**. Make sure you practice all this stuff so that you're on track for the next few Lessons.

Lesson 3.1.3

Circles

California Standards:

Measurement and Geometry 2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures, and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

What it means for you:

You'll find the circumference and area of circles using formulas.

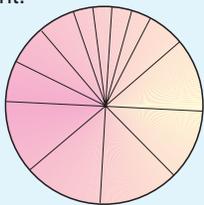
Key words:

- pi (π)
- radius
- diameter
- irrational number
- circumference

Check it out:

Circles are not polygons — they don't have any straight sides.

A circle is formed from the set of all points that are an equal distance from a given center point.



Check it out:

There's a special π button on your calculator that will allow you to do very precise calculations. Otherwise, use the approximate values of 3.14 or $\frac{22}{7}$ in your calculations.

You've already met the special irrational number π or "pi". Now you're going to use it to find the circumference and area of circles.

Circles Have a Radius and a Diameter

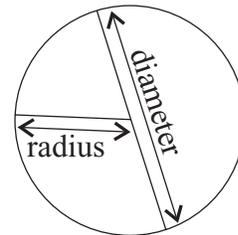
The distance of any point on a circle from the center is called the **radius**.

The distance from one side of the circle to the other, through the center point, is called the **diameter**.

Notice the **diameter is always twice the radius**.

$$\text{diameter} = 2 \cdot \text{radius}$$

$$d = 2r$$



Example 1

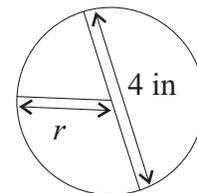
If a circle has a diameter of 4 in., what is its radius?

Solution

Use the formula: $d = 2r$

Rearrange to give r in terms of d , so $r = \frac{d}{2}$

Substitute d from question: $r = 4 \div 2 = 2$ in.



Guided Practice

1. If a circle has a radius of 2 in., what is its diameter?
2. A circle has a diameter of 12 m. What is its radius?

Circumference is the Perimeter of a Circle

The **circumference** is the **distance around the edge** of a circle.

This is similar to the perimeter of a polygon.

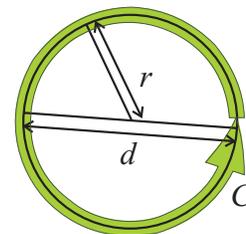
There's a **formula** to find the circumference.

$$\text{Circumference} = \pi \cdot \text{diameter}$$

$$C = \pi d$$

Because the diameter = $2 \times$ radius,
Circumference = $2 \times \pi \times$ radius

$$C = 2\pi r$$

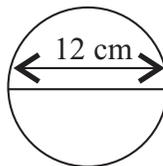


Check it out:

π is the **ratio** between the circumference and the diameter of a circle.
For any size of circle:
 $\text{circumference} \div \text{diameter} = \pi$.

Example 2

Find the circumference of the circle below. Use the approximation, $\pi \approx 3.14$.



Solution

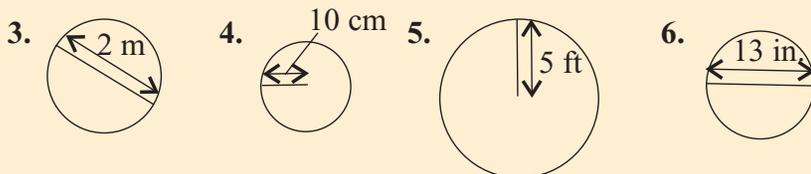
Use the formula that has diameter in it.

Substitute in the values and evaluate with your calculator.

$$C = \pi d \approx 3.14 \times 12 \text{ cm} = 37.68 \text{ cm} \approx \mathbf{37.7 \text{ cm}}.$$

Guided Practice

Find the circumference of the circles in Exercises 3–6.



7. Find the radius of a circle that has a circumference of 56 ft.

8. Find the diameter of a circle that has a circumference of 7 m.

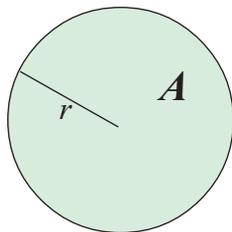
Check it out:

There are two formulas for the circumference. If you're given the radius in the question, use $C = 2\pi r$; if you're given the diameter, use $C = \pi d$.

The Area of a Circle Involves π Too

The **area** of a circle is the **amount of surface it covers**.

The area of a circle is **related to π** — just like the circumference. There's a **formula** for it:



$$\text{Area} = \pi \cdot (\text{radius})^2$$

$$A = \pi r^2$$

Check it out:

π is the **ratio** between the area and the square of the radius.
For any size of circle:
 $\text{area} \div (\text{radius})^2 = \pi$

Example 3

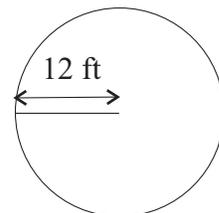
Find the area of the circle opposite, using $\pi \approx 3.14$.

Solution

Use the formula: $A = \pi r^2$

Substitute in the values and evaluate the area

$$A \approx 3.14 \times (12 \text{ ft})^2 = 3.14 \times 144 \text{ ft}^2 = 452.16 \text{ ft}^2 \approx \mathbf{452 \text{ ft}^2}$$



If you know the **area** of a circle you can calculate its **radius**:

Example 4

The area of a circle is 200 cm^2 . What is the radius of this circle?
Use $\pi \approx 3.14$.

Solution

The question gives the area, and you need to find the radius. This means rearranging the formula for the area of a circle to get r by itself.

$$A = \pi r^2$$

$$\frac{A}{\pi} = r^2$$

Divide both sides by π

$$r = \sqrt{\frac{A}{\pi}}$$

Take the square root of both sides

$$r = \sqrt{\frac{200}{\pi}} \approx \sqrt{63.7} \approx 8 \text{ cm}$$

Substitute in the values and find the radius

Don't forget:

$$r^2 = r \times r, \text{ not } 2r.$$

Don't forget:

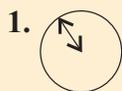
You don't use the diameter in area problems, so it's useful to make sure you find the radius before attempting any other calculations. Remember, the radius is half the diameter.

Guided Practice

- Find the area of a circle that has a diameter of 12 in.
- Find the area of a circle that has a radius of 5 m.
- If a circle has an area of 45 in^2 , what is its radius?

Independent Practice

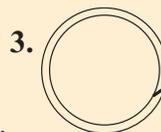
In Exercises 1–3, find the area of the circles shown.



$$r = 8 \text{ m}$$



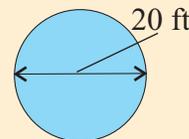
$$d = 2 \text{ in.}$$



$$C = 45 \text{ cm}$$

4. Find the circumference and area of a circle with a diameter of 6 m.

5. Lakesha has measured the diameter of her new whirlpool bath as 20 ft. Find its surface area.



- Find the circumference of the base of a glass with a 1.5 inch radius.
- Find the area of the base of the glass in Exercise 6.
- A circle has an area of 36 cm^2 . Find its radius and circumference.

Now try these:

Lesson 3.1.3 additional questions — p443

Round Up

This Lesson is all about **circles**, and how to find their **circumferences** and **areas**. There are a few formulas that you need to master — make sure you practice **rearranging** them.

Lesson 3.1.4

Areas of Complex Shapes

California Standards:

Measurement and
Geometry 2.2

Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

What it means for you:

You'll use the area formulas for regular shapes to find the areas of more complex shapes.

Key words:

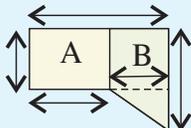
- complex shape
- addition
- subtraction

Don't forget:

Areas should always have a squared unit. Check what it should be and make sure your answer includes it.

Check it out:

You could also have split the shape into a rectangle and a trapezoid.



You get the same answer however you split the shape.

You've practiced finding the areas of regular shapes. Now you're going to use what you've learned to find areas of more complex shapes.

Complex Shapes Can Be Broken into Parts

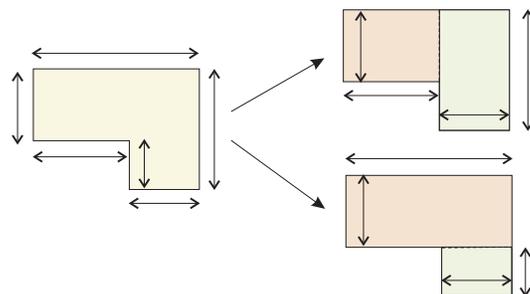
There are no easy formulas for finding the areas of **complex shapes**. However, complex shapes are often made up from **simpler shapes** that you know how to find the area of.

To find the area of a complex shape you:

- 1) **Break it up** into shapes that you know how to find the area of.
- 2) Find the area of each part **separately**.
- 3) **Add the areas** of each part together to get the **total area**.

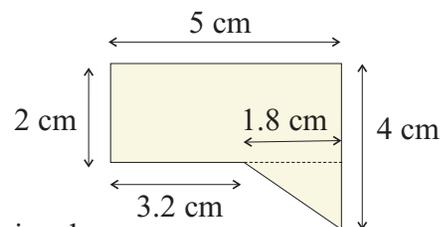
Shapes can often be broken up in **different ways**.

Whichever way you choose, you'll get the **same total area**.



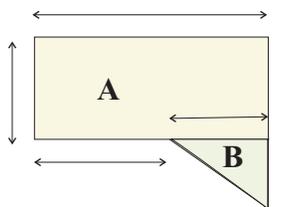
Example 1

Find the area of this shape.



Solution

Split the shape into a rectangle and a triangle.



Area A is a rectangle.

$$\text{Area A} = bh = 5 \text{ cm} \times 2 \text{ cm} = 10 \text{ cm}^2.$$

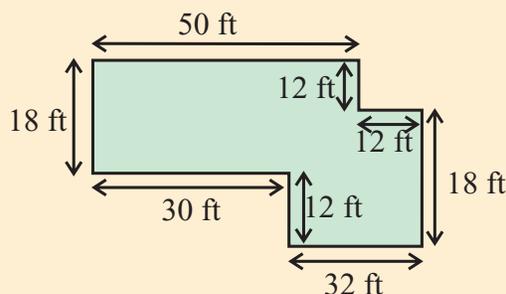
Area B is a triangle.

$$\text{Area B} = \frac{1}{2}bh = \frac{1}{2} \times 2 \text{ cm} \times 1.8 \text{ cm} = 1.8 \text{ cm}^2.$$

$$\text{Total area} = \text{area A} + \text{area B} = 10 \text{ cm}^2 + 1.8 \text{ cm}^2 = \mathbf{11.8 \text{ cm}^2}$$

✓ Guided Practice

1. Find the area of the complex shape below.



Don't forget:

It's easy to count some pieces of the shape twice. Always draw the shape split into its parts first so you know exactly what you're dealing with.

You Can Find Areas by Subtraction Too

So far we've looked at complex shapes where you **add together** the areas of the different parts.

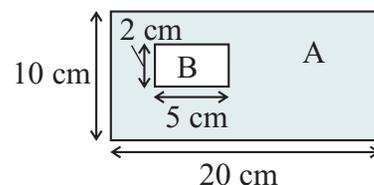
For some shapes, it's easiest to find the area of a **larger shape** and **subtract** the area of a **smaller shape**.

Check it out:

Most problems can be solved by either addition or subtraction of areas. Use whichever one looks simpler.

Example 2

Find the shaded area of this shape.



Solution

First calculate the area of rectangle A, then subtract the area of rectangle B.

$$\text{Area A} = lw = 20 \times 10 = 200 \text{ cm}^2$$

$$\text{Area B} = lw = 5 \times 2 = 10 \text{ cm}^2$$

$$\text{Total area} = \text{area A} - \text{area B} = 200 \text{ cm}^2 - 10 \text{ cm}^2 = \mathbf{190 \text{ cm}^2}$$

Since there are **many stages** to these questions, always **explain** what you're doing and set your work out **clearly**.

Example 3

Find the shaded area of this shape.

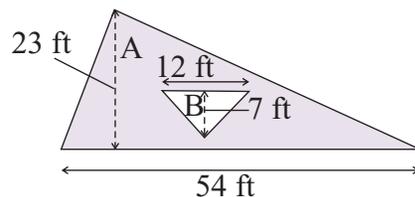
Solution

First calculate the area of triangle A, then subtract the area of triangle B.

$$\text{Area A} = \frac{1}{2}bh = \frac{1}{2} \times 54 \text{ ft} \times 23 \text{ ft} = 621 \text{ ft}^2$$

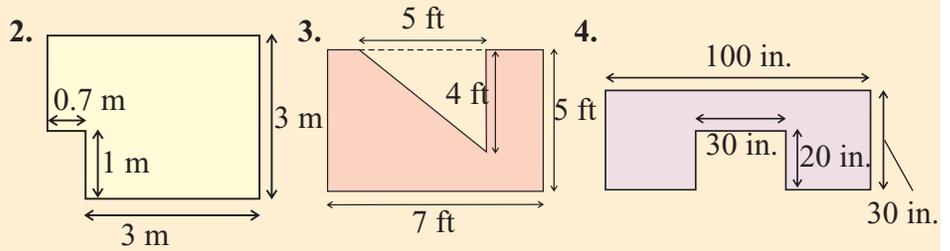
$$\text{Area B} = \frac{1}{2}bh = \frac{1}{2} \times 12 \text{ ft} \times 7 \text{ ft} = 42 \text{ ft}^2$$

$$\text{Total area} = \text{area A} - \text{area B} = 621 \text{ ft}^2 - 42 \text{ ft}^2 = \mathbf{579 \text{ ft}^2}$$



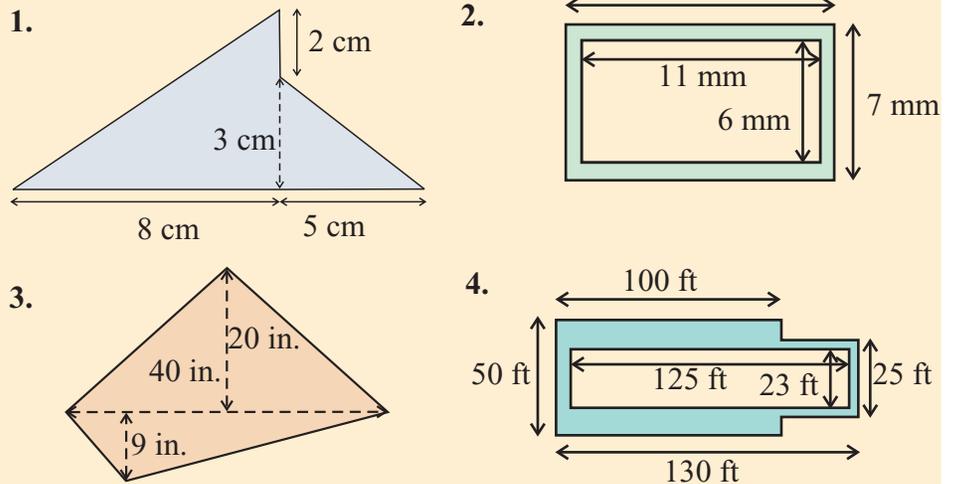
✓ Guided Practice

Use subtraction to find the areas of the shapes in Exercises 2–4.



✓ Independent Practice

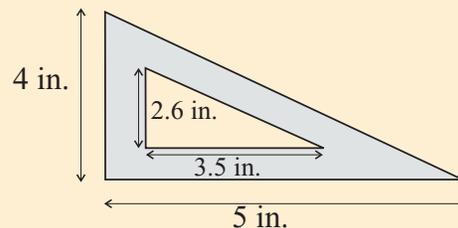
Use either addition or subtraction to find the areas of the following shapes.



5. Damion needs his window frame replacing. If the outside edge of the frame is a rectangle measuring 3 ft \times 5 ft and the pane of glass inside is a rectangle measuring 2.6 ft by 4.5 ft, what is the total area of the frame that Damion needs?

6. Aisha has a decking area in her backyard. Find its area, if the deck is made from six isosceles triangles of base 4 m and height 5 m.

7. Find the area of the metal bracket opposite.



Now try these:

Lesson 3.1.4 additional questions — p444

Round Up

You can find the *areas of complex shapes* by splitting them up into shapes you know formulas for — squares, rectangles, triangles, trapezoids, parallelograms... Take care to include every piece though.

Lesson 3.1.5

More Complex Shapes

California Standards:

Measurement and
Geometry 2.2

Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

What it means for you:

You'll find the areas of complex shapes by using the area formulas for simple shapes, including those for circles and trapezoids, and you'll also find perimeters of complex shapes.

Key words:

- sector
- semicircle
- circle
- complex shapes
- trapezoid

Don't forget:

If you're given the diameter of a circle in an area question, you need to halve it to find the radius before using the area formula.

Don't forget:

A little square in a corner means that the angle is a right angle.



In the last Lesson you found the areas of *complex shapes* by breaking them down into rectangles and triangles. Complex shapes can sometimes be broken down into other shapes — such as *parts of circles or trapezoids*. That's what you'll practice in this Lesson. You'll also look at finding the *perimeters* of complex shapes.

Complex Shapes Can Contain Circles

Some complex shapes involve **circles**, or **fractions of circles**.

To calculate the area, you first have to decide what **fraction** of the full circle is in the shape — for example, a half or a quarter.

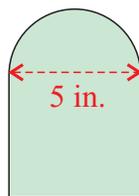
Once you know what fraction of the circle you want, find the area of the **whole circle**, and then **multiply** that area by the fraction of the circle in the shape. For example, **a semicircle has half the area of a full circle**.

Example 1

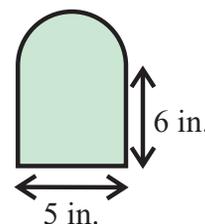
Find the area of the complex shape opposite.

Solution

Split the shape into a **semicircle** and a **rectangle**.



The semicircle has half the area of a full circle, and has a diameter of 5 in. This means its radius is 2.5 in.



$$\text{Area of full circle} = \pi r^2 = \pi \times (2.5 \text{ in})^2 = \pi \times 6.25 = 19.6 \text{ in}^2$$

$$\begin{aligned} \text{Area of semicircle} &= 0.5 \times \text{area of full circle} \\ &= 0.5 \times 19.6 \text{ in}^2 = 9.8 \text{ in}^2 \end{aligned}$$

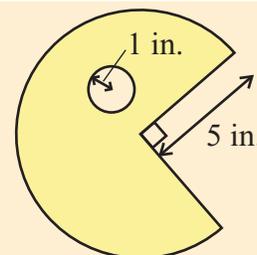
$$\text{Area of rectangle} = lw = 5 \text{ in.} \times 6 \text{ in.} = 30 \text{ in}^2$$

$$\text{Total area} = \text{area of semicircle} + \text{area of rectangle} = 9.8 \text{ in}^2 + 30 \text{ in}^2$$

$$\text{Total area} = \mathbf{39.8 \text{ in}^2}$$

✓ Guided Practice

1. Find the area of the complex shape opposite.



Look Out for Trapezoids and Parallelograms Too

Some complex shapes need to be broken into **more parts** than others. Often it's **not obvious** what the **best** way to break them up is.

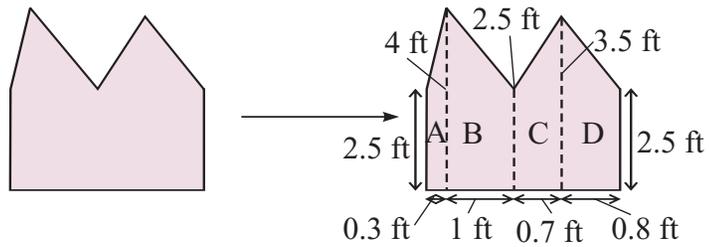
If you only look for triangles and rectangles you could miss the easiest way to solve the problem — look out for **trapezoids** and **parallelograms** too.

Don't forget:

If you see a problem that looks too tricky, don't panic! Break it down into small manageable chunks, instead of attempting the whole thing right away.

Example 2

Find the area of the complex shape below by breaking it down into trapezoids.



Solution

The equation for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.

$$\text{Area of trapezoid A} = \frac{1}{2} \times 0.3 \text{ ft} \times (2.5 \text{ ft} + 4 \text{ ft}) = 0.975 \text{ ft}^2$$

$$\text{Area of trapezoid B} = \frac{1}{2} \times 1 \text{ ft} \times (4 \text{ ft} + 2.5 \text{ ft}) = 3.25 \text{ ft}^2$$

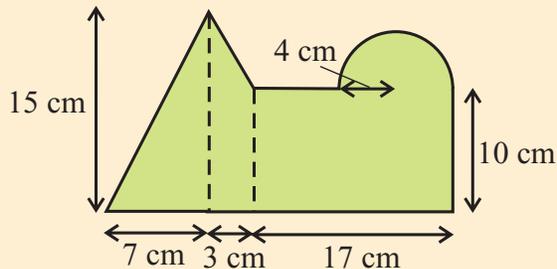
$$\text{Area of trapezoid C} = \frac{1}{2} \times 0.7 \text{ ft} \times (2.5 \text{ ft} + 3.5 \text{ ft}) = 2.1 \text{ ft}^2$$

$$\text{Area of trapezoid D} = \frac{1}{2} \times 0.8 \text{ ft} \times (3.5 \text{ ft} + 2.5 \text{ ft}) = 2.4 \text{ ft}^2$$

$$\text{Total area} = 0.975 \text{ ft}^2 + 3.25 \text{ ft}^2 + 2.1 \text{ ft}^2 + 2.4 \text{ ft}^2 = \mathbf{8.725 \text{ ft}^2}$$

✓ Guided Practice

2. Find the area of the complex shape below.



Finding the Perimeter of Complex Shapes

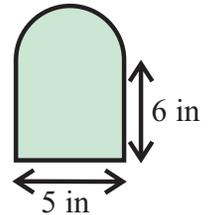
The perimeter is the **distance** around the edge of a shape.

To find the perimeter of a complex shape, you need to **add the lengths of each side**.

It's likely that you won't be given the lengths of **all** the sides, so you may need to **find** some lengths yourself — for example, the circumference of a semicircle, which is half the circumference of a full circle.

Example 3

Find the perimeter of the complex shape opposite.



Solution

The circumference of a semicircle is half the circumference of a full circle of the same radius.

$$\begin{aligned} \text{Circumference of semicircle} &= \frac{1}{2} \cdot \text{circumference of full circle} \\ &= \frac{1}{2} \pi d = \frac{1}{2} \times \pi \times 5 \text{ in.} = 7.85 \text{ in.} \end{aligned}$$

$$\text{Perimeter of 3 sides of rectangle} = 6 \text{ in.} + 5 \text{ in.} + 6 \text{ in.} = 17 \text{ in.}$$

$$\text{Total perimeter} = \text{circumference of semicircle} + \text{perimeter of rectangle.}$$

$$\text{Total perimeter} = 7.85 \text{ in.} + 17 \text{ in.} = \mathbf{24.85 \text{ in.}}$$

Don't forget:

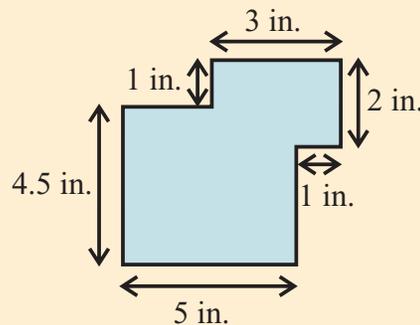
Remember — don't include the lengths of sides that don't form the outline of the final shape. For example, the fourth side of the rectangle isn't included here.

Don't forget:

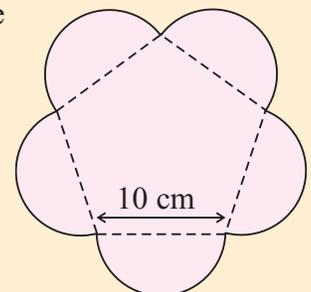
Perimeter has units of length (for example, feet). Area has units of length squared (for example, ft²).

Guided Practice

3. Find the perimeter of the shape below.

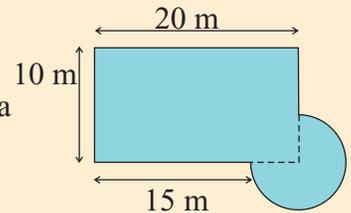


4. Davina has made a flower shape out of some wood by cutting out a regular pentagon and sticking a semicircle to each side of the pentagon, as shown. She wants to make a border for her shape by sticking some ribbon all around the edge. Find the length of ribbon that Davina will need.

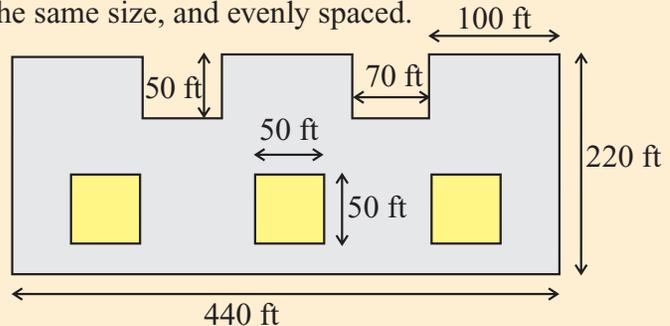


Independent Practice

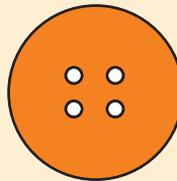
1. Kia's swimming pool is rectangular in shape with a circular wading pool at one corner, as shown. Find the total surface area of Kia's pool and the distance around the edge.



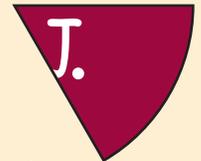
2. Find the area of the face of the castle below (don't include the windows). Assume that all the windows are the same size and that all the turrets are the same size, and evenly spaced.



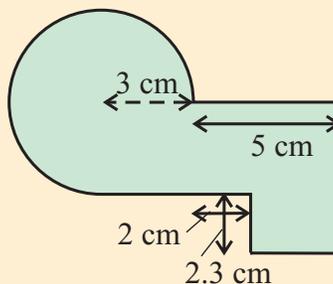
3. Find the area of the button shown below if each hole has a diameter of 0.1 in. and the button has a diameter of 1.2 in.



4. T.J. has five friends coming to his 13th birthday party. He bakes a cake that is 12 inches in diameter. At the party, T.J. and his friends divide the cake equally between them, into identically shaped slices, as shown. Find the perimeter and area of the base of each slice of cake.



5. Find the perimeter and area of the shape below.



Check it out:

If there is a rectangle overlapping a circle, always try to find the area of the rectangle, then the area of the rest of the circle. You'll get in a mess if you try to solve it the other way around!

Now try these:

Lesson 3.1.5 additional questions — p444

Round Up

Now you know everything you need to know about finding the area and perimeter of complex shapes. The first step is to look at the shape and decide on the *easiest way* to break it up into *simple shapes*.

Section 3.2 introduction — an exploration into: Coordinate 4-in-a-row

Ordered pairs are used to represent points on the *coordinate plane*. The goal of this game is to get as many points *in a line* as possible — the lines can be *vertical*, *horizontal*, or *diagonal*. You score for each row of *four or more* points that you make — the scoring system is below.

Scoring System

4-in-a-row	1 point
5-in-a-row	2 points
6-in-a-row	3 points (or more)

Example

You need part of a coordinate plane, like shown. Players **take turns** calling out coordinates.

For example:

Player 1 (x): (4, 2) — right 4, up 2.

Player 2 (•): (9, 3) — right 9, up 3.

Player 1 (x): (5, 3) — right 5, up 3.

Player 2 (•): (6, 4) — right 6, up 4.

Player 1 (x): (3, 1) — right 3, up 1.

Now, Player 1 needs to get **(2, 0)** to get **4-in-a-row** and score **1 point**.

But it's Player 2's turn next, and if they choose (2, 0), they'll **block** the point.

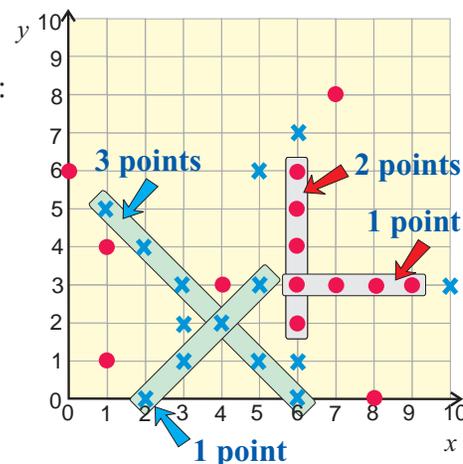
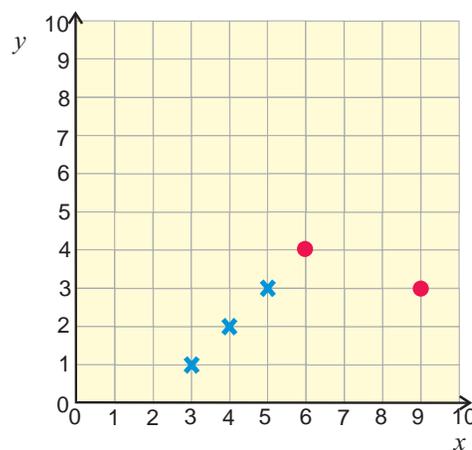
After a while, your coordinate plane will look a bit like this:

Player 1 (x) has 4 points in total so far and

Player 2 (•) has 3 points so far.

So **Player 1** is winning at the moment.

If a player calls out a point that's already taken, or plots a different one to that which they called, they **lose their turn**.



Exercises

1. Play the game with another person. The person whose birthday is next goes first.
2. Play the game again. This time, players are allowed to pick two points in each turn.

Round Up

You can pinpoint a certain place on the *coordinate plane* using a *pair of coordinates*, and by plotting several points you can form a *straight line*.

Lesson 3.2.1

Section 3.2 Plotting Points

California Standards:

Measurement and
Geometry 3.2

Understand and use **coordinate graphs** to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

What it means for you:

You'll see how to use a grid system to plot numbered points.

Key words:

- coordinate
- x -axis
- y -axis
- quadrant

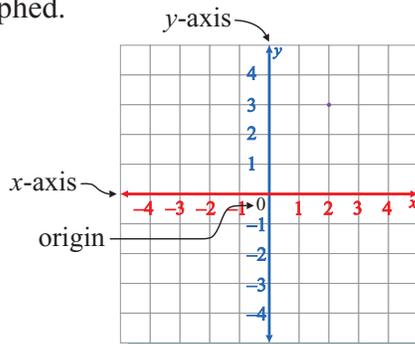
Check it out:

Coordinates are sometimes called ordered pairs. This means that the order of the numbers matters — $(1, 2)$ is different from $(2, 1)$.

When you draw a graph you draw it on a **coordinate plane**. This is a flat grid that has a horizontal axis and a vertical axis. You can describe where any point on the plane is using a **pair of numbers** called **coordinates**.

You Plot Coordinates on a Coordinate Plane

The **coordinate plane** is a two-dimensional (flat) area where points and lines can be graphed.



The plane is formed by the intersection of a **vertical number line**, or **y -axis**, and a **horizontal number line**, or **x -axis**. They cross where they are **both equal to 0** — a point called the **origin**.

Coordinates Describe Points on the Plane

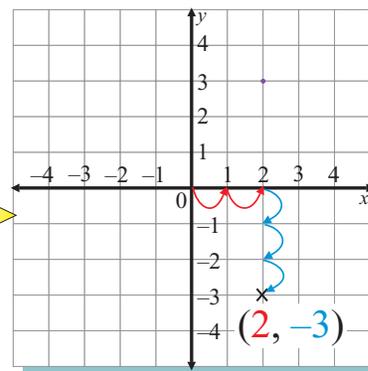
The **x and y coordinates** of a point describe where on the plane it lies. The coordinates are written as **(x, y)** .

x -coordinate → **$(2, -3)$** ← **y -coordinate**

When you plot points on the coordinate plane you plot them in relation to the **origin**, which has coordinates of **$(0, 0)$** .

- The **x -coordinate** tells you how many spaces along the **x -axis** to go. Negative values mean you go **left**. Positive values mean you go **right**.
- The **y -coordinate** tells you how many spaces up or down the **y -axis** to go. Positive values mean you go **up**. Negative values mean you go **down**.

So a point with the coordinates $(2, -3)$ will be **two** units to the **right** of the origin, and **three** units **below**.



Example 1

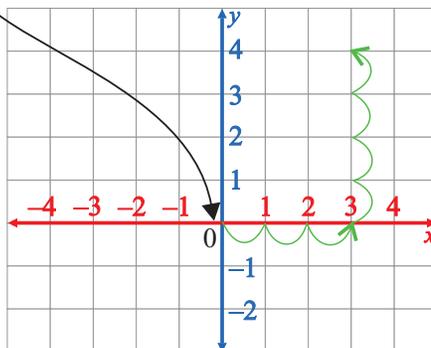
Plot the point with the coordinates (3, 4).

Solution

Step 1: start at the origin, (0, 0).

Step 2: move right along the x -axis 3 units.

Step 3: now move straight up 4 units and plot the point.

**Check it out:**

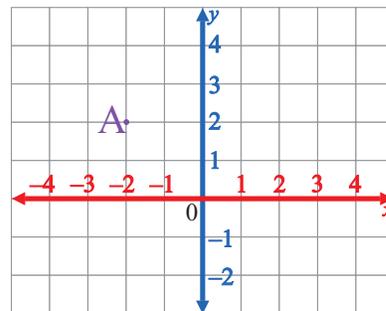
A positive x -coordinate tells you to move right along the x -axis, while a negative x -coordinate tells you to move left along the x -axis.

A positive y -coordinate tells you to move up the y -axis, while a negative y -coordinate tells you to move down the y -axis.

When you are reading the coordinates of a point on a graph you can use the same idea.

Example 2

What are the coordinates of point A?

**Solution**

Start at (0, 0). To get to point A on the graph you need to move 2 units to the left. So the x -value of your coordinate is -2 .

Then you need to go 2 units straight up. So the y -value is 2.

The coordinates of the point A are $(-2, 2)$.

✓ Guided Practice

Plot and label each of the coordinate pairs in Exercises 1–6 on a coordinate plane.

1. (1, 4)

2. (2, -3)

3. (-1 , -2)

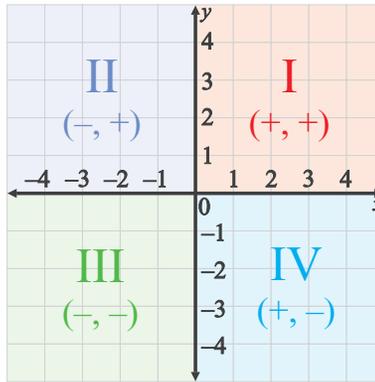
4. (-4 , 2)

5. (0, 3)

6. (-4 , 0)

The Coordinate Plane is Divided into Four Quadrants

The **x-axis** and **y-axis** divide the coordinate plane into four sections. Each of these sections is called a **quadrant**. The quadrants are represented by Roman numerals, and are labeled **counterclockwise**.



The **signs** of the x and y values are **different** in each quadrant. For instance, in **quadrant I** both the x and y values are **positive**. But in **quadrant II** the x value is **negative** and the y value is **positive**.

You can tell which quadrant a point will fall in by looking at the signs of the x and y coordinates.

Example 3

Which quadrant is the point $(1, -4)$ in?

Solution

The x -value is 1. This is positive, so the point must be in quadrant I or IV. The y -value is -4 . This is negative, so the point must be in quadrant IV.

The point $(1, -4)$ is in quadrant IV.

Check it out:

A point that is on either the x -axis or the y -axis is not in any of the quadrants.

Example 4

Which quadrant is the point $(-3, -6)$ in?

Solution

Both coordinates are **negative**, so **the point $(-3, -6)$ is in quadrant III.**

✓ Guided Practice

In Exercises 7–14 say which quadrant the point lies in.

- | | |
|-----------------|-------------|
| 7. (1, 3) | 8. (−2, −4) |
| 9. (7, −2) | 10. (−3, 6) |
| 11. (−2, −2) | 12. (2, 2) |
| 13. (−1.5, 2.5) | 14. (1, −1) |

✓ Independent Practice

In Exercises 1–6 say which quadrant the point is in.

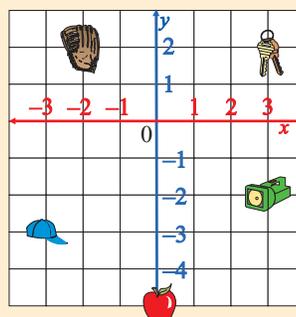
- | | |
|------------|---------------|
| 1. (1, 1) | 2. (−1, −1) |
| 3. (−1, 2) | 4. (−2, 1) |
| 5. (3, −2) | 6. (−61, 141) |

7. Do the coordinate pairs (−3, 4) and (4, −3) correspond to the same point on the plane?

Plot each of the points in Exercises 8–13 on a coordinate plane.

- | | |
|--------------|-------------|
| 8. (0, 0) | 9. (3, 2) |
| 10. (−2, −3) | 11. (−3, 4) |
| 12. (1, −2) | 13. (0, 4) |

14. Sophie and Jorge are playing a game. Sophie marks out a coordinate plane on the beach, and buries some objects at different points. Jorge has to use the map below to find the objects.



- What are the coordinates of the keys?
- What are the coordinates of the baseball cap?
- What are the coordinates of the apple?
- How many units are there between the keys and the flashlight?
- How many units are there between the baseball mitt and the keys?

Now try these:

Lesson 3.2.1 additional questions — p445

Round Up

Coordinates allow you to describe where points are plotted — they're written as pairs of numbers, such as (1, −5). The first number tells you the horizontal or x-coordinate. The second tells you the vertical or y-coordinate. Plotting points on coordinate planes is a big part of drawing graphs, and will be used a lot in the rest of this Chapter and in the next Chapter.

Lesson 3.2.2

Drawing Shapes on the Coordinate Plane

California Standards:

Measurement and
Geometry 3.2

Understand and use
coordinate graphs to plot
simple figures, determine
lengths and areas related to
them, and determine their
image under translations and
reflections.

What it means for you:

You'll see how to draw
shapes on a grid by plotting
points and joining them.

Key words:

- coordinate
- area
- perimeter

Check it out:

You have to join the points in
order. So pentagon DEFGH
must have D joined to E, E
joined to F, F joined to G, etc...
If you joined the points in a
different order you'd get a
completely different shape.

Check it out:

You can tell something about
the shape you're drawing just
from the number of
coordinates you're given. If
you are only given three points
to plot, the shape must be a
triangle. If you are given four
points, then it's probably a
quadrilateral (though it could
be a triangle — if three of the
points are in a line).

In the last Lesson you saw how to *plot points on the coordinate plane*.
If you plot several points and then *join them up*, you get a *shape*.

You Can Make Shapes by Joining Points

You can **draw a shape** on the coordinate plane by **plotting points** and
joining them together. The coordinates are the **corners** of the shape.

Example 1

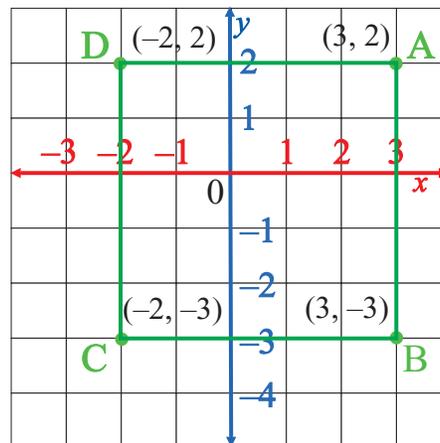
Plot the shape ABCD on the coordinate plane, where A is (3, 2),
B is (3, -3), C is (-2, -3), and D is (-2, 2).

Name the shape you have drawn.

Solution

Step 1: **Plot** and **label** the
points A, B, C, and D.

Step 2: **Join** the points **in order**.
So A joins to B, B joins to C, C
joins to D, and D joins to A.



The shape has four sides of equal length and four right angles —
so it's a **square**.

Guided Practice

Plot the shapes in Exercises 1–4 on the coordinate plane and name the
shapes.

1. JKL J(3, 1) K(2, 3) L(1, 1)
2. EFG E(1, 2) F(2, -3) G(-3, -2)
3. RSTU R(-2, 1) S(2, 1) T(3, -1) U(-3, -1)
4. PQRS P(-1, 3) Q(3, 3) R(1, 0) S(-3, 0)

Use the Shape's Properties to Find Missing Points

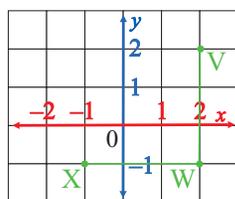
Sometimes you might be given a shape to graph with the coordinates of a corner **missing**. You can use the **properties** of the shape to work out the missing pair of coordinates.

Example 2

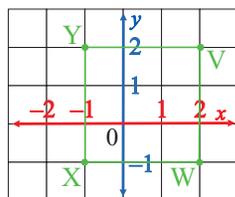
VWXY is a square on the coordinate plane, where V is $(2, 2)$, W is $(2, -1)$, and X is $(-1, -1)$. What are the coordinates of point Y?

Solution

First **plot** points V, W, and X and **join** them in order.



You know that VWXY is a **square**. So it must have **four equal-length sides** that meet at **right angles**. The lines VW and WX are both 3 units long. So point Y must be 3 units left of V, and 3 units above X. Add it to the graph, and form the square.



Now read the coordinates of Y from the graph: **point Y is at $(-1, 2)$** .

Don't forget:

An isosceles triangle is one that has at least two sides the same length and at least two angles of equal measure.

Guided Practice

In Exercises 5–8 find the missing point.

- | | |
|---------------------------|--|
| 5. Square KLMN | K $(-2, 2)$ L $(2, 2)$ M $(2, -2)$ N $(?, ?)$ |
| 6. Rectangle CDEF | C $(1, 3)$ D $(1, -1)$ E $(-2, -1)$ F $(?, ?)$ |
| 7. Parallelogram ABCD | A $(-3, -2)$ B $(1, -2)$ C $(3, 1)$ D $(?, ?)$ |
| 8. Isosceles triangle RST | R $(1, -2)$ S $(0, 1)$ T $(?, ?)$ |

Find Lengths Using Absolute Value

Once you've **plotted** a shape on the coordinate plane you can find out its **area** or **perimeter** using the **formulas** that you saw in Section 3.1.

But first you'll need to find some **lengths** on the coordinate plane — such as the side lengths of the shapes.

You could do this by counting squares on the diagram. Another way of doing this is to use ***x*- and *y*-coordinate values**.

This is shown in the example below.

Don't forget:

If you need a reminder, then all the perimeter formulas that you might need are in Lesson 3.1.1. All the area formulas that you might need are in Lesson 3.1.2.

Example 3

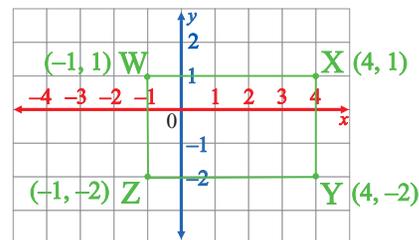
Plot the rectangle WXYZ on the coordinate plane, where W is $(-1, 1)$, X is $(4, 1)$, Y is $(4, -2)$, and Z is $(-1, -2)$.

What are the perimeter and area of WXYZ?

Solution

First plot WXYZ on the coordinate plane.

To find the perimeter and area, you need to know the width and the length.



The sides WX and YZ give the length. They're the same, so find either. To find the **length** (l) using side WX, **subtract** the x -coordinate of W from the x -coordinate of X:

$$l = |4 - (-1)| = 5.$$

Sides WZ and XY give the width. They're the same, so find either. To find the **width** (w) using side XY, subtract the y -coordinate of Y from the y -coordinate of X:

$$w = |1 - (-2)| = 3.$$

Now just plug the length and width values into the formulas for perimeter and area.

$$\text{Perimeter of WXYZ} = 2(l + w) = 2(5 + 3) = 16 \text{ units}$$

$$\text{Area of WXYZ} = l \cdot w = 5 \cdot 3 = 15 \text{ units}^2$$

Don't forget:

The bars around the calculation here show that the absolute value is being calculated.

This means it doesn't matter which coordinate you subtract from which — you'll always get the correct, positive answer.

✓ Guided Practice

9. What is the perimeter of square ABCD, where A is (1, 1), B is (3, 1), C is (3, 3), and D is (1, 3)?
10. What are the perimeter and area of rectangle EFGH, where E is (-2, 1), F is (3, 1), G is (3, -2), and H is (-2, -2)?
11. What is the area of triangle JKL, when J is (-1, -3), K is (3, -3), and L is (1, 0)?

✓ Independent Practice

Plot and name the shapes in Exercises 1–3 on the coordinate plane.

1. ABC A(2, 3) B(3, -3) C(-2, -1)
2. TUVW T(4, -1) U(0, -1) V(0, 2) W(4, 2)
3. EFGH E(-2, -2) F(-1, 0) G(1, 0) H(2, -2)

4. Anthony is marking out a pond in his yard. It is going to be perfectly square. He is marking it out on a grid system, and has put the first three marker stakes in at (-1, -3), (-1, 1), and (3, 1). At what coordinates should he put in the last stake?

In Exercises 5–7, find the missing pair of coordinates.

5. Square CDEF C(1, 2) D(4, 2) E(4, -1) F(?, ?)
6. Rectangle TUVW T(-3, 3) U(-2, 3) V(-2, -2) W(?, ?)
7. Parallelogram KLMN K(1, 0) L(-2, 0) M(-1, 2) N(?, ?)
8. What is the perimeter of rectangle BCDE, where B is (-2, 4), C is (3, 4), D is (3, 2), and E is (-2, 2)?
9. What is the area of triangle JKL, where J is (1, 2), K is (4, -1), and L is (7, -1)?
10. What are the area and perimeter of rectangle PQRS, where P is (0, 0), Q is (3, 0), R is (3, -2), and S is (0, -2)?
11. A school has decided to set aside an area of their playing field as a nature reserve. A plan is made using a grid with 10-foot units. The coordinates of the corners of the area set aside are (0, 0), (4, 0), (2, -2), and (-2, -2). What area will the nature reserve cover?

Now try these:

Lesson 3.2.2 additional questions — p445

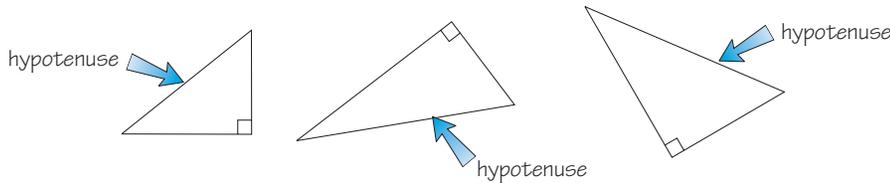
Round Up

Drawing shapes on the coordinate plane just means plotting their corners from coordinates and joining them together. You can even use the known properties of some shapes to figure out the coordinates of any missing corners. Once you've got the shapes plotted, you can use the standard formulas to work out their perimeters and areas.

Section 3.3 introduction — an exploration into: Measuring Right Triangles

There's a special relationship between the *leg-lengths* and the *hypotenuse-length* in a *right triangle*. The purpose of the Exploration is to discover this relationship.

The **hypotenuse** of a right triangle is the side **directly across from the right-angle**. The other sides are called **legs**. Some right triangles are shown below with their hypotenuses labeled.



Example

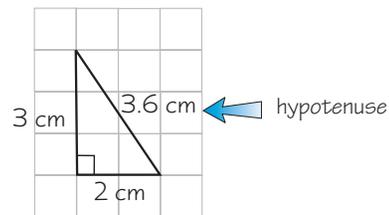
On grid paper, draw a right triangle. Measure the length of each leg and the length of the hypotenuse.

Solution

You can draw any triangle, as long as it has a **right-angle**.

Legs = 2 cm and 3 cm

Hypotenuse = 3.6 cm



Exercises

- Draw 5 right triangles on grid paper. Label them A-E. Then label the hypotenuse on each.
- For each right triangle, measure the length of each leg and the length of the hypotenuse. Measure in centimeters and record your measurements in a copy of this table.

Triangle	Leg 1 (cm)	Leg 2 (cm)	Hypotenuse (cm)
Example	2	3	3.6
A			
B			
C			
D			
E			

- Explain how the length of a right triangle's hypotenuse compares to the lengths of its legs.
- Explain how the sum of the legs of each right triangle compares to the hypotenuse length.
- Add three new columns to your table, like this:

Triangle	Leg 1 (cm)	Leg 2 (cm)	Hypotenuse (cm)	Leg 1 squared	Leg 2 squared	Hypotenuse squared
Example	2	3	3.6	4	9	13

Complete these columns, and then compare the squared side lengths for each triangle. What patterns do you notice?

Round Up

You should now have discovered how the *leg-lengths* of a right triangle are related to the *hypotenuse-length*. This is known as the *Pythagorean Theorem* — you'll be using it in this Section.

Lesson 3.3.1

Section 3.3 The Pythagorean Theorem

California Standards:

Measurement and Geometry 3.3

Know and understand the **Pythagorean theorem** and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments **and, in some situations, empirically verify the Pythagorean theorem by direct measurement.**

What it means for you:

You'll learn about an equation that you can use to find a missing side length of a right triangle.

Key words:

- Pythagorean theorem
- right triangle
- hypotenuse
- legs
- right angle

Check it out:

A right angle is an angle of exactly 90° . In diagrams, a right angle is shown as a small square in the corner like this:



Any other angle is shown as a piece of a circle like this:

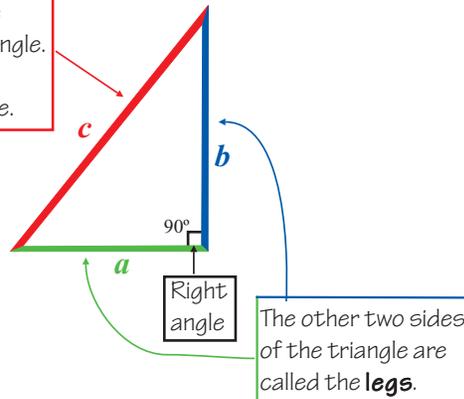


You will have come across *right triangles* before — they're just *triangles* that have one corner that's a 90° angle. Well, there's a special formula that links the side lengths of a right triangle — it comes from the *Pythagorean theorem*.

The Pythagorean Theorem is About Right Triangles

A **right triangle** is any triangle that has a 90° angle (or right angle) as one of its corners. You need to know the names of the parts of a right triangle:

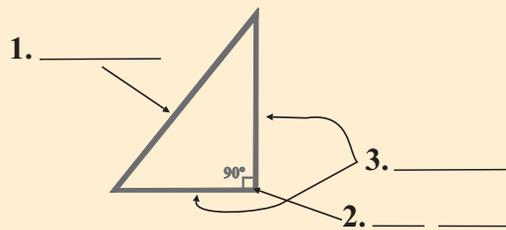
The **hypotenuse** is the longest side of the triangle. It's the side directly opposite the right angle.



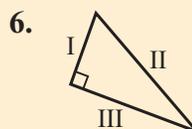
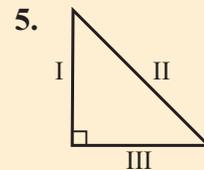
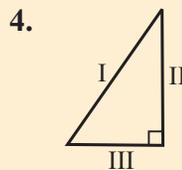
In diagrams of right triangles, the **hypotenuse** is usually labeled as c , and the two **legs** as a and b . It doesn't matter which leg you label a , and which you label b .

Guided Practice

Complete the missing labels on the diagram.



In Exercises 4–7 say which side of the right triangle is the hypotenuse.



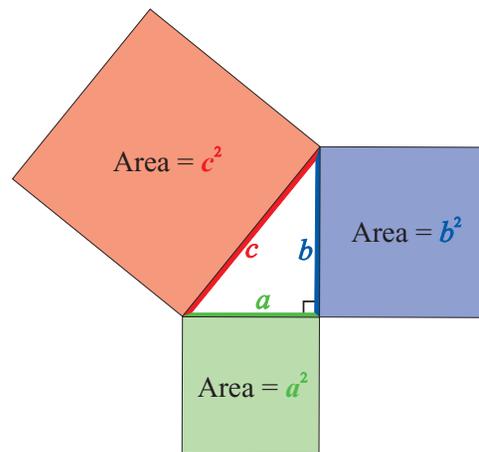
The Theorem Links Side Lengths of Right Triangles

Pythagoras was a Greek mathematician who lived around 500 B.C. A famous theorem about right triangles is named after him. It's called the **Pythagorean theorem**:

For any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs.

This all sounds very complicated, but it's not so bad once you know what it actually means.

Look again at the right triangle. Now add three squares whose side lengths are the same as the side lengths of the triangle:



What the Pythagorean theorem is saying is that the area of the red square is the same as the area of the blue square plus the area of the green square.

$$\begin{array}{c} \text{Area } c^2 \\ \square \end{array} = \begin{array}{c} \text{Area } a^2 \\ \square \end{array} + \begin{array}{c} \text{Area } b^2 \\ \square \end{array}$$

So this is what the Pythagorean theorem looks like written algebraically:

For any right triangle:
$$c^2 = a^2 + b^2$$

It means that if you know the **lengths** of **two sides** of a **right triangle**, you can always **find the length** of the **other side** using the equation.

You Can Check the Theorem Using a Right Triangle

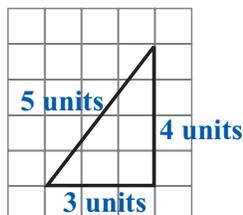
Don't forget:

It's really important to remember that the Pythagorean theorem **only** works on right triangles. It won't work on any other type.

You can check for yourself that the theorem works by **measuring** the **side lengths** of **right triangles**, and putting the values into the **equation**.

Example 1

Use the right triangle below to verify the Pythagorean theorem.



Solution

$$a = 3 \text{ units}$$

$$b = 4 \text{ units}$$

$$c = 5 \text{ units}$$

$$c^2 = a^2 + b^2$$

$$5^2 = 3^2 + 4^2$$

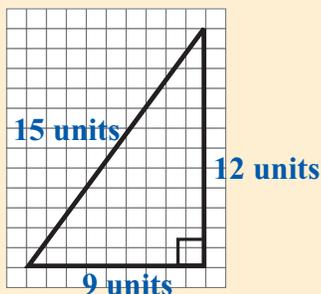
$$25 = 9 + 16$$

$$25 = 25$$

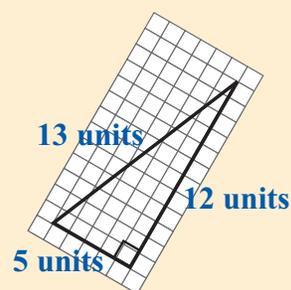
Guided Practice

Use the right triangles in Exercises 8–11 to verify the Pythagorean theorem.

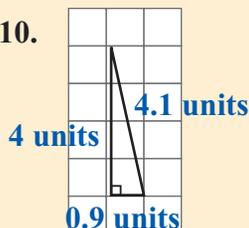
8.



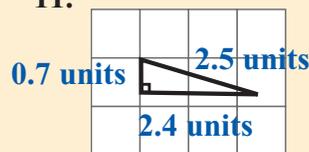
9.



10.

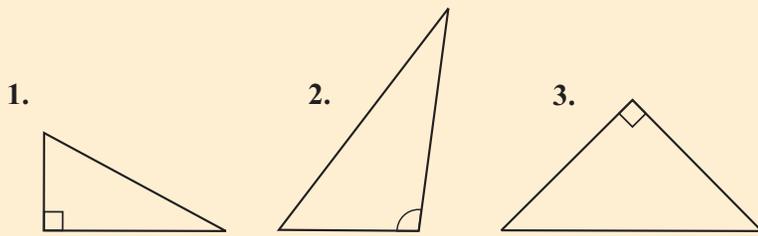


11.

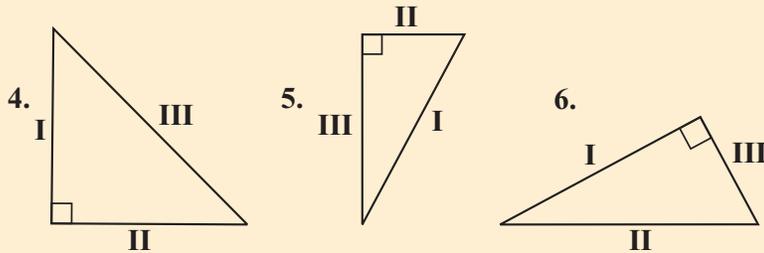


Independent Practice

In Exercises 1–3 say whether the triangle is a right triangle or not.



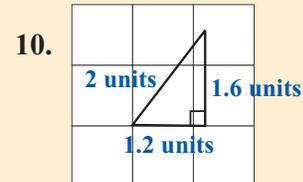
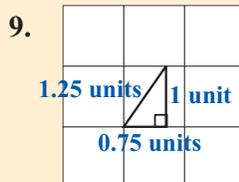
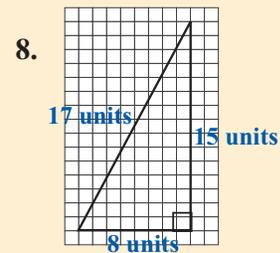
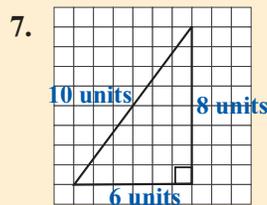
In Exercises 4–6 say which side of the right triangle is the hypotenuse.



Don't forget:

The hypotenuse is always the longest side of a right triangle.

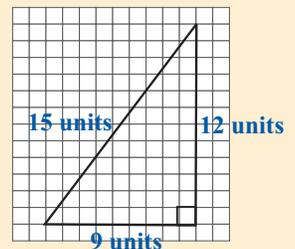
Use the triangles in Exercises 7–10 to verify the Pythagorean theorem.



Now try these:

Lesson 3.3.1 additional questions — p445

11. Victor used the triangle shown on the right to try to verify the Pythagorean theorem. Explain why his work is wrong.



Victor's work:

$$9^2 = 12^2 + 15^2$$

$$81 = 144 + 225$$

$$81 = 369$$

Round Up

The *Pythagorean theorem* describes the *relationship* between the *lengths* of the *hypotenuse* and the *legs* of a *right triangle*. It means that when you *know* the lengths of *two* of the sides of a right triangle, you can *always* find the length of the *third* side. You'll get a lot of practice at using it in the next few Lessons.

Lesson 3.3.2

Using the Pythagorean Theorem

California Standards:

Measurement and Geometry 3.2

Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

Measurement and Geometry 3.3

Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What it means for you:

You'll see how to use the Pythagorean theorem to find missing side lengths of right triangles.

Key words:

- Pythagorean theorem
- hypotenuse
- legs
- square root

Don't forget:

Finding the square root of a number is the opposite of squaring it. The symbol: $\sqrt{\quad}$ represents the positive square root of the number inside it. For more about square roots, see Section 2.6.

In the last Lesson, you met the *Pythagorean theorem* and saw how it linked the *lengths* of the *sides* of a *right triangle*.

In this Lesson, you'll practice using the theorem to work out *missing side lengths* in right triangles.

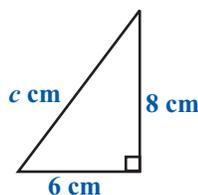
Use the Pythagorean Theorem to Find the Hypotenuse

If you know the **lengths** of the **two legs** of a **right triangle** you can use them to find the **length** of the **hypotenuse**.

The theorem says that $c^2 = a^2 + b^2$, where c is the length of the **hypotenuse**, and a and b are the lengths of the **two legs**. So if you know the lengths of the legs you can put them into the equation, and **solve** it to find the length of the hypotenuse.

Example 1

Use the Pythagorean theorem to find the length of the hypotenuse of the right triangle shown below.



Solution

$$c^2 = a^2 + b^2$$

First write out the equation

$$c^2 = 6^2 + 8^2$$

Substitute in the side lengths that you know

$$c^2 = 36 + 64$$

$$c^2 = 100$$

Simplify the expression

$$c = \sqrt{100}$$

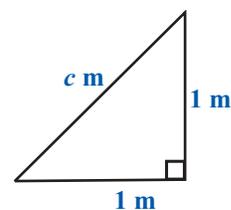
Take the square root of both sides

$$c = 10 \text{ cm}$$

A lot of the time your solution won't be a **whole number**. That's because the last step of the work is taking a **square root**, which often leaves a **decimal** or an **irrational number** answer.

Example 2

Use the Pythagorean theorem to find the length of the hypotenuse of the right triangle shown.

**Check it out:**

Before you start, it's very important to work out which side of the triangle is the longest side — the hypotenuse. Otherwise you'll get an incorrect answer.

Solution

$$c^2 = a^2 + b^2 \quad \text{First write out the equation}$$

$$c^2 = 1^2 + 1^2 \quad \text{Substitute in the side lengths that you know}$$

$$c^2 = 1 + 1$$

$$c^2 = 2 \quad \text{Simplify the expression}$$

$$c = \sqrt{2} \text{ m} \quad \text{Cancel out the squaring by taking the square root}$$

If you do this calculation on a calculator, you'll see that $\sqrt{2} \text{ m}$ is approximately equal to **1.4 m**.

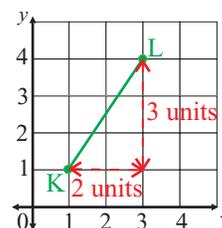
The Pythagorean theorem is also useful for finding **lengths on graphs** that aren't horizontal or vertical.

Example 3

Find the length of the line segment KL.

Solution

Draw a horizontal and vertical line on the plane to make a right triangle — then use the method above.



$$c^2 = a^2 + b^2 \quad \text{First write out the equation}$$

$$c^2 = 3^2 + 2^2 \quad \text{Substitute in the side lengths that you know}$$

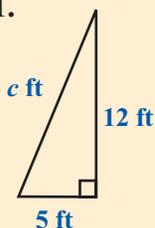
$$c^2 = 9 + 4 = 13 \quad \text{Simplify the expression}$$

$$c = \sqrt{13} \text{ units} \approx \mathbf{3.6 \text{ units}} \quad \text{Cancel out the squaring by taking the square root}$$

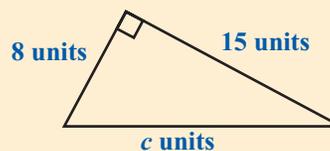
✓ Guided Practice

Use the Pythagorean theorem to find the length of the hypotenuse in Exercises 1–3.

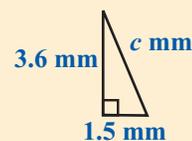
1.



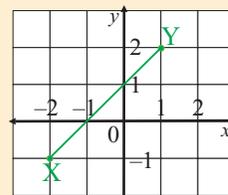
2.



3.



4. Use the Pythagorean theorem to find the length of the line segment XY.



You Can Use the Theorem to Find a Leg Length

If you know the length of the **hypotenuse** and one of the **legs**, you can use the **theorem** to find the length of the “**missing**” leg. You just need to **rearrange the formula**:

Don't forget:

To keep an equation balanced, you have to do the same thing to both sides. Here b^2 is subtracted from each side. For a reminder on equations see Section 1.2.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 &= c^2 - b^2 \end{aligned}$$

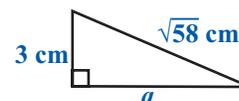
Subtract b^2 from both sides to get the a^2 term by itself.

Remember that it doesn't matter which of the **legs** you call a and which you call b . But the **hypotenuse** is always c .

Now you can **substitute** in values to find the missing leg length as you did with the hypotenuse.

Example 4

Find the missing leg length in this right triangle.



Solution

$$c^2 = a^2 + b^2$$

$$a^2 = c^2 - b^2$$

$$a^2 = (\sqrt{58})^2 - 3^2$$

$$a^2 = 58 - 9$$

$$a^2 = 49$$

$$a = \sqrt{49}$$

$$a = 7 \text{ cm}$$

First write out the equation

Rearrange it

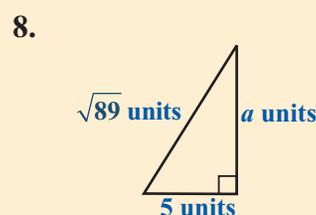
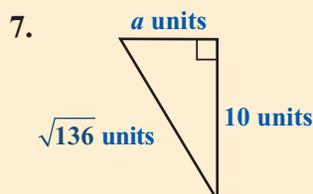
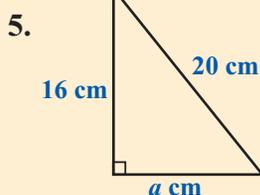
Substitute in the side lengths that you know

Simplify the expression

Take the square root of both sides

Guided Practice

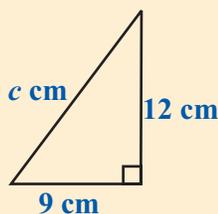
Use the Pythagorean theorem to calculate the missing leg lengths in Exercises 5–8.



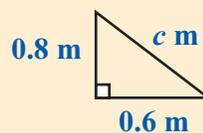
Independent Practice

Use the Pythagorean theorem to find the value of c in Exercises 1–5.

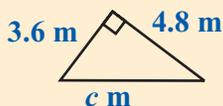
1.



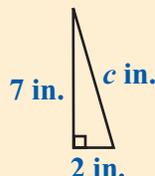
2.



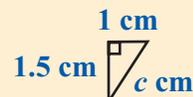
3.



4.

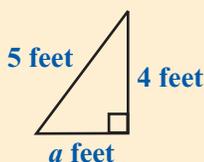


5.

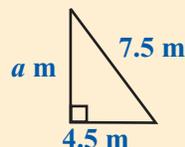


Calculate the value of a in Exercises 6–10.

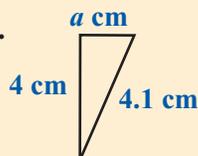
6.



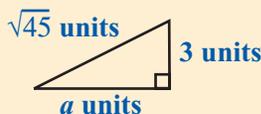
7.



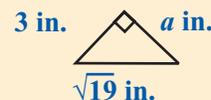
8.



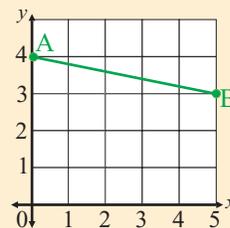
9.



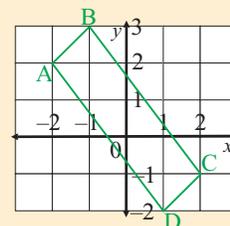
10.



11. Find the length of line AB.



12. Find the perimeter of quadrilateral ABCD.



Now try these:

Lesson 3.3.2 additional questions — p446

Round Up

The *Pythagorean theorem* is really useful for finding *missing side lengths* of right triangles. If you know the lengths of *both legs* of a triangle, you can use the formula to work out the length of the *hypotenuse*. And if you know the lengths of the *hypotenuse* and *one of the legs*, you can rearrange the formula and use it to work out the length of the *other leg*.

Lesson 3.3.3

Applications of the Pythagorean Theorem

California Standards:

Measurement and Geometry 3.3

Know and understand the **Pythagorean theorem** and its converse and use it to **find the length of the missing side of a right triangle and the lengths of other line segments** and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What it means for you:

You'll see how the Pythagorean theorem can be used to find lengths in more complicated shapes and in real-life situations.

Key words:

- Pythagorean theorem
- right triangle
- hypotenuse
- legs
- right angle

Check it out:

When you use the Pythagorean theorem, it's important to remember that the longest side of the right triangle is the hypotenuse. Your first step in any problem involving the Pythagorean theorem should be to decide which side is the hypotenuse.

In the last two Lessons you've seen what the *Pythagorean theorem* is, and how you can use it to find *missing side lengths in right triangles*. Now you'll see how it can be used to help find missing lengths in *other shapes* too — by breaking them up into right triangles. It can help solve *real-life measurement problems* too.

Use the Pythagorean Theorem in Other Shapes Too

You can use the **Pythagorean theorem** to find lengths in lots of **shapes** — you just have to **split** them up into **right triangles**.

Here's a reminder of the formula.

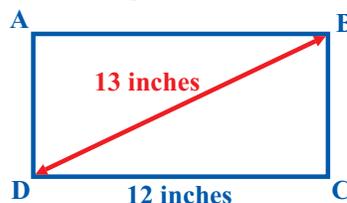
$$c^2 = a^2 + b^2$$

Which rearranges to: $a^2 = c^2 - b^2$

(c is the hypotenuse length, and a and b are the leg lengths.)

Example 1

Find the area of rectangle ABCD, shown below.



Solution

The formula for the area of a rectangle is **Area = length × width**.

You know that the **length** of the rectangle is **12 inches**, but you don't know the rectangle's **width, BD**.

But you do know the length of the **diagonal BC** and since all the corners of a rectangle are 90° angles, you know that **BCD** is a **right triangle**.

You can use the **Pythagorean theorem** to find the length of side **BC**.

$$BC^2 = BD^2 - CD^2$$

$$BC^2 = 13^2 - 12^2$$

$$BC^2 = 169 - 144$$

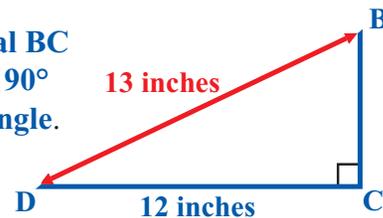
$$BC^2 = 25$$

$$BC = \sqrt{25} = 5 \text{ inches}$$

BC is the width of the rectangle. Now you can find its area.

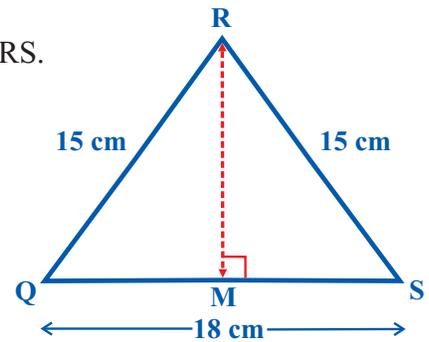
$$\text{Area} = \text{length} \times \text{width}$$

$$= 12 \text{ inches} \times 5 \text{ inches} = 60 \text{ inches}^2$$



Example 2

Find the area of isosceles triangle QRS.



Don't forget:

Isosceles triangles have two sides of equal length, and two angles of equal size.

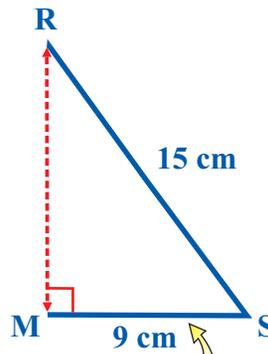
Solution

The formula for the area of a triangle is $\text{Area} = \frac{1}{2} \text{base} \times \text{height}$.

The **base** of the triangle is **18 cm**, but you don't know its **height**, **MR**.

Isosceles triangles can be split up into two right triangles.

Here's one:



You can use the **Pythagorean theorem** to find the length of side **MR**.

$$MR^2 = RS^2 - MS^2$$

$$MR^2 = 15^2 - 9^2$$

$$MR^2 = 225 - 81$$

$$MR^2 = 144$$

$$MR = \sqrt{144} = 12 \text{ cm}$$

This is half the base of the original triangle.

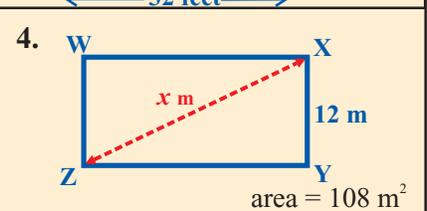
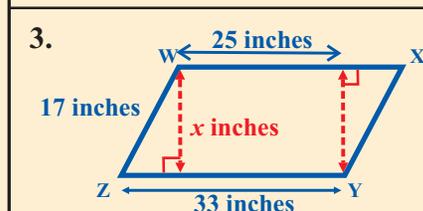
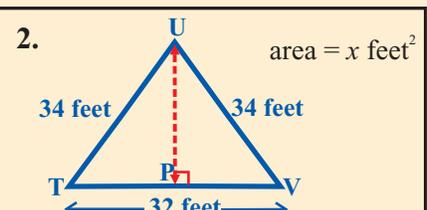
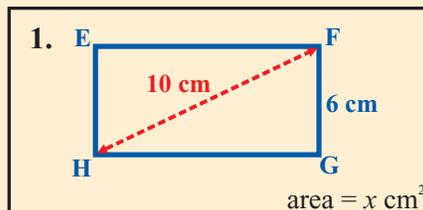
Now put the value of MR into the area formula:

$$\text{Area} = \frac{1}{2} \text{base} \times \text{height}$$

$$= \frac{1}{2} (18 \text{ cm}) \times 12 \text{ cm} = 9 \text{ cm} \times 12 \text{ cm} = 108 \text{ cm}^2$$

Guided Practice

In Exercises 1–4 use the Pythagorean theorem to find the missing value, x .



Don't forget:

If you need a reminder of the area formulas of any of the shapes in this Lesson, see Section 3.1.

The Pythagorean Theorem Has Real-Life Applications

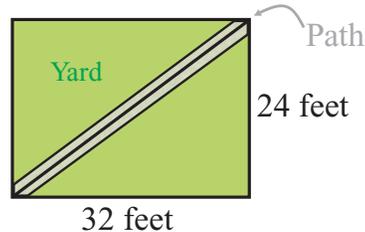
Because you can use the Pythagorean theorem to find lengths in many different shapes, it can be useful in lots of real-life situations too.

Example 3

Monique's yard is a rectangle 24 feet long by 32 feet wide. She is laying a diagonal gravel path from one corner to the other. One sack of gravel will cover a 10-foot stretch of path. How many sacks will she need?

Solution

The first thing you need to work out is the length of the path. It's a good idea to draw a diagram to help sort out the information.



You can see from the diagram that the path is the hypotenuse of a right triangle. So you can use the Pythagorean theorem to work out its length.

$$c^2 = a^2 + b^2$$

$$c^2 = 32^2 + 24^2$$

$$c^2 = 1024 + 576$$

$$c^2 = 1600$$

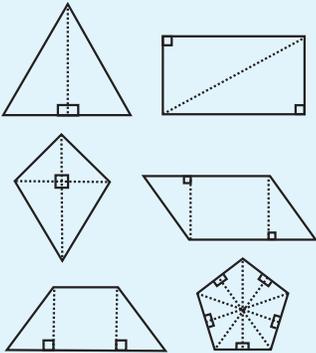
$$c = \sqrt{1600} = 40 \text{ feet}$$

The question tells you that one sack of gravel will cover a 10-foot length of path. To work out how many are needed, divide the path length by 10.

$$\text{Sacks needed} = 40 \div 10 = 4 \text{ sacks}$$

Check it out:

Right triangles are found in many different shapes. You can use the Pythagorean theorem on any shape that contains right triangles.



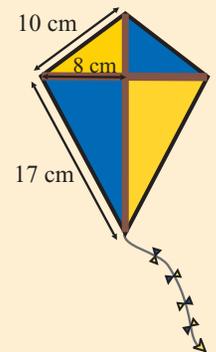
Guided Practice

5. Rob is washing his upstairs windows. He puts a straight ladder up against the wall. The top of the ladder is 8 m up the wall. The bottom of the ladder is 6 m out from the wall. How long is the ladder?

6. To get to Gabriela's house, Sam walks 0.5 miles south and 1.2 miles east around the edge of a park. How much shorter would his walk be if he walked in a straight line across the park?

7. The diagonal of Akil's square tablecloth is 4 feet long. What is the area of the tablecloth?

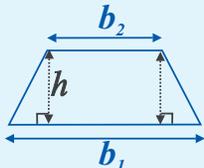
8. Megan is making the kite shown in the diagram on the right. The crosspieces are made of thin cane. What length of cane will she need in total?



Independent Practice

In Exercises 1–4 use the Pythagorean theorem to find the missing value, x .

Don't forget:



The formula for the area of a trapezoid is:

$$A = \frac{1}{2}h(b_1 + b_2)$$

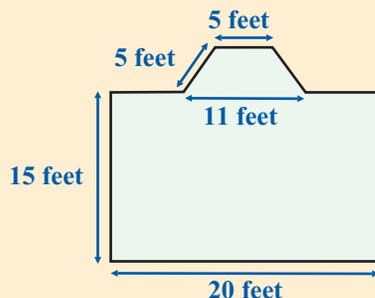
Now try these:

Lesson 3.3.3 additional questions — p446

<p>1. area = $x \text{ cm}^2$</p>	<p>2. area = 300 in^2</p>
<p>3. $x \text{ inches}$</p>	<p>4. $x \text{ feet}$</p>

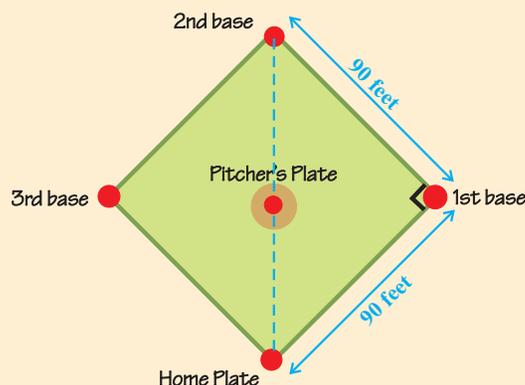
5. A local radio station is getting a new radio mast that is 360 m tall. It has guy wires attached to the top to hold it steady. Each wire is 450 m long. Given that the mast is to be put on flat ground, how far out from the base of the mast will the wires need to be anchored?

6. Luis is going to paint the end wall of his attic room, which is an isosceles triangle. The attic is 7 m tall, and the length of each sloping part of the roof is 15 m. One can of paint covers a wall area of 20 m^2 . How many cans should he buy?



7. Maria is carpeting her living room, shown in the diagram on the left. It is rectangular, but has a bay window. She has taken the measurements shown on the diagram. What area of carpet will she need?

8. The diagram on the right shows a baseball diamond. The catcher throws a ball from home plate to second base. What distance does the ball travel?



Round Up

You can break up a lot of shapes into right triangles. This means you can use the Pythagorean theorem to find the missing lengths of sides in many different shapes — it just takes practice to be able to spot the right triangles.

Lesson 3.3.4

Pythagorean Triples & the Converse of the Theorem

California Standards:

Measurement and Geometry 3.3

Know and understand the **Pythagorean theorem and its converse** and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What it means for you:

You'll learn about the groups of whole numbers that make the Pythagorean theorem true, and how to use the converse of the theorem to find out if a triangle is a right triangle.

Key words:

- Pythagorean theorem
- Pythagorean triple
- converse
- right triangle
- acute
- obtuse

Don't forget:

The longest side of a right triangle is always the hypotenuse. The other two sides are the legs.

Don't forget:

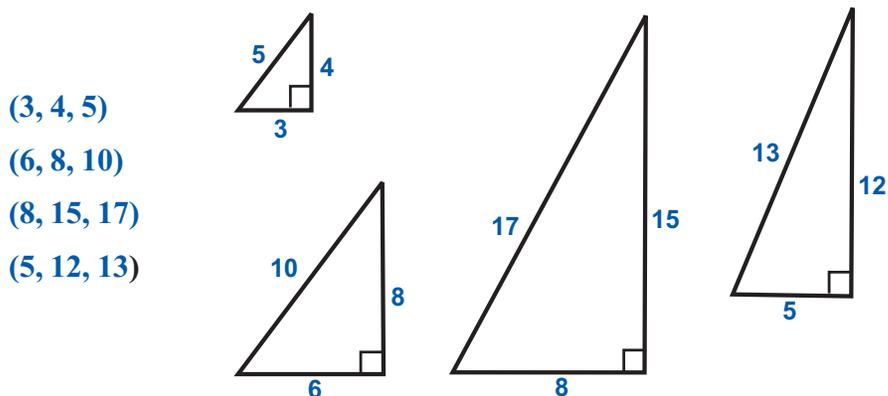
Lengths are always positive, so you can't have a negative integer in a Pythagorean triple. Positive integers can also be called **whole numbers**.

Up to now you've been using the Pythagorean theorem on triangles that you've been told are **right triangles**. But if you don't know for sure whether a triangle is a right triangle, you can use the theorem to decide. It's kind of like using the theorem backwards — and it's called using the **converse of the theorem**.

Pythagorean Triples are All Whole Numbers

You can draw a **right triangle** with any length **legs** you like, so the list of **side lengths** that can make the equation $c^2 = a^2 + b^2$ true never ends. Most sets of side lengths that fit the equation include at least one **decimal** — that's because finding the length of the hypotenuse using the equation involves taking a **square root**.

There are some sets of side lengths that are **all integers** — these are called **Pythagorean triples**. You've seen a lot of these already. For example:



You can find more Pythagorean triples by **multiplying** each of the numbers in a triple by the same number. For example:

$$\begin{array}{l} (3, 4, 5) \xrightarrow{\times 2} (6, 8, 10) \\ (3, 4, 5) \xrightarrow{\times 3} (9, 12, 15) \\ (3, 4, 5) \xrightarrow{\times 4} (12, 16, 20) \end{array} \left. \vphantom{\begin{array}{l} (3, 4, 5) \\ (3, 4, 5) \\ (3, 4, 5) \end{array}} \right\} \text{These are all} \\ \text{Pythagorean triples.}$$

To test if three integers are a **Pythagorean triple**, put them into the equation $c^2 = a^2 + b^2$, where c is the biggest of the numbers.

If they make the equation **true**, they're a **Pythagorean triple**.

If they **don't**, they're **not**.

Don't forget:

The greatest number is substituted for c .

Example 1

Are the numbers (72, 96, 120) a Pythagorean triple?

Solution

To see if the numbers are a Pythagorean triple, put them into the equation.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 120^2 &= 72^2 + 96^2 \\ 14,400 &= 5184 + 9216 \\ 14,400 &= 14,400 \qquad \text{— so which is true} \end{aligned}$$

These numbers are a Pythagorean triple.

✓ Guided Practice

Are the sets of numbers in Exercises 1–6 Pythagorean triples or not? If they are not, give a reason why not.

- | | |
|-----------------------|------------------|
| 1. 5, 12, 13 | 2. 0.7, 0.9, 1.4 |
| 3. 8, 8, $\sqrt{128}$ | 4. 25, 60, 65 |
| 5. 6, 9, 12 | 6. 18, 80, 82 |

The Converse of the Pythagorean Theorem

The **Pythagorean theorem** says that the **side lengths** of any **right triangle** will satisfy the equation $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the leg lengths.

You can also say the **opposite** — if a triangle's **side lengths** satisfy the equation, it is a **right triangle**. This is called the **converse** of the theorem:

The converse of the Pythagorean theorem:

If the side lengths of a triangle a , b , and c , where c is the largest, satisfy the equation $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Check it out:

The converse of a theorem or statement is formed by changing it around.

For example:

If the statement is, "if a shape is a square, then it has four equal-length sides,"

then its converse is, "if a shape has four equal length sides, then it is a square."

The converse of a true statement isn't always true — this one isn't. A rhombus is a shape with four equal-length sides — but it's not a square.

Example 2

A triangle has side lengths 2.5 cm, 6 cm, and 6.5 cm. Is it a right triangle?

Solution

Put the side lengths into the equation $c^2 = a^2 + b^2$, and evaluate both sides.

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{The longest side of a right triangle} \\ 6.5^2 &= 2.5^2 + 6^2 && \text{is the hypotenuse, } c. \\ 42.25 &= 6.25 + 36 \\ 42.25 &= 42.25 && \text{— which is true} \end{aligned}$$

It is a right triangle.

Don't forget:

A right triangle has one 90° angle.



An obtuse triangle has one angle between 90° and 180° .



An acute triangle has all three angles under 90° .



Don't forget:

When you evaluate c^2 and $a^2 + b^2$, remember that c is the longest side length — the hypotenuse is always the longest side.

Test Whether a Triangle is Acute or Obtuse

If a triangle isn't a right triangle, it must either be an **acute triangle** or an **obtuse triangle**. By seeing if c^2 is greater than or less than $a^2 + b^2$, you can tell what type of triangle it is.

If $c^2 > a^2 + b^2$ then the triangle is obtuse.

If $c^2 < a^2 + b^2$ then the triangle is acute.

Example 3

A triangle has side lengths of 2 ft, 2.5 ft, and 3 ft. Is it right, acute, or obtuse?

Solution

Check whether $c^2 = a^2 + b^2$, with $c = 3$, $a = 2$ and $b = 2.5$.

$$c^2 = 3^2 = 9 \quad \text{and}$$

$$a^2 + b^2 = 2^2 + 2.5^2 = 4 + 6.25 = 10.25$$

$$9 < 10.25 \quad c^2 < a^2 + b^2, \text{ so this is an acute triangle.}$$

✓ Guided Practice

Are the side lengths in Exercises 7–12 of right, acute, or obtuse triangles?

7. 50, 120, 130

8. 8, 9, 10

9. 3, 4, 6

10. 12, 6, $\sqrt{180}$

11. 0.25, 0.3, 0.5

12. 0.5, 0.52, 0.55

✓ Independent Practice

1. "Every set of numbers that satisfies the equation $c^2 = a^2 + b^2$ is a Pythagorean triple." Explain if this statement is true or not.

Say if the side lengths in Exercises 2–7 are Pythagorean triples or not.

2. 8, 15, 17

3. 1, 1, $\sqrt{2}$

4. 0.3, 0.4, 0.5

5. 300, 400, 500

6. 12, 29, 40

7. 15, 36, 39

8. In triangle ABC, side AB is longest.

If $AB^2 > AC^2 + BC^2$ then what kind of triangle is ABC?

Are the following side lengths those of right, acute, or obtuse triangles?

9. 5, 10, 14

10. 10, 11, 12

11. 12, 16, 20

12. 5.1, 5.3, 9.3

13. 2.4, 4.5, 5.1

14. 3.7, 4.7, 5.7

15. Justin is going to fit a new door. He measures the width of the door frame as 105 cm, the height as 200 cm, and the diagonal of the frame as 232 cm. Is the door frame perfectly rectangular?

Now try these:

Lesson 3.3.4 additional questions — p447

Round Up

The converse of the *Pythagorean theorem* is a kind of "backward" version. You can use it to prove whether a triangle is a right triangle or not — and if it's not, you can say if it's *acute* or *obtuse*.

Section 3.4 introduction — an exploration into: Transforming Shapes

Geometric figures, like triangles, rectangles and so on, can be *plotted* on the coordinate plane. The purpose of this Exploration is to *predict and discover* what changes will occur when the *coordinates* of a figure are *changed* in a particular way.

If you change the coordinates of figure, ABC, you normally label the changed figure **A'B'C'**.

Example

Plot triangle ABC on a coordinate plane. A(-1, 1), B(1, 5), C(2, 3).

Add 5 to each *x*-value of the coordinates, and plot the new triangle A'B'C'.

Describe the change.

Solution

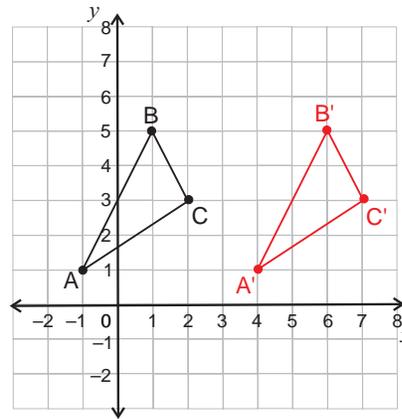
The new coordinates are

$$A'(-1 + 5, 1) = A'(4, 1)$$

$$B'(1 + 5, 5) = B'(6, 5)$$

$$C'(2 + 5, 3) = C'(7, 3)$$

The shape has moved 5 units to the right.



Exercises

1. Draw a coordinate plane. Your *x* and *y* axes should both go from -8 to +8. Plot triangle PQR — P(-4, 2), Q(-1, 4), R(4, 3).
2. Predict how triangle PQR will change if 3 is added to the *y*-values of each coordinate pair. Then test your prediction by performing the change.
3. Draw a coordinate plane. Your *x* and *y* axes should both go from -6 to +6. Plot trapezoid EFGH on the coordinate plane. E(-3, -2), F(5, -2), G(2, -5), H(-1, -5).
4. Predict how trapezoid EFGH will change if the *y*-values are changed from negative to positive. Then test your prediction by performing the change.
5. Draw a coordinate plane. Your *x* and *y* axes should both go from -8 to +8. Plot rectangle RSTU — R(1, -2), S(7, -2), T(7, -4), U(1, -4).
6. Predict how rectangle RSTU will change if the signs of the *x*-values are changed to the opposite sign. Then test your prediction by performing the change.
7. What was the effect on the size and shape of all of the figures after the changes were made to the coordinates?

Round Up

You've looked at *two* types of *transformation* in this Exploration — *translations* and *reflections*. In *translations*, the shape is *slid* across the grid. In *reflections*, it's "*flipped*" over.

Lesson 3.4.1

Section 3.4 Reflections

California Standards:

Measurement and Geometry 3.2

Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

What it means for you:

You'll learn what it means to reflect a shape. You'll also see how to draw and describe reflections.

Key words:

- reflection
- image
- flip
- prime
- coordinates
- x -axis/ y -axis

Check it out:

A' is read as "A prime."

The next few Lessons are about *transformations*. A transformation is a way of *changing a shape*. For example, it could be *flipping, stretching, moving, enlarging, or shrinking the shape*.

The first type of transformation you're going to meet is *reflection*.

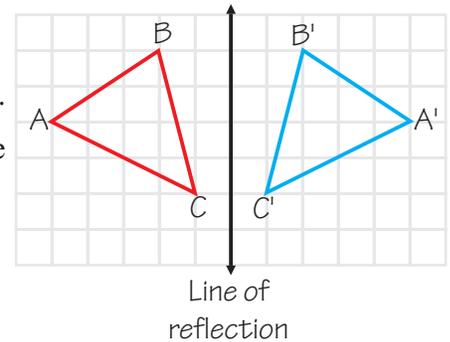
A Reflection Flips a Figure Across a Line

A **reflection** takes a shape and makes a **mirror image** of it on the other side of a given line.

Here triangle **ABC** has been **reflected** or "flipped" across the line of reflection.

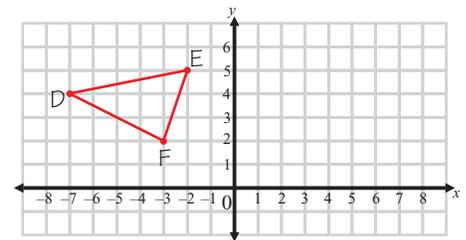
The reflections of points A, B, and C are labeled **A'**, **B'**, and **C'**.

The whole reflected triangle A'B'C' is called the **image** of ABC.



Example 1

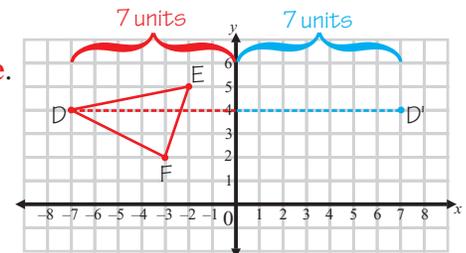
Reflect triangle DEF across the y -axis.



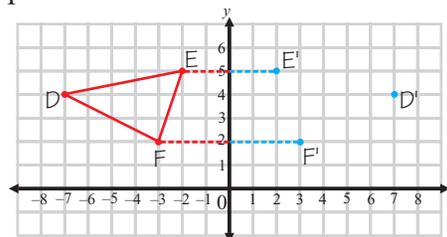
Solution

Step 1: Pick a point to reflect. Point **D** is **7 units** away from the y -axis.

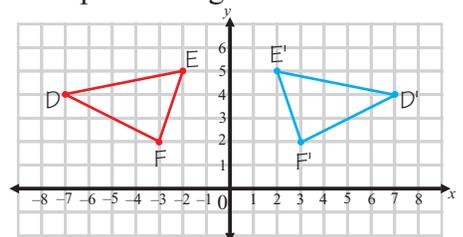
Move **across** the y -axis and find the point **7 units** away on the **other side**. This is where you plot the point **D'**.



Step 2: Repeat step 1 for points **E** and **F**.



Step 3: **Join** the points to complete triangle **D'E'F'**.

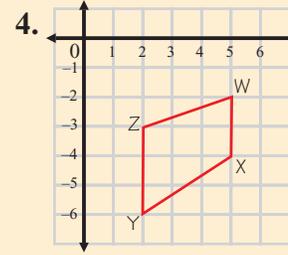
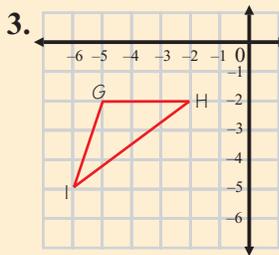
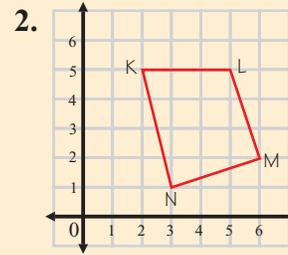
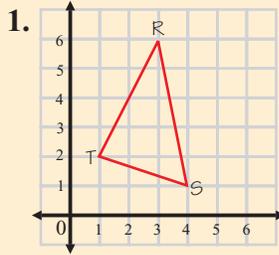


✓ Guided Practice

In Exercises 1–4, copy each shape onto a set of axes, then draw its reflections across the y -axis and the x -axis. Draw a new pair of axes for each Exercise, ranging from -6 to 6 in both directions.

Check it out:

Suppose you're drawing more than one image of a shape called ABC. The first image should be called A'B'C', the second is A''B''C'', the third is A'''B'''C''', and so on.



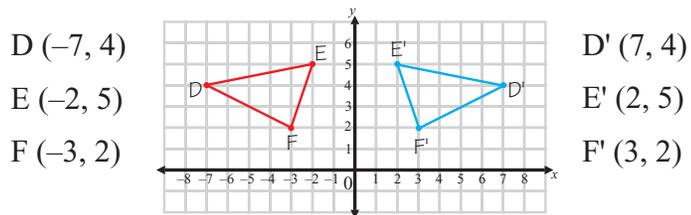
Reflections Change Coordinate Signs

A reflection across the x -axis changes (x, y) to $(x, -y)$.

A reflection across the y -axis changes (x, y) to $(-x, y)$.

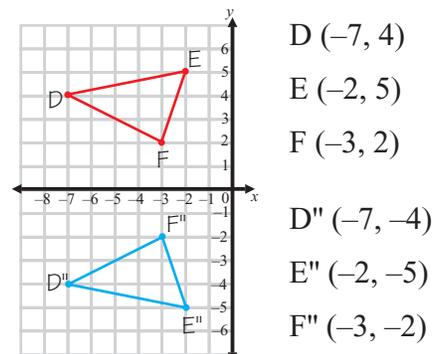
To see this, look again at the reflection from Example 1.

The **coordinates** of the corners of the triangles are shown below.



When DEF is reflected across the y -axis, the y -coordinate **stays the same** and the x -coordinate **changes** from negative to positive.

If you reflect DEF across the x -axis, the x -coordinate **stays the same** and the y -coordinate **changes** from positive to negative.



✓ Guided Practice

In Exercises 5–8, give the coordinates of the image produced.

5. A: (5, 2), (4, 7), (6, 1). Triangle A is reflected over the x -axis.
6. B: (9, 9), (−4, 8), (−2, 6). Triangle B is reflected over the y -axis.
7. C: (−2, 10), (2, 10), (5, 5), (0, −3), (−5, 5).
Pentagon C is reflected over the x -axis.
8. Pentagon C from Exercise 7 is reflected over the y -axis.

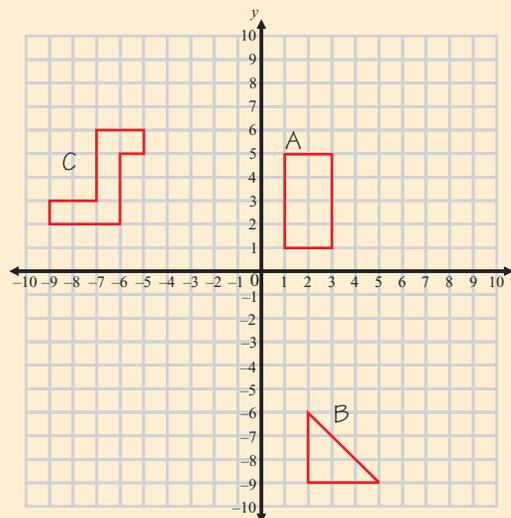
Exercises 9–11 give the coordinates of the corners of a figure and its reflected image. Describe each reflection in words.

9. D: (5, 2), (6, 3), (8, 1), (4, 1); D' (5, −2), (6, −3), (8, −1), (4, −1)
10. E: (−6, −1), (−3, −6), (−9, −4); E' (6, −1), (3, −6), (9, −4)
11. F: (0, 0), (0, 5), (3, 3); F' (0, 0), (0, 5), (−3, 3)

✓ Independent Practice

Copy the grid and figures shown below, then draw the reflections described in Exercises 1–6.

1. Reflect A across the x -axis. Label the image A'.
2. Reflect A across the y -axis. Label the image A''.
3. Reflect B across the x -axis. Label the image B'.
4. Reflect B across the y -axis. Label the image B''.
5. Reflect C across the x -axis. Label the image C'.
6. Reflect C across the y -axis. Label the image C''.



Now try these:

Lesson 3.4.1 additional questions — p447

In Exercises 7–9, copy the figures onto graph paper and reflect each one over the line of reflection shown.

- 7.
- 8.
- 9.

Round Up

Don't forget that a **reflection** makes a **back-to-front** image — like the image you see when you look in a mirror. Unless the original is **symmetrical**, the image **shouldn't** be the same way around as the original. If it is the same way around, that's a **translation**, not a reflection. You'll learn about translations in the next Lesson.

Lesson 3.4.2

Translations

California Standards:

Measurement and
Geometry 3.2

Understand and use
coordinate graphs to plot
simple figures, determine
lengths and areas related to
them, and determine their
image under translations
and reflections.

What it means for you:

You'll see how to draw and
describe translations of
shapes.

Key words:

- translation
- slide
- image
- coordinates

Don't forget:

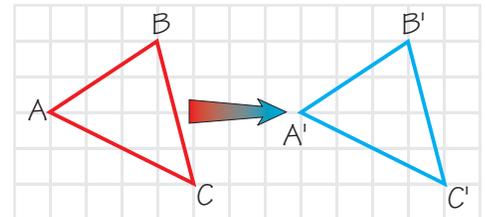
The image of point A is the
point A' (A prime).

A **translation** is another type of transformation. When you translate a shape, you slide it around. You can translate a shape up, down, left, or right, or any combination of these.

A Translation Slides a Figure

A **translation** takes a shape and **slides** every point of that shape a **fixed distance** in the same **direction**.

The image is the **same size** and **shape**, and the **same way around** as the original figure.



Example 1

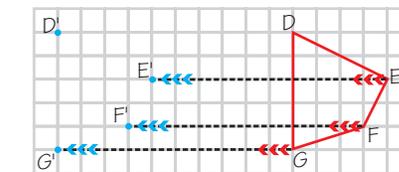
Translate DEFG 10 units to the left.

Solution

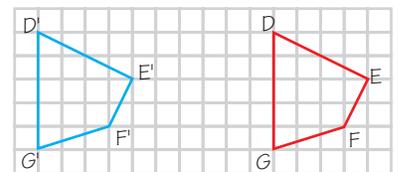
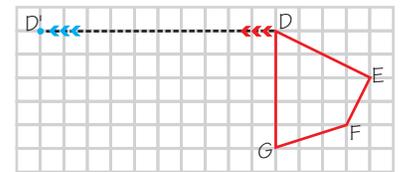
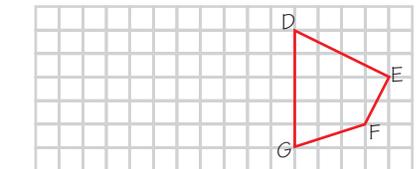
Step 1: Pick a point to translate — we'll start with point **D**.

Move **across** the grid and find the point **10 units** to the **left** of point **D**. This is where you plot the point **D'**.

Step 2: Repeat step 1 for points **E**, **F**, and **G**.



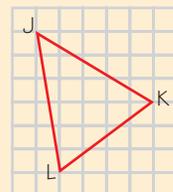
Step 3: **Join** the points to complete **D'E'F'G'**.



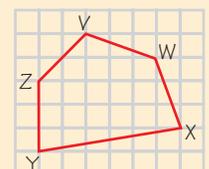
Guided Practice

In Exercises 1–4, copy each shape onto graph paper. Remember to leave enough space to draw the translations.

1. Translate JKL up 7 units.
2. Translate JKL left 8 units and down 1 unit.



3. Translate VWXYZ left 9 units.



4. Translate VWXYZ down 3 units and right 7 units.

You Can Describe Translations with Coordinates

When you **translate** a shape, the **coordinates** of every point change by the **same amount**. So you can use coordinates to **describe** the translation.

Example 2

Translate the triangle LMN using the translation:

$$(x, y) \rightarrow (x + 4, y - 3)$$

Solution

The question tells you how to change the coordinates of the points of LMN.

Start by finding the coordinates of L, M, and N:

$$L = (1, 5)$$

$$M = (4, 5)$$

$$N = (3, 1)$$

Now you can apply the transformation:

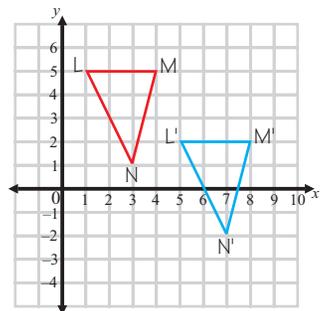
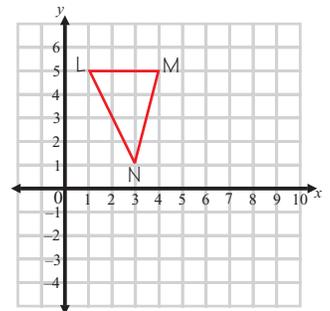
$$(x, y) \rightarrow (x + 4, y - 3)$$

$$L(1, 5) \rightarrow L'(1 + 4, 5 - 3) = (5, 2)$$

$$M(4, 5) \rightarrow M'(4 + 4, 5 - 3) = (8, 2)$$

$$N(3, 1) \rightarrow N'(3 + 4, 1 - 3) = (7, -2)$$

Once you've figured out the coordinates of the image L'M'N', you can draw it on the coordinate grid.



Guided Practice

In Exercises 5–8, copy the shapes and axes shown onto graph paper.

Apply the following translations to triangle PQR:

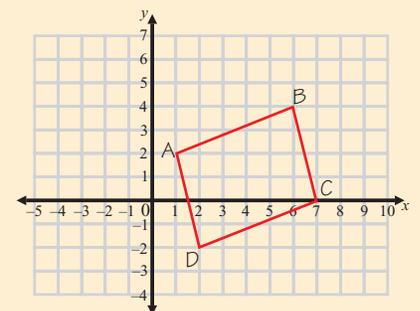
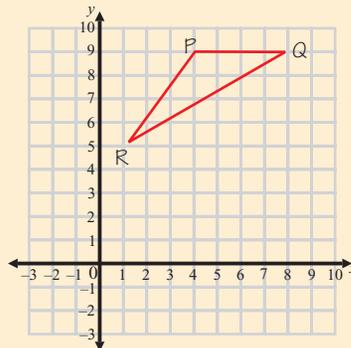
5. $(x, y) \rightarrow (x + 2, y - 4)$

6. $(x, y) \rightarrow (x - 3, y - 6)$

Apply the following translations to the quadrilateral ABCD:

7. $(x, y) \rightarrow (x - 4, y)$

8. $(x, y) \rightarrow (x + 2, y + 2)$



Check it out:

A lot of people prefer to use coordinates to do translations. If you use the method of counting how many squares to move, it's easy to miscount and put a point in the wrong place.

Find the Translation by Looking at the Coordinates

When you look at a shape and its translated **image**, you can figure out what **translation** was used to make the image.

Example 3

Describe the translation from QRST to Q'R'S'T' in coordinate notation.

Solution

Find the coordinates of Q and Q'.

$$Q = (-8, 2) \quad Q' = (-2, 5)$$

The **x-coordinate** has changed from -8 to -2 .
This is an **increase of 6**.

The **y-coordinate** has changed from 2 to 5 , so it has **increased by 3**.

So the translation is $(x, y) \rightarrow (x + 6, y + 3)$

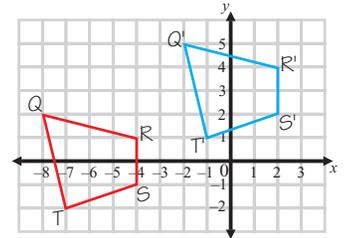
You can check that this is the right answer by seeing if this translation changes R, S, and T to R', S', and T'.

$$R(-4, 1) \rightarrow (-4 + 6, 1 + 3) = (2, 4) = R'$$

$$S(-4, -1) \rightarrow (-4 + 6, -1 + 3) = (2, 2) = S'$$

$$T(-7, -2) \rightarrow (-7 + 6, -2 + 3) = (-1, 1) = T'$$

It does — so you have the right answer.



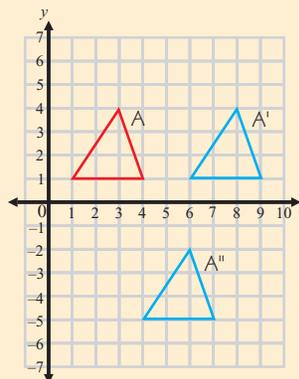
✓ Guided Practice

In Exercises 9–14, describe the following translations in coordinates.

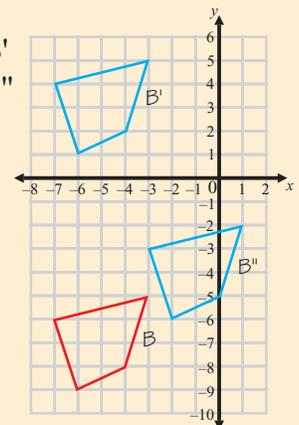
Check it out:

You don't just use the prime symbol to indicate the image of a single point. Sometimes it's used when the whole shape is named by a single letter, like in these Exercises.

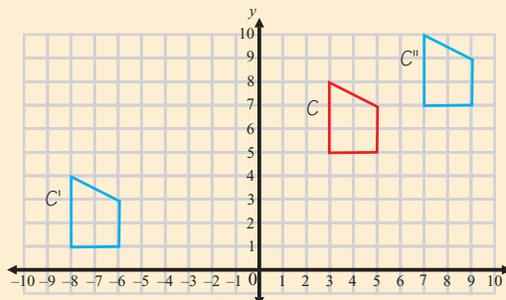
9. A to A'
10. A to A''



11. B to B'
12. B to B''



13. C to C'
14. C to C''



Independent Practice

Copy the shapes and axes shown onto graph paper for Exercises 1–6.

1. Translate K 5 units to the left. Label the image K'.

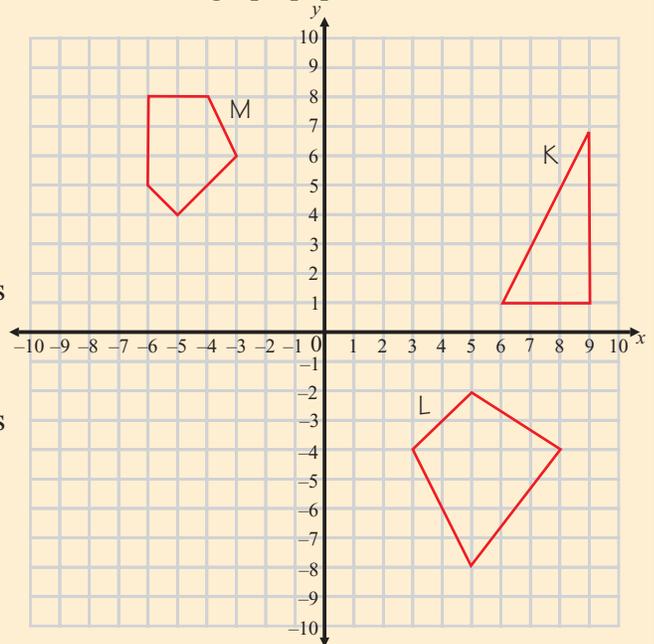
2. Translate K 7 units left and 7 units down. Label the image K''.

3. Translate L 12 units up. Label the image L'.

4. Translate L 13 units left and 2 units down. Label the image L''.

5. Translate M 1 unit up and 3 units right. Label the image M'.

6. Translate M 3 units left and 4 units down. Label the image M''.



Use coordinate notation to describe the following translations:

7. K to K'

8. K to K''

9. L to L'

10. L to L''

11. M to M'

12. M to M''

Now try these:

Lesson 3.4.2 additional questions — p447

Copy the axes and triangle shown onto graph paper for Exercises 13–16.

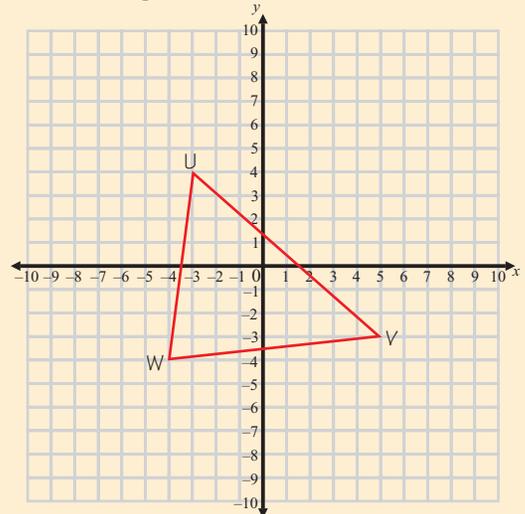
Apply the following translations to triangle UVW.

13. $(x, y) \rightarrow (x + 4, y + 5)$

14. $(x, y) \rightarrow (x - 2, y + 4)$

15. $(x, y) \rightarrow (x + 1, y - 6)$

16. $(x, y) \rightarrow (x - 5, y - 5)$



Round Up

Coordinates are really useful for *drawing* translations, and can help you *check* your answers.

But don't forget that one of the most important checks is to *look* at the *image* you've drawn and see if it looks the same as the original.

Lesson 3.4.3

Scale Factors

California Standards:

Measurement and
Geometry 1.2

Construct and read
drawings and models made
to scale.

What it means for you:

You'll learn how to use scale factors to produce images that have the same shape as another figure, but a different size.

Key words:

- scale factor
- image
- multiply

Check it out:

A scale factor of 1 leaves the shape exactly the same size.

In this Section so far you've seen two types of *transformation* — reflections and translations. These both give an *image* that's the *same size* as the original.

Another type of transformation changes the size of shapes. The *scale factor* tells you by *how much* the size changes.

A Scale Factor of More Than 1 Makes a Shape Bigger

Sometimes an image is **identical** to the original apart from its **size**. The **scale factor** tells you how much larger or smaller the image is.

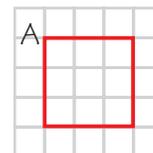
The scale factor is a **number**. You **multiply** all the lengths in the **original** by the scale factor to get the lengths in the **image**.

So:

$$\text{original length} \times \text{scale factor} = \text{image length}$$

Example 1

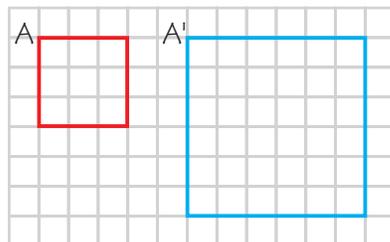
Draw an image of square A using a scale factor of 2.



Solution

The sides of A are 3 units long.

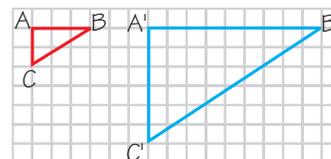
So if you apply a scale factor of 2, the sides of the image will be $3 \times 2 = 6$ units long.



So the image **A'** is a square with side length **6 units**.

Example 2

What scale factor has been used to enlarge ABC to A'B'C'?



Solution

The scale factor is given by dividing any length in the **image** by the corresponding length in the **original**.

$$\text{Length of } A'B' = 9$$

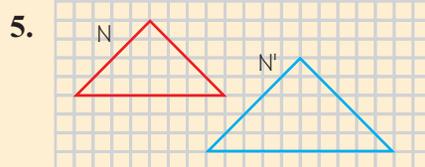
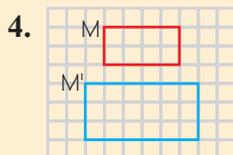
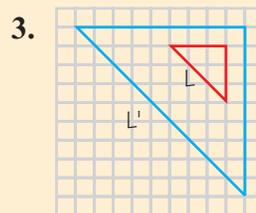
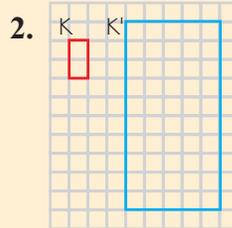
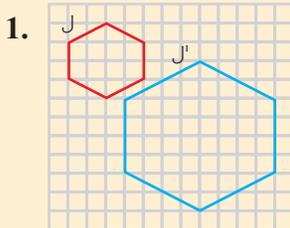
$$\text{Length of } AB = 3$$

So the scale factor is:

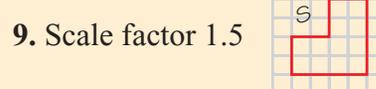
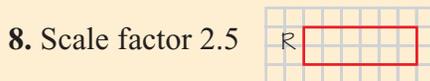
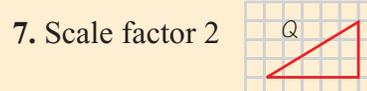
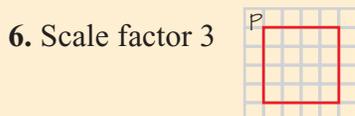
$$\text{Length of } A'B' \div \text{Length of } AB = 9 \div 3 = 3$$

✓ Guided Practice

In Exercises 1–5, find the scale factor that has produced each image.



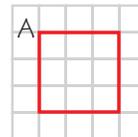
Copy the shapes shown in Exercises 6–9 onto graph paper. Draw the image produced by applying the given scale factor.



A Scale Factor of Less Than 1 Makes a Shape Smaller

Example 3

Draw an image of square A using a scale factor of $\frac{1}{3}$.



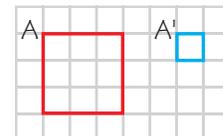
Solution

The sides of A are 3 units long.

So if you apply a scale factor of $\frac{1}{3}$, the sides of the image will be

$$3 \times \frac{1}{3} = 1 \text{ unit long.}$$

So the image A' is a square with side length **1 unit**.

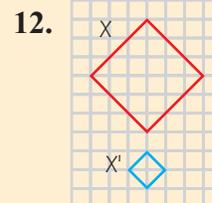
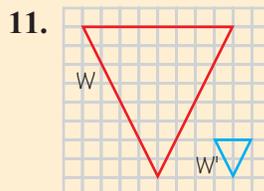
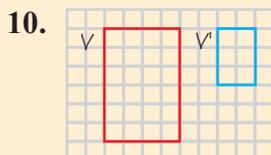


Check it out:

Decimals could also be used for scale factors. The same method applies — multiply the original dimension by the decimal scale factor to find the dimension of the image.

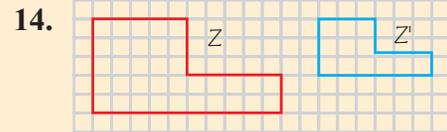
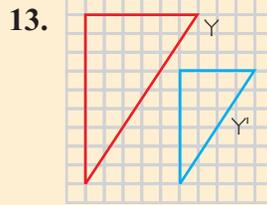
✓ Guided Practice

In Exercises 10–14, find what scale factor has produced each image.

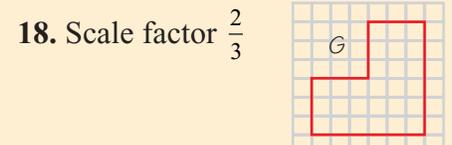
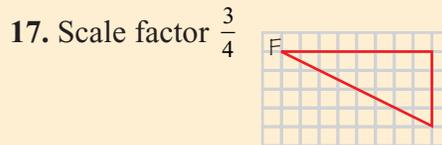
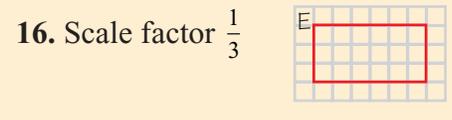
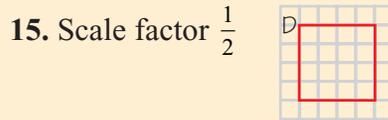


Don't forget:

Go back to Lesson 2.3.3 if you need a reminder on multiplying fractions by integers.

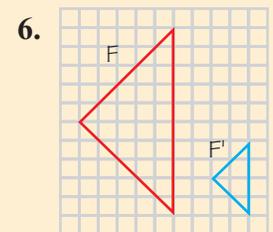
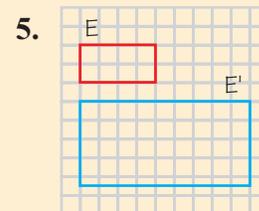
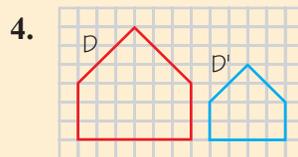
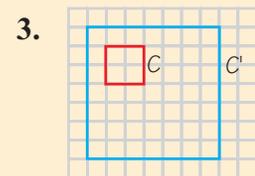
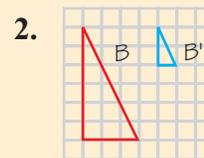
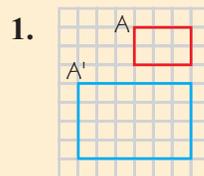


Copy the shapes shown in Exercises 15–18 onto graph paper. Draw the image produced by applying the given scale factor.

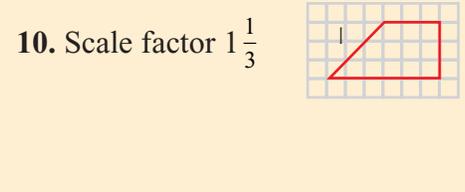
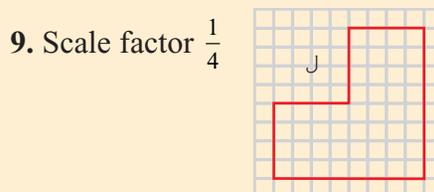
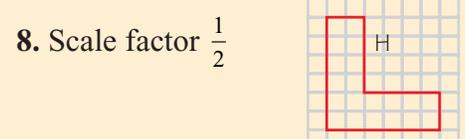
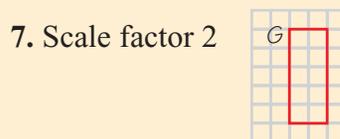


Independent Practice

In Exercises 1–6, find what scale factor has produced each image.



Copy the shapes shown in Exercises 7–10 onto graph paper. Draw the image produced by applying the given scale factor.



Now try these:

Lesson 3.4.3 additional questions — p448

Round Up

The *scale factor* tells you how much bigger or smaller than the original object an image is. You'll use scale factors to make and understand *scale drawings*, which you'll learn about in the next Lesson.

Lesson 3.4.4

Scale Drawings

California Standards:

Measurement and
Geometry 1.2

Construct and read
drawings and models made
to scale.

What it means for you:

You'll learn how to draw pictures that accurately represent real places or objects. You'll also use drawings to find information about the places or objects they represent.

Key words:

- scale drawing
- scale factor
- measurement
- distance
- scale

Don't forget:

Scales are often written as ratios. Ratios are a way of comparing two numbers — you should have learned about them in grade 6. You can also write a ratio as a fraction, so the scale can be

written as $\frac{1 \text{ inch}}{4 \text{ feet}}$.

The ratio of the length on the drawing to the real-life length must be equivalent to the scale ratio. That's why you can write the proportion as

$$\frac{1 \text{ inch}}{4 \text{ feet}} = \frac{x}{24 \text{ feet}}$$

Scale drawings often show *real objects or places* — maps are good examples of scale drawings. All the *measurements on the drawing are related to the real-life measurements by the same scale factor*.

So if you know the scale factor, you can *figure out what the real-life measurements are*.

To Make Scale Drawings You Need Real Measurements

To make a **scale drawing** of an object or place, you need two things.

First, you need the **real-life measurements** of what you're going to draw.

Second, you need a **scale**. This will tell you what the distances on the drawing **represent**. The scale is usually written as a **ratio**.

If **1 inch** on the drawing represents **10 feet** in real life, the scale is **1 inch : 10 feet**.

Example 1

A rectangular yard has a length of 24 feet and a width of 20 feet. Make a scale drawing using a scale of 1 inch : 4 feet.

Solution

You need to find the **length and width of the yard in the drawing**.

To convert the real-life **length** into a length for the drawing, set up a proportion using the scale given.

Let x be the length the yard in the drawing should be.

$$\frac{\text{Drawing length}}{\text{Real-life length}} = \frac{1 \text{ inch}}{4 \text{ feet}} = \frac{x}{24 \text{ feet}}$$

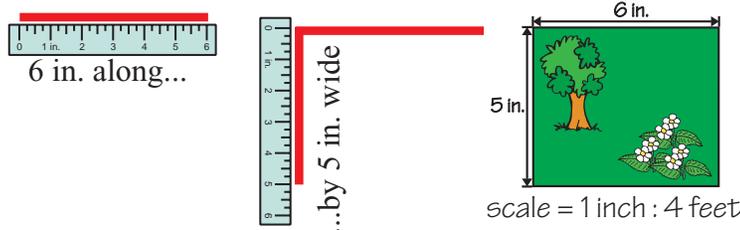
$$\frac{1 \text{ inch} \times 24 \text{ feet}}{4 \text{ feet}} = x \times \frac{\cancel{24 \text{ feet}}}{\cancel{24 \text{ feet}}} = x \quad \text{Multiply both sides by 24 ft}$$

$$\text{So, } x = 1 \text{ in.} \times \frac{24 \text{ feet}}{4 \text{ feet}} = 1 \text{ in.} \times 6 = \mathbf{6 \text{ in.}}$$

Repeat the process using y for the **width** of the drawing and you find:

$$y = 1 \text{ in.} \times \frac{20 \text{ feet}}{4 \text{ feet}} = 1 \text{ in.} \times 5 = \mathbf{5 \text{ in.}}$$

You can use these measurements to make a **scale drawing**.



✓ Guided Practice

In Exercises 1–4, make the following scale drawings:

1. A square of side length 4 m, using the scale 1 cm : 1 m.
2. A rectangle measuring 40 in. by 60 in., using the scale 1 in. : 20 in.
3. A rectangular room measuring 6 ft by 12 ft, using the scale 1 in. : 3 ft.
4. A circular pond with diameter 3 m, using the scale 1 cm : 2 m.

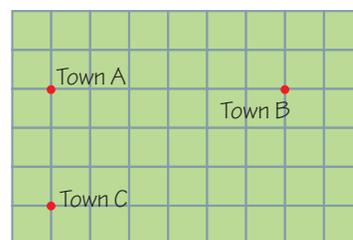
You Can Use Scale Drawings to Find Actual Lengths

The **size** of real objects can be found by **measuring** scale drawings.

Example 2

This map shows three towns.
Find the real-life distances between:

- Town A and Town B
- Town A and Town C



Scale — 1 grid square : 2.5 miles

Solution

The distance between Town A and Town B on the map is **6 grid squares**.

The scale tells us that 1 grid square represents 2.5 miles, so the distance between Town A and Town B is 6×2.5 miles = **15 miles**.

Town A and Town C are **3 grid squares** apart on the map.
In real life this is equal to 3×2.5 miles = **7.5 miles**.

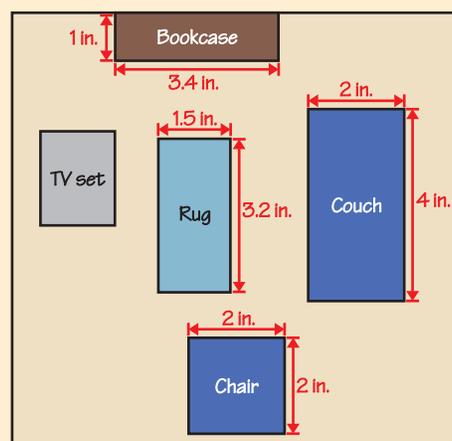
✓ Guided Practice

This picture shows a scale drawing of the living room in Lashona's house.

The scale used is 2 in. : 3 feet.

In Exercises 5–8, find the real-life measurements of:

5. The chair
6. The couch
7. The bookcase
8. The rug



Check it out:

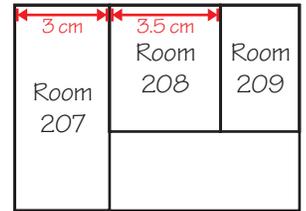
When you're buying new furniture, such as kitchen units, you might use a scale drawing of the room to decide where you want the furniture (or which furniture would fit).

You Can Sometimes Find Real Lengths Without a Scale

If you know **one** of the real-life lengths shown on a scale drawing, then you can **figure out** the others without a scale.

Example 3

This scale drawing shows three classrooms at Gabriel's school. Gabriel measures the drawing. His measurements are shown in red.



Gabriel knows Room 207 is 4.2 m wide in real life. What is the real-life width of Room 208?

Solution

You can find the answer by **setting up a proportion**, similar to the one in Example 1. Use x for the real-life width of Room 208.

$$\frac{\text{Real-life width}}{\text{Drawing width}} = \frac{4.2 \text{ m}}{3 \text{ cm}} = \frac{x}{3.5 \text{ cm}}$$

$$x = 4.2 \text{ m} \times \frac{3.5 \text{ cm}}{3 \text{ cm}} = 4.9 \text{ m}$$

So the width of Room 208 is 4.9 m in real life.

Check it out:

Another way of doing this is to find the scale factor that's been used and use it to find the other real-life lengths, as you did before.

The scale factor is the ratio between the real-life length and the drawing length.

So in Example 3, the scale factor would be $4.2 \text{ m} \div 3 \text{ cm}$, which is 1.4 m/cm .

To find the real-life width of room 208, you'd just multiply the drawing width by the scale factor — $3.5 \text{ cm} \times 1.4 \text{ m/cm} = 4.9 \text{ m}$. This gives the same answer as the other method.

✓ Guided Practice

Use the map below to answer Exercises 9–14.



It is 18 miles from Town D to Town E. Calculate the distance from:

- | | |
|----------------------|----------------------|
| 9. Town D to Town F | 10. Town F to Town G |
| 11. Town G to Town J | 12. Town H to Town J |
| 13. Town D to Town G | |

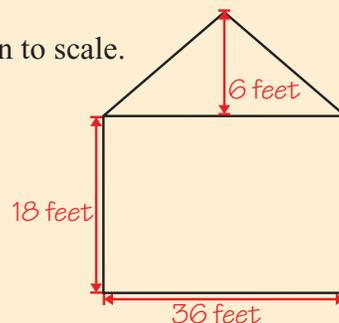
14. Find the number that completes the following sentence:
The scale on this map is 1 grid square : ____ miles.

Independent Practice

1. A sail for a boat is in the shape of a right triangle. The actual height of the sail is 18 feet, and it has a base of 12 feet. Make a scale drawing of the sail using a scale of 1 cm : 3 ft.

2. This sketch of a house has not been drawn to scale.

Make a scale drawing of the house using a scale of 1 cm : 6 ft.

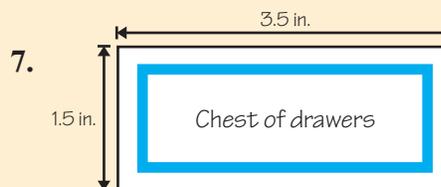
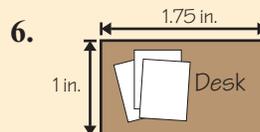
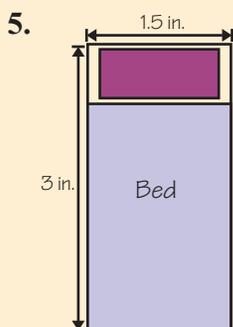


3. A scale model of a town uses a scale of 1 inch : 30 feet. Find the actual height of a building that is 2.5 in. tall in the model.

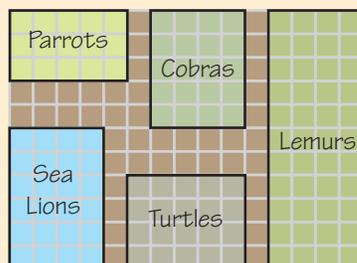
4. On a map, 2 inches represents 45 miles. What does one inch represent on this map?

Amanda is drawing a plan of her bedroom using a scale of 1 in. : 2 ft. Exercises 5–7 show objects from the plan.

Calculate the real-life dimensions of the objects.



The scale drawing below shows part of a zoo. The parrot enclosure measures 40 m by 24 m. Find the following real-life measurements:



8. The length and width of the sea lion enclosure.

9. The length and width of the lemur enclosure.

10. The perimeter of the turtle enclosure.

11. The area of the cobra enclosure.

Now try these:

Lesson 3.4.4 additional questions — p448

Round Up

Pictures that are drawn to scale can be very useful. If maps weren't made to scale, they would be much harder to use. And if plans and blueprints for buildings or machines weren't done as scale drawings, it would be difficult to build them the right size and shape.

Lesson 3.4.5

Perimeter, Area, and Scale

California Standards:

Measurement and Geometry 1.2

Construct and read drawings and models made to scale.

Measurement and Geometry 2.0

Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale.

What it means for you:

You'll see the effect that multiplying by a scale factor has on perimeter and area.

Key words:

- perimeter
- area
- scale factor
- image

Don't forget:

The formula for the perimeter of a rectangle is $P = 2(l + w)$. For more about perimeter see Section 3.1.

Check it out:

You'll need to use the Pythagorean theorem to help you find the perimeter of these triangles. See Section 3.3 for more.

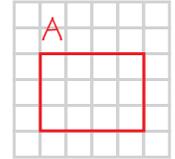
So far you've been looking at how length and width are altered by applying a scale factor. In this Lesson, you're going to see how applying scale factors affects perimeter and area.

Applying a Scale Factor Changes the Perimeter

When you change the size of a shape, the perimeter changes too.

Example 1

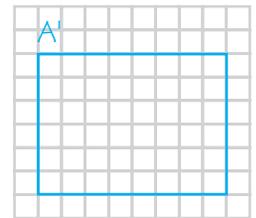
Gilberto draws an image of rectangle A using a scale factor of 2. Find the perimeter of rectangle A. What is the perimeter of the image A'?



Solution

Rectangle A is 3 units wide and 4 units long. So A' will be $2 \times 3 = 6$ units wide and $2 \times 4 = 8$ units long.

The perimeter of A is $2(3 + 4) = 2 \times 7 = 14$ units. The perimeter of A' is $2(6 + 8) = 2 \times 14 = 28$ units.



In the example above, the perimeter of the image is double the perimeter of the original. This is because all the lengths that you add together to find the perimeter have been multiplied by 2.

The perimeter of the image is the perimeter of the original multiplied by the scale factor. This is true for any shape and any scale factor, so:

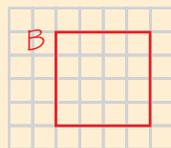
$$\text{original perimeter} \times \text{scale factor} = \text{image perimeter}$$

Guided Practice

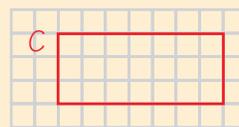
In Exercises 1–6, find the perimeter of the image you would get if you applied the given scale factor to the figure shown.

Give your answers in units. You do not need to draw the images.

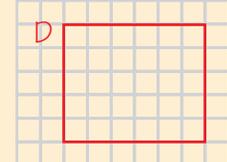
1. Scale factor $\frac{1}{2}$



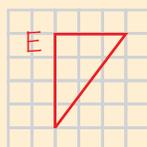
2. Scale factor 3



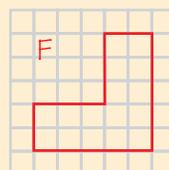
3. Scale factor 5



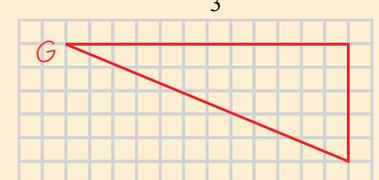
4. Scale factor 2



5. Scale factor 2.5



6. Scale factor $\frac{1}{3}$

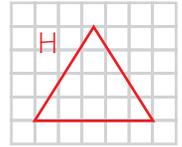


Areas Also Change When You Apply Scale Factors

Area also gets larger or smaller as a figure changes **size**.

Example 2

Chelsea uses a scale factor of 2 to draw an image of triangle H. Find the area of H and of the image H'.



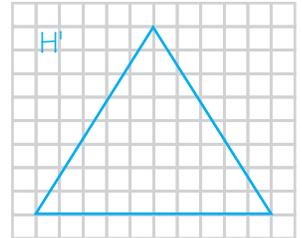
Solution

Triangle H has a base of **5 units** and height of **4 units**.

So the base of H' will be $2 \times 5 = 10$ **units** and its height will be $2 \times 4 = 8$ **units**.

The area of H is $\frac{1}{2}(5 \times 4) = \frac{1}{2} \times 20 = 10$ **units²**.

The area of H' is $\frac{1}{2}(10 \times 8) = \frac{1}{2} \times 80 = 40$ **units²**.



Don't forget:

The formula for the area of a triangle is $A = \frac{1}{2}(b \times h)$.
For more about area see Section 3.1.

In the example above, the area of the **image** is **4 times** the area of the **original**. The **lengths** that you multiply together to find the area have both been **multiplied by 2**, so the area is multiplied by $2 \times 2 = 4$.

The area of the image is the area of the original **multiplied by the scale factor squared**. This is true for **any shape** and **any scale factor**.

$$\text{original area} \times (\text{scale factor})^2 = \text{image area}$$

Example 3

Alejandra draws an image of shape J. She uses a scale factor of 3. If the area of shape J is 5 cm^2 , what is the area of the image J'?

Solution

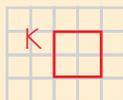
$$\begin{aligned} \text{The area of the image} &= (\text{area of the original}) \times (\text{scale factor})^2 \\ &= 5 \text{ cm}^2 \times 3^2 \\ &= 5 \text{ cm}^2 \times 9 = 45 \text{ cm}^2 \end{aligned}$$

✓ Guided Practice

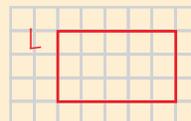
In Exercises 7–12, find the area of the image you would get if you applied the given scale factor to the figure shown.

Give your answers in units². You do not need to draw the images.

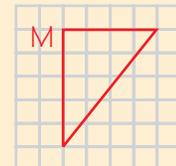
7. Scale factor 4



8. Scale factor 3



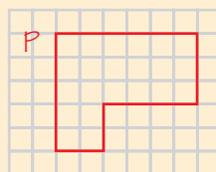
9. Scale factor 10



10. Scale factor 2



11. Scale factor 5



12. Scale factor 20



✓ Independent Practice

Roger draws a figure with a perimeter of 8 units.

Find the perimeter of the image if Roger multiplies his figure by:

1. Scale factor 2
2. Scale factor 11
3. Scale factor 4.5
4. Scale factor 0.25

Daesha draws a figure with an area of 10 cm^2 .

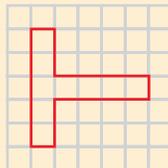
Find the area of the image if Daesha multiplies her figure by:

5. Scale factor 2
6. Scale factor 3
7. Scale factor 7.5
8. Scale factor 0.5

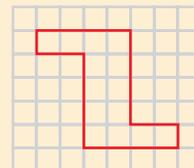
In Exercises 9–10, find the perimeter of the image you would get if you applied the given scale factor to the figure shown.

Give your answers in units. You do not need to draw the images.

9. Scale factor 9



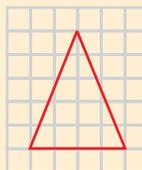
10. Scale factor $\frac{1}{4}$



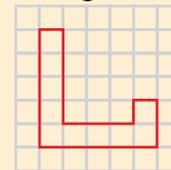
In Exercises 11–12, find the area of the image you would get if you applied the given scale factor to the figure shown.

Give your answers in units^2 . You do not need to draw the images.

11. Scale factor 2



12. Scale factor 7



Now try these:

Lesson 3.4.5 additional questions — p449

Don't forget:

You'll need to find square roots in some of these Exercises. You learned about them in Section 2.5.

What scale factor has been used in the following transformations?

13. Perimeter of original = 13 cm, perimeter of image = 26 cm
14. Perimeter of original = 22 in., perimeter of image = 77 in.
15. Perimeter of original = 50 in., perimeter of image = 5 in.
16. Perimeter of original = 15 cm, perimeter of image = 3.75 cm
17. Area of original = 10 in^2 , area of image = 90 in^2
18. Area of original = 1 cm^2 , area of image = 36 cm^2
19. Area of original = 8 in^2 , area of image = 128 in^2
20. Area of original = 5 cm^2 , area of image = 125 cm^2

Round Up

The effects of *scale factor* on *perimeter* and *area* can be confusing, but they do make sense.

Try to remember them, because understanding them is an important part of *geometry* in general.

Congruence and Similarity

California Standards:

Measurement and
Geometry 3.4

Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

What it means for you:

You'll learn the meaning of the terms congruent and similar. You'll find out how to tell if two shapes are congruent, similar, or neither.

Key words:

- congruent
- similar
- size
- shape
- scale factor

Don't forget:

If a shape is flipped over, it's called a reflection. A shape and its reflection are congruent.

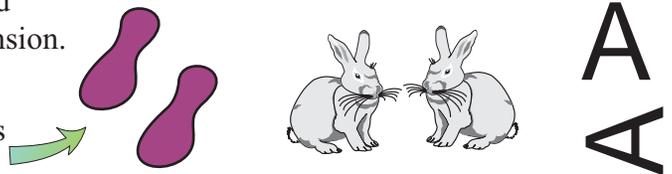
Congruent figures are shapes that are exactly the same size and shape as each other. That means that if you could lift them off the page, there would always be a way to make them fit exactly on top of each other, just by flipping them over or turning them around.

Congruent Figures Have the Same Size and Shape

Two figures are **congruent** if they match perfectly when you place them on top of each other.

They can be **turned around** or **flipped over**, but they always have the same **size, shape,** and **length** of each dimension.

These pairs of shapes are all congruent.



Example 1

Which of these pairs of shapes are congruent? Which are not, and why?



Solution

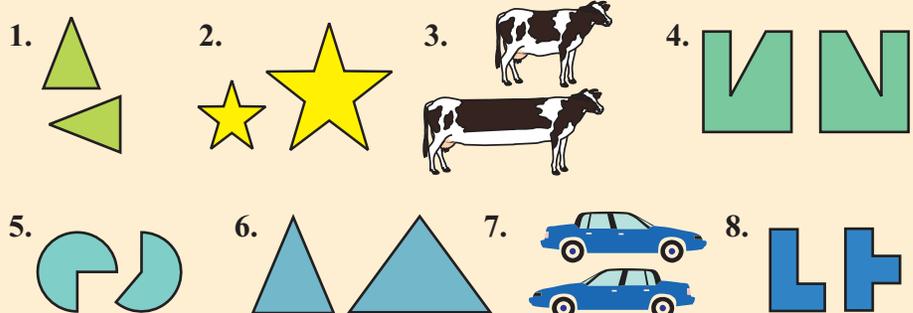
In pairs 1 and 4, each shape is identical to the other, but upside down. So pairs **1** and **4** are **congruent**.

Pair **2** is also **congruent**, as each shape is a mirror image of the other.

The rectangles in pair **3** are the same shape but they're **not** the same **size**, so they're **not congruent**.

Guided Practice

In Exercises 1–8, say whether or not each pair of shapes is congruent. If they are not, give a reason why not.

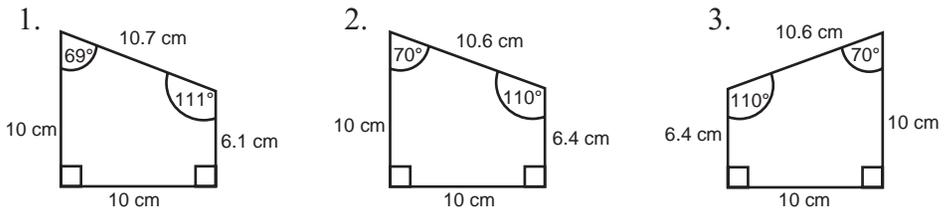


Congruent Polygons Have Matching Sides and Angles

Sometimes two **polygons** might look quite **alike**. You can tell for sure if they're **congruent** if you know the **measures** of their sides and angles.

Example 2

Which two of these quadrilaterals are congruent?



Solution

Quadrilaterals 1 and 2 look alike, but you can see from the angle measures and side lengths that they're **not identical**.

The angle measures tell you that Quadrilateral 3 is a **mirror image** of Quadrilateral 2.

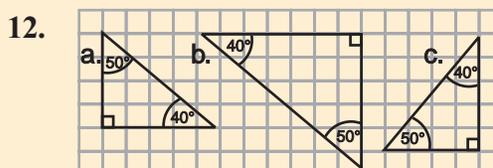
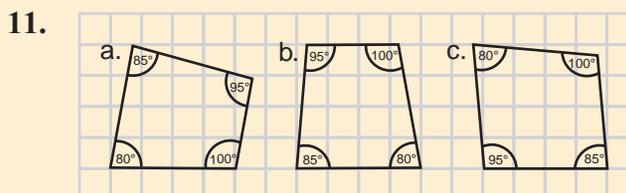
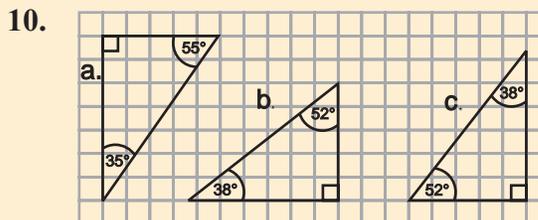
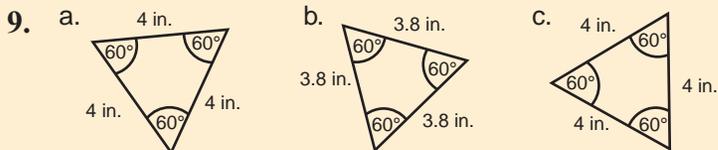
So Quadrilaterals **2 and 3** are **congruent**.

Check it out:

The angles of congruent polygons have the same measures, in the same order. Suppose that, going clockwise around its vertices, a quadrilateral has angles of 70° , 80° , 100° , and 110° . Then a congruent polygon's vertices will have the same measures in the same order, either clockwise or counterclockwise.

✓ Guided Practice

In Exercises 9–12, say which two out of each group of shapes are congruent. Give a reason why the other one is not.



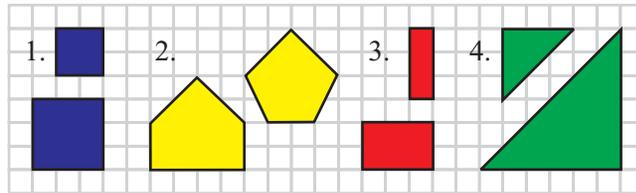
Similar Figures Can Be Different Sizes

Similar figures have angles of the same measure and have the same shape as each other, but they can be different sizes.

So two figures are **similar** if you can **apply a scale factor** and get a congruent pair.

Example 3

Which of these pairs of shapes are similar?



Solution

Pair 1 is a **similar pair**. They are both squares, and the only difference is the **size**.

Pair 2 is **not** a similar pair. The shapes are different — they have different angles.

Pair 3 is **not** a similar pair. You can't multiply either of them by any **scale factor** to get a rectangle **congruent** to the other one.

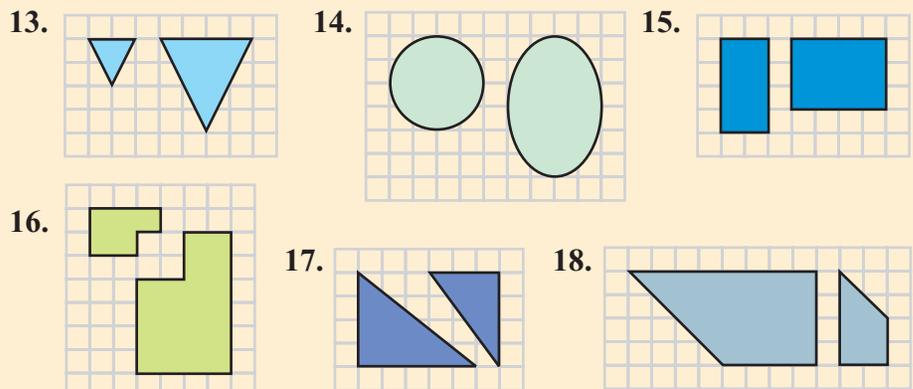
Pair 4 is a **similar pair**. If you multiply the smaller triangle by a scale factor of **2**, you will get a triangle **congruent** to the larger one.

Check it out:

You could also turn pair 4 from Example 3 into a congruent pair by applying a scale factor of $\frac{1}{2}$.

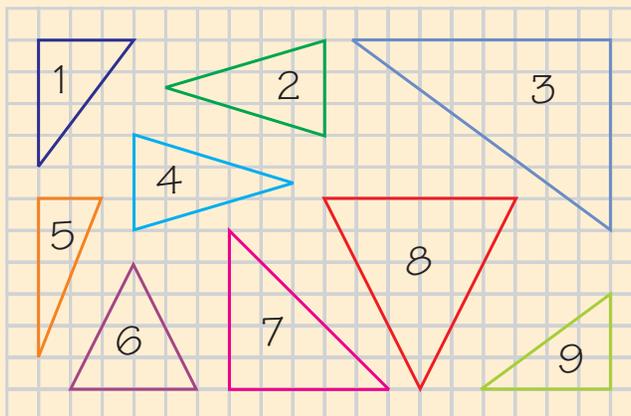
✓ Guided Practice

In Exercises 13–18, say whether or not each pair of shapes is similar.



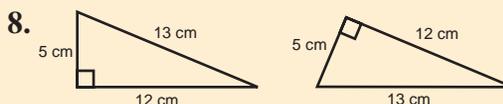
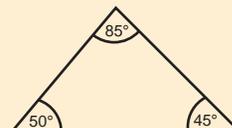
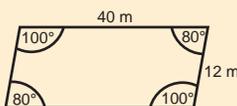
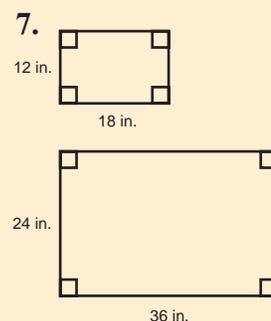
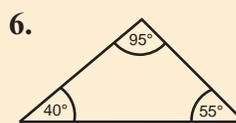
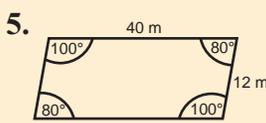
Independent Practice

Use the triangles shown below to answer Exercises 1–4.



- Which triangle is congruent to triangle 1?
- Which triangle is similar to triangle 6?
- Which triangle is congruent to triangle 4?
- Which two triangles are similar to triangle 3?

In Exercises 5–8, identify each pair of shapes as congruent, similar, or neither. Explain your answers.



Now try these:

Lesson 3.4.6 additional questions — p449

- Explain the difference between congruency and similarity when examining two figures.
- Triangle ABC has sides measuring 5 in., 6 in., and 8 in. Write the side lengths of a triangle that would be similar to ABC.
- "You can tell whether two shapes are congruent just by looking at the lengths of the sides. It is not necessary to look at the measures of the angles."
Is this statement true or false? Give a reason why.

Round Up

You'll learn more about *congruence* and *similarity* — particularly with triangles — in later grades. For now, make sure you know what each term means, and don't forget which is which.

Lesson 3.5.1

Section 3.5 Constructing Circles

California Standards:

Measurement and
Geometry 3.1

Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; **central angles, radii, diameters, and chords of circles**) by using a **compass and straightedge**.

What it means for you:

You'll learn what chords and central angles of circles are, and how to draw them.

Key words:

- circle
- compass
- radius
- chord
- central angle
- arc

Don't forget:

360° is the measure of a full circle.

Check it out:

The part-circle you make if you don't join the ends of your curve in a full circle is called an arc.

Don't forget:

You open your compass to the length of the radius — not the diameter.
If you're asked to draw a circle with a certain diameter, halve it to find the radius.

You already know a lot about *circles*. Earlier in this Chapter, you learned about the *radius*, *diameter*, *circumference*, and *area* of circles. In this Lesson, you'll learn some more words relating to circles. You'll also draw circles and mark features on them using a *compass*.

A Compass Can Help You to Draw Shapes

You might have used a **compass** in math lessons in earlier grades. A compass is a tool that can help you **draw** many types of shape.

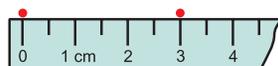
The easiest shape to draw with a compass is a **circle**.

Example 1

Use a compass and ruler to construct a circle with radius 3 cm.

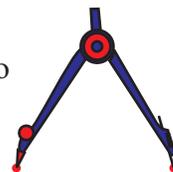
Solution

Step 1: Draw a **point** that will be the **center**.

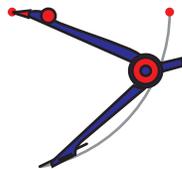


Step 2: Draw another point **3 cm** away from the center.

Step 3: Open the compass to the **length** between the two points. This length is the **radius** of the circle.

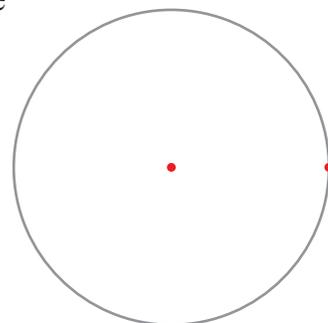


Step 4: Slowly sweep the **pencil end** of the compass 360° .



Keep the **pointed end** of the compass on top of the **center point**.

Make sure the ends of the curve **join** to make a complete circle.



✓ Guided Practice

Use a compass and ruler to construct the following circles:

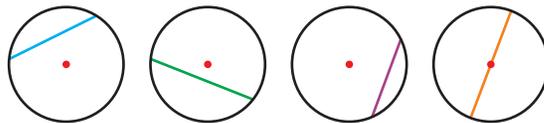
1. Radius 4 cm
2. Radius 2.5 cm
3. Diameter 10 cm
4. Diameter 7 cm

A Chord Joins Two Points of a Circle

Check it out:

A line segment is a straight line between two points. You'll learn more about line segments next Lesson.

A **chord** is a **line segment** that joins two points on the **circumference** of a **circle**. The **length** of a chord can be less than or equal to the length of the **diameter**.



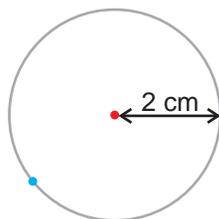
A **compass** can help you to draw chords.

Example 2

Use a compass and ruler to construct a circle, then draw a chord of length 3 cm.

Solution

Step 1: Start by drawing a **circle**.

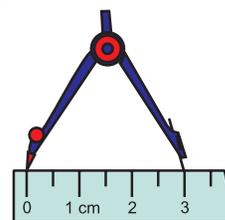
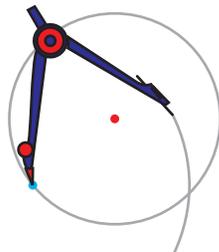


Remember that the length of the chord is less than or equal to the diameter. So the **diameter** must be at least **3 cm** — which means the **radius** must be at least $3 \div 2 = 1.5$ cm.

A circle of radius **2 cm** will do nicely.

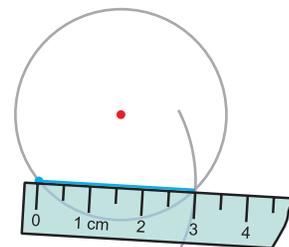
Step 2: Mark a **point** on the circle. This will be one **endpoint** of the chord.

Step 3: Open the compass to a length of **3 cm** — the length of the chord you want to draw.



Step 4: Put the **pointed end** of the compass on the point you marked on the circle. Draw an **arc** that crosses the circle.

Step 5: Draw a straight line from the point you drew in Step 2 to the point where the arc **crosses** the circle. Measure the chord you've drawn to **check** that it is 3 cm long.



Check it out:

In step 4 of this method, there will be two possible places where your arc could cross the circle. You only need it to cross at one of these points.

Guided Practice

In Exercises 5–10, use a ruler and compass to construct the following circles and chords

5. Circle of radius 3 cm, chord of length 5 cm
6. Circle of radius 1.5 cm, chord of length 2 cm
7. Circle of radius 2.2 cm, chord of length 3.5 cm
8. Circle of diameter 11 cm, chord of length 10 cm
9. Circle of diameter 6.8 cm, chord of length 4 cm
10. Circle of diameter 5.2 cm, chord of length 3.2 cm

A Central Angle is Formed by Two Radii

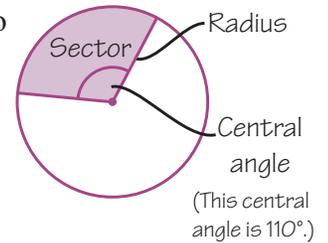
Don't forget:

Radii is the plural of radius.

A **central angle** of a circle is an angle made by two **radii** of the circle.

The size of a central angle is between 0° and 360° .

When a circle is divided by radii, the parts that it splits into are called **sectors**.

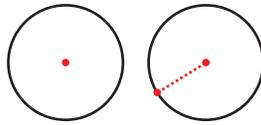


Example 3

Construct a central angle of a circle with a measure of 130° .

Solution

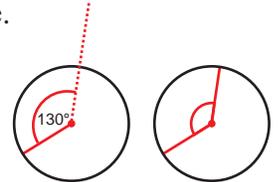
Step 1: Start by using a compass to draw a **circle**.



Step 2: You can now use a ruler or straightedge to join any point on the circle to the center.

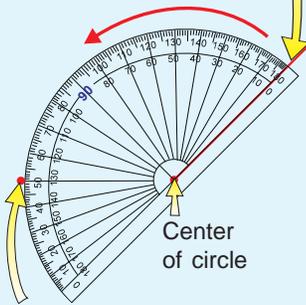
This is a **radius** of the circle.

Step 3: Use a **protractor** to make another radius at an angle of 130° to the first one.



Don't forget:

1. Place the baseline of the protractor on the radius, with the middle of the baseline on the center of the circle.
2. Count around to 130° on the scale that starts at zero.



3. Make a dot next to the correct number of degrees.
4. Join this dot to the center of the circle with a straight line.

Now try these:

Lesson 3.5.1 additional questions — p450

Guided Practice

For each of Exercises 11–16, construct a circle and draw central angles with the following measures:

11. 90°

12. 75°

13. 45°

14. 125°

15. 150°

16. 10°

Independent Practice

In Exercises 1–2, draw the following onto a circle of radius 5 cm:

1. A chord of length 7 cm
2. A central angle measuring 60°

In Exercises 3–4, draw the following onto a circle of diameter 5.8 cm:

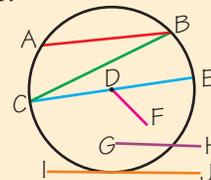
3. A chord of length 3.5 cm
4. A central angle measuring 85°

5. Terrell is constructing a circle with a diameter of 6 inches.

He opens his compass so that it is 6 inches wide.

Explain what error Terrell has made.

6. Identify the chords in this circle.



Round Up

If you need to draw a **circle**, always use a **compass**. It's pretty much impossible to draw a **perfect circle** without one. **Practice** drawing circles, chords, and central angles until you're really confident.

Lesson 3.5.2

Constructing Perpendicular Bisectors

California Standards:

Measurement and
Geometry 3.1

Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

What it means for you:

You'll learn what midpoints and perpendicular bisectors are, and how to draw them.

Key words:

- line segment
- midpoint
- bisect
- perpendicular bisector
- right angle

Check it out:

This method of constructing line segments will be used for other more complex drawings over the next few Lessons.

You should have gotten used to using a *compass* to draw *circles* and *chords* in the last Lesson. Now you can start using it for more *complex drawings* that don't have anything to do with circles. For starters, this Lesson shows you a neat way to use a compass to divide a *line segment* exactly in half.

A Line Segment is Part of a Line

If you **join** two **points** on a page using a straightedge, you make what you'd normally call a **line**. But to mathematicians, a **line** carries on **forever** in both directions.

When you join two points, you draw **part of a line**. In math that's called a **line segment**.

This line segment joins points **A** and **B**.

A and B are called the **endpoints**.

The line segment is called **AB**.



You can use a compass to make an accurate copy of a line segment **without** measuring its length.

Example 1

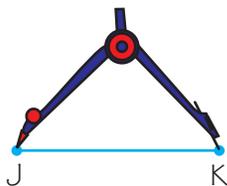
Use a compass and straightedge to copy the line segment JK. Label the copy LM.



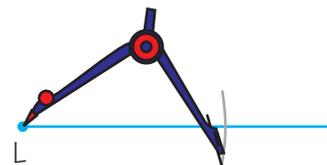
Solution

Step 1: Draw a line segment longer than JK.

Label one of its endpoints L.



Step 2: Open up your compass to the length of JK.



Step 3: With the compass point on L, draw an arc that crosses your new segment.



Step 4: The point where the arc crosses the line segment is point M, the second endpoint of your new line segment.

Guided Practice

In Exercises 1–4, use a ruler to draw a line segment with the length given, then copy it using a compass.

1. 5 cm

2. 6.5 cm

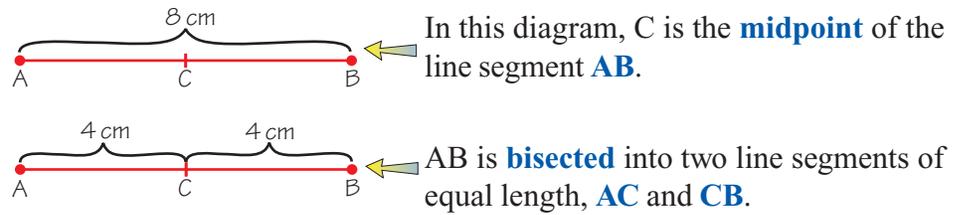
3. 3.9 cm

4. 2.6 cm

You'll use these line segments again for Guided Practice Exercise 11.

The Midpoint Splits a Line Segment in Half

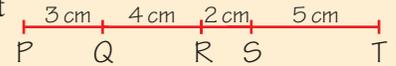
The midpoint of a line segment is the point that divides it into **two** line segments of **equal measure**. Dividing a line segment into two equal parts like this is called **bisecting** the line segment.



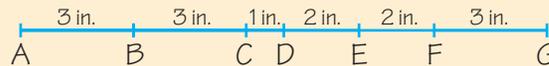
Guided Practice

5. The line segment DE is 8 in. long. F is the midpoint of DE. How long is the line segment DF?

6. Which is the midpoint of line segment PT — point Q, point R, or point S?



Use the diagram below to answer Exercises 7–10.



7. Which point is the midpoint of line segment AG?
8. Which point is the midpoint of line segment BE?
9. Which point the midpoint of line segment DF?
10. Which line segment is point B the midpoint of?

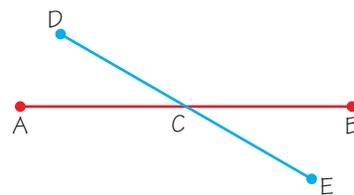
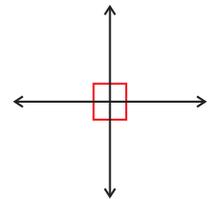
Don't forget:

A right angle measures 90° . Right angles are usually marked with a little square.



A Perpendicular Bisector Crosses the Midpoint at 90°

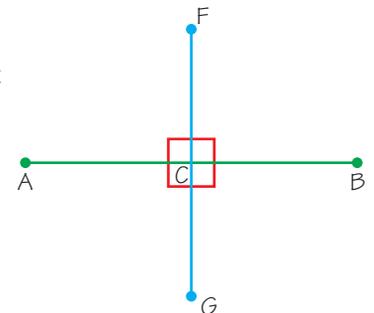
Two lines are **perpendicular** if the angles that are made where they cross each other are **right angles**.



A **bisector** is a line or line segment that crosses the **midpoint** of another line segment. In this diagram, C is the midpoint of AB, so DE is a bisector of AB.

A **perpendicular bisector** of a line segment is a bisector that passes through the **midpoint** at a **right angle**.

In this diagram, FG is a perpendicular bisector of AB.



You need a **compass** to draw a **perpendicular bisector**.

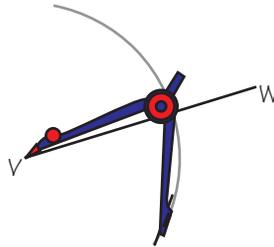
Example 2

Use a compass and straightedge to draw the perpendicular bisector of VW . Label the bisector YZ .

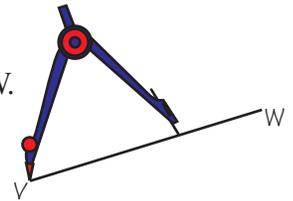


Solution

Step 1: Place compass point on V . Open the compass more than half the length of VW .

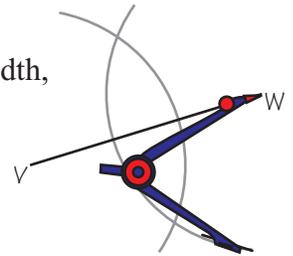


Step 2: Sweep a large arc that goes above and below line segment VW .



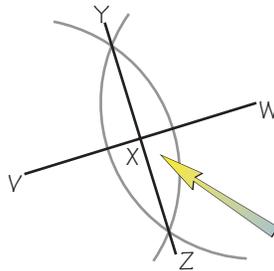
Step 3: Keeping the compass open to the same width, place the compass point on W and repeat step 2.

The two arcs should cross in two places. If they don't you might need to extend them.



Step 4: Draw a line segment that passes through the points where the arcs cross.

This is the perpendicular bisector of VW , so label its endpoints Y and Z .



The bisector crosses VW at the midpoint, X .

Check it out:

This method has two uses. It is the way to draw a perpendicular bisector using a compass and straightedge, but it also shows you where the midpoint is.

Guided Practice

11. Use a compass and straightedge to draw the perpendicular bisectors of each of the line segments you copied in Exercises 1–4. Label the midpoint of each line segment.

Independent Practice

S is the midpoint of line segment RT . The length of RS is 12.6 in.

1. What is the length of RT ? 2. What is the length of ST ?

In Exercises 3–8, draw a line segment of the given length, then construct its perpendicular bisector. Mark the midpoint X .

3. 8 cm 4. 5 in. 5. 4.5 cm
6. 9.3 cm 7. 3.5 in. 8. 6.5 in.

Now try these:

Lesson 3.5.2 additional questions — p450

Round Up

A *compass* isn't just useful for drawing *circles* and *arcs*. You also use it to find midpoints and draw perpendicular bisectors. This is something else that you need to practice until you are happy with it.

Lesson 3.5.3

Perpendiculars, Altitudes, and Angle Bisectors

California Standards:

Measurement and
Geometry 3.1

Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

What it means for you:

You'll learn what perpendiculars, altitudes, and angle bisectors are, and how to draw them.

Key words:

- perpendicular
- altitude
- triangle
- angle bisector

Don't forget:

If you need a reminder of how to construct a perpendicular bisector, look at Lesson 3.5.2.

Last Lesson you learned to draw *perpendicular bisectors*, which cross other line segments exactly in their center, at 90° . Now you're going to use the skills you learned to draw other *perpendiculars* too.

You're also going to learn about *angle bisectors*. These do just what it sounds like they do — *divide an angle exactly in half*.

Perpendicular Line Segments Meet at Right Angles

You've seen how to construct a **perpendicular bisector**. That's a line segment that crosses the **midpoint** of another line segment at a 90° angle.

A line or line segment that makes a 90° angle with another line segment, but isn't a **bisector**, is sometimes called a **perpendicular**.

You can **construct** a perpendicular that passes through a specific point using a **compass** and **straightedge**.

Example 1

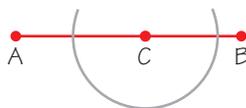
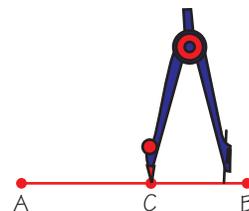


Use a compass and straightedge to construct a line segment perpendicular to AB that passes through point C.

Solution

Step 1: Put the compass point on C.

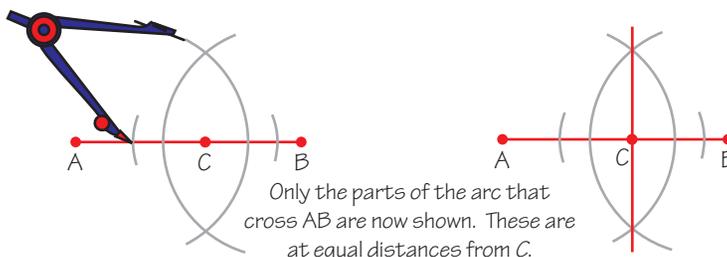
Open the compass to a length between C and the nearest end of the line segment — B in this case.



Step 2: Draw an arc that crosses AB twice.

(In fact, you only need to draw the parts of this arc where it crosses AB — see the next step.)

Step 3: Follow the steps for constructing a perpendicular bisector. Use the points where the arc crosses the line segment as endpoints.



Only the parts of the arc that cross AB are now shown. These are at equal distances from C.

The point you need to pass through isn't always on the line segment you want to cross.

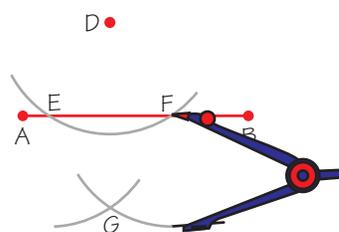
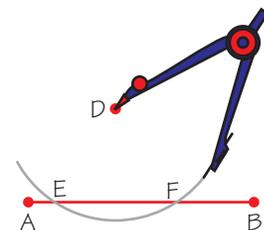
Example 2



Use a compass and straightedge to construct a line segment perpendicular to AB that passes through point D.

Solution

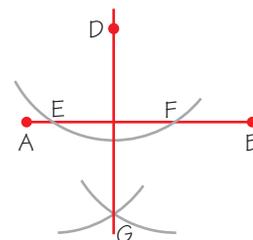
Step 1: Put the compass point on D and draw an arc that crosses AB twice. Call the points where AB crosses the arc E and F.



Step 2: With the compass point on E, draw an arc on the opposite side of AB to point D.

Step 3: Move the compass point to F. Keep the compass setting the same and draw an arc that crosses the one you drew in step 2. Call the point where the two arcs cross G.

Step 4: Draw a line segment passing through D and G.



✓ Guided Practice

Draw a line segment, JK, that is 12 cm long. Mark the following points on the line. Draw a perpendicular through each of those points.

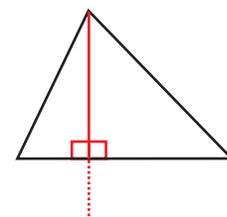
- Point L, 3 cm away from J
- Point M, 2 cm away from K
- Point N, 5 cm away from J
- Point O, 5.5 cm away from K

5. Draw a line segment, PQ, that is 8 cm long. Draw one point, S, above PQ and one point, T, below PQ. Construct a perpendicular to PQ that passes through S and another that passes through T.

An Altitude is a Line Showing the Height of a Triangle

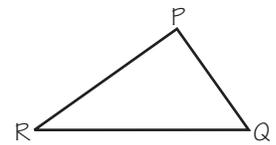
An **altitude** of a **triangle** is a line segment that starts from one corner of the triangle and crosses the opposite side at a 90° angle.

The way to **construct** an altitude is very similar to the method for constructing a **perpendicular** through a point not on the line — just use the **corner** of the triangle as the point.

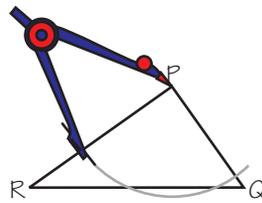


Example 3

Use a compass and straightedge to construct an altitude from P through RQ.

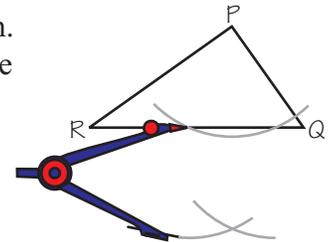


Solution

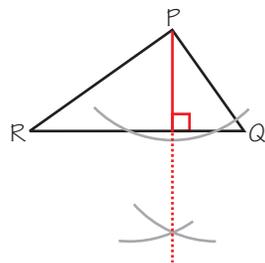


Step 1: Put the compass point on P. Draw an arc that crosses RQ in two places.

Step 2: Keep the compass open the same width. Put the compass point at one of the points where the arc crosses RQ. Draw an arc below RQ.



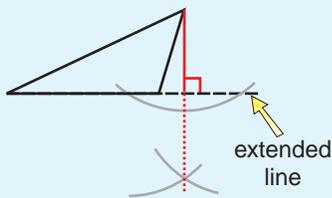
Step 3: Repeat step 2 from the other point where the arc and RQ cross. Make sure the two new arcs cross.



Step 4: Draw a line segment from P to the opposite side of the triangle, toward the point where the two arcs cross.

Check it out:

Drawing an altitude from one of the acute corners of an obtuse triangle is a little more tricky. You need to extend the opposite side of the triangle for the method to work.



Don't forget:

An acute triangle has three angles of less than 90° .
A right triangle has one angle of exactly 90° .
An obtuse triangle has one angle of more than 90° .

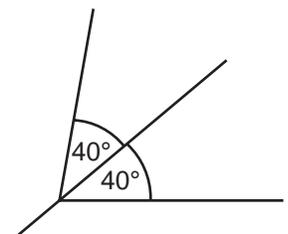
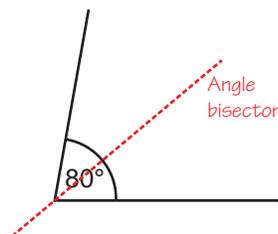
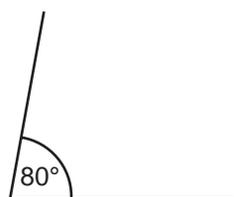
Guided Practice

In Exercises 6–8, you need to draw triangles. Start each one by drawing a line, AB, that is 6 cm long. Choose the lengths of the other sides to suit the question.

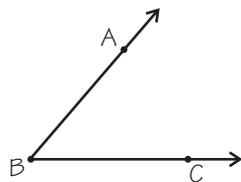
6. Draw an acute triangle ABC. Construct an altitude from C.
7. Draw a right triangle ABC. Construct an altitude from C.
8. Draw an obtuse triangle ABC. Construct an altitude from C.

An Angle Bisector Divides an Angle Exactly in Half

An **angle bisector** is a line or line segment that **divides** an angle into two new angles of **equal measure**.



Example 4



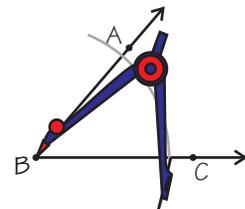
Use a compass and straightedge to bisect the angle ABC.

Check it out:

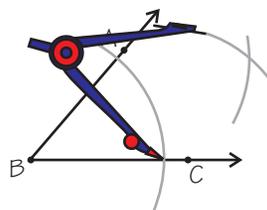
The method for constructing an angle bisector works for any angle — acute, obtuse, or right.

Solution

Step 1: Put the compass point on B and draw an arc that crosses the line segments AB and BC.

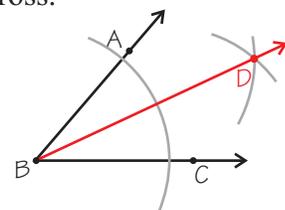


Step 2: Put the compass point where the arc crosses AB and draw a new arc in the middle of the angle. Keep the compass open at the width you've just used.



Step 3: Using the same compass width, repeat step 2 with the compass point at the spot where the first arc crosses BC. Make sure the two new arcs cross.

Step 4: Join point B to the point where the two arcs cross. The angle ABC has been bisected into two equal angles, ABD and DBC.



Guided Practice

Use a protractor to draw the following angles. Bisect them using a compass and straightedge.

9. 90°

10. 65°

11. 20°

12. 129°

Independent Practice

1. Draw a 10 cm long line segment UV. Mark two points on the line segment, and label them W and X. Mark a point Y above the line. Draw perpendiculars to UV through W, X, and Y.
2. Draw an acute triangle and an obtuse triangle. Construct altitudes from all three corners of each triangle. What is different about the points where the three altitudes meet (or will meet if extended)?
3. Draw one acute, one right, and one obtuse triangle. Start each one by drawing a line that is 5 cm long. Construct an angle bisector for the largest angle in each triangle.

Now try these:

Lesson 3.5.3 additional questions — p450

Round Up

This Lesson gives you *two* methods for the price of *one*. The method for drawing an *altitude* of a *triangle* is the same as for drawing a *perpendicular* through a point that's not on the line. Remember, watch out for those tricky *obtuse triangles*, where you might need to *extend* one side.

Lesson 3.6.1

Geometrical Patterns and Conjectures

California Standards:

Mathematical Reasoning
1.2

Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

Mathematical Reasoning
2.4

Make and test conjectures by using both inductive and deductive reasoning.

Measurement and
Geometry 3.3

Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What it means for you:

You'll make conjectures, or "educated guesses," about problems and use counterexamples and reasoning to decide whether your conjectures are true or false.

Key words:

- conjecture
- limiting case
- justifying
- reasoning
- instance

Check it out:

There's no real right or wrong answer with conjectures. The only rule is that you should be able to explain why you've made that conjecture, and why you think it's likely to be true.

In this Lesson, you're going to learn about *testing* and *justifying conjectures*. You make conjectures all the time in math, and also in everyday life. A conjecture is just an educated guess that is based on some good reason.

A Conjecture is an Educated Guess

A **conjecture** is what's called an **educated guess** or an **unproved opinion**, such as, "it'll rain soon because there are gray clouds." This is unproved because we don't actually know whether it will rain soon or not.

You can make conjectures about mathematical situations such as **patterns** or **data**. For example, given the pattern 2, 4, 6... you could make a conjecture that the pattern increases by 2 each time.

There are two main types of conjecture in mathematical patterns:

- **Specific** conjectures about a **new instance** of a pattern.
- **General** conjectures about a pattern.

Example 1

Make three specific conjectures and three general conjectures about the pattern below.



Solution

Specific conjectures:

1. Instance 4 will have 13 dots.
2. Instance 4 will be in the shape of a cross.
3. Instance 5 will have 17 dots.

These are specific conjectures because they describe instances 4 and 5 only.

General conjectures:

1. Each instance is the shape of a plus sign.
2. Each instance has rotational symmetry.
3. Each instance has four more dots than the instance before it.

These are general conjectures because they describe the entire pattern.

There are usually **lots** of conjectures you could make about a pattern, and you have to select which you think are the **most important** to mention.

Not every conjecture has to be true, but if you make a conjecture you should either **think it is true**, or think it has the **possibility** of being true. So, although we don't know that instance 4 will definitely have 13 dots, we make the conjecture because it seems the most sensible guess based on what we know so far.

✓ Guided Practice

1. Below is the first three instances of a dot pattern.
Make at least one specific and one general conjecture.



Instance 1



Instance 2



Instance 3

A Counterexample Shows that a Conjecture is False

It only takes **one instance** where the conjecture **doesn't** apply to show that the conjecture is **not true**. For example, if you made the conjecture that it never rains on Mars, only one drop of rain would have to fall on Mars to prove you wrong.

You can show that some math conjectures aren't true by finding a counterexample. A counterexample is a single case that makes a conjecture false.

To **find a counterexample**, consider some instances of the situation. Try to think about any **extreme** or **limiting cases**. These may be cases such as negative numbers, zero, or the most regular or irregular shapes.

Don't forget:

Limiting cases are generally the ones that are most out of the ordinary.

For example, one limiting case for the parallelogram is the rhombus, which has all sides the same length.

Example 2

Test the following conjecture about rectangles:

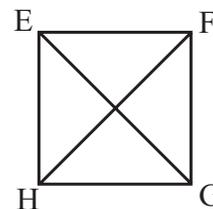
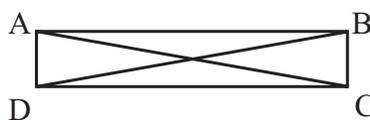
“The diagonals of a rectangle are never perpendicular.”

Solution

Perpendicular means that the lines are at 90° to each other.

First find the **limiting cases**.

For rectangles, try the case where the rectangle is very long and thin, and the case when the rectangle is square.



For the conjecture to be true, it must be true for **every possible case**. So if this conjecture is true, it must hold for every possible rectangle.

The special case of the long and thin rectangle clearly **does not** have perpendicular diagonals. But the special case of the square **does** have perpendicular diagonals.

So the conjecture is **false** since it isn't true for every case.

The square is a **counterexample**.

Deciding whether a conjecture is true or not by looking at specific limiting examples is called **justifying through cases**.

✓ Guided Practice

Don't forget:

Quadrilaterals are four-sided shapes.

2. Consider the following conjecture:

“All quadrilaterals have four right angles.”

Decide whether it is false or could possibly be true by examining limiting cases.

You Can Justify Conjectures Through Reasoning

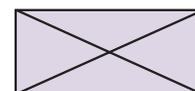
It's hard to show that a conjecture is **definitely** true — there could always be an example that you haven't found that would disprove the conjecture.

Justifying through reasoning means that you use **algebra** or **principles** to show that a conjecture is true for **all possible cases**.

Example 3

Test the following conjecture about rectangles:

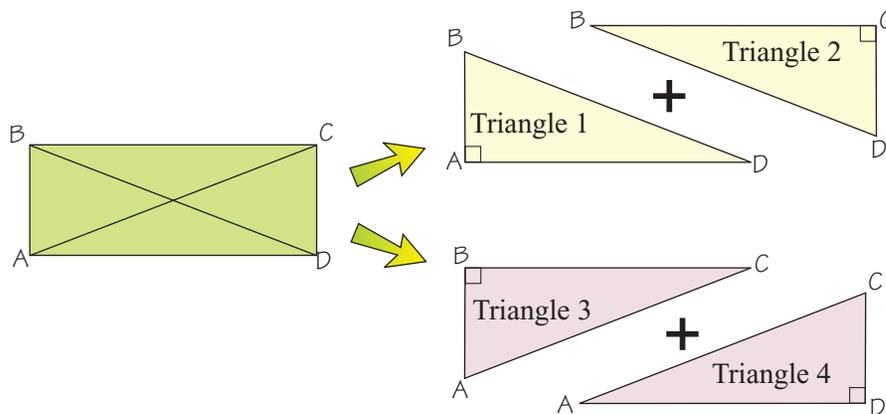
“The diagonals of a rectangle are congruent.”



Solution

Congruent means the same in size and shape.

Split the rectangle along both the diagonals to make **4 triangles**, where each diagonal becomes the **hypotenuse** of a right triangle.



The **Pythagorean theorem** says that the square of the length of the hypotenuse is equal to the sum of the squares of the two legs.

$$\text{Triangle 1: } (BD)^2 = (AB)^2 + (AD)^2$$

$$\text{Triangle 3: } (AC)^2 = (AB)^2 + (BC)^2 = (AB)^2 + (AD)^2$$

Rectangles have two pairs of equal sides, so $BC = AD$.

$$(BD)^2 = (AC)^2, \text{ so } \mathbf{BD \text{ and } AC \text{ must be the same length.}}$$

This means the diagonals of the rectangle are the same length, and so they're congruent.

Don't forget:

Go back to Lesson 3.3.1 if you need a reminder of the Pythagorean theorem.

✓ Guided Practice

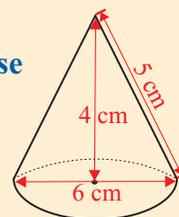
Check it out:

You can use the Pythagorean theorem to justify whether this conjecture is true or false too.

3. Consider the following conjecture:

“The vertical height of a cone will be 4 cm, if the base has diameter 6 cm and the slant height is 5 cm.”

Decide whether the conjecture is true or false by justification through reasoning.



✓ Independent Practice

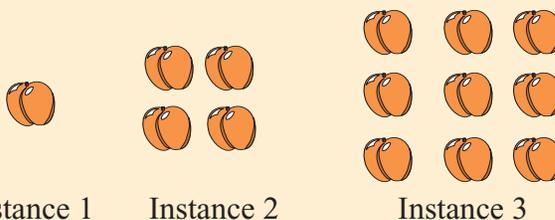
1. Make two specific conjectures and two general conjectures about the following number sequence.

“1, 9, 25, 49...”

Use some of your conjectures to find the next two numbers in the series.

2. If n is an odd number, make a conjecture about $n + 1$.

The pattern below shows the first three instances of a pattern. Use the pattern to answer Exercises 3–5.



Instance 1

Instance 2

Instance 3

3. Make three specific conjectures about the pattern.

4. Make three general conjectures about the pattern.

5. Draw the next two instances of the pattern.

6. Consider the following conjecture:

“Parallelograms never have a line of symmetry.”

Decide if this conjecture is true or false by testing limiting cases.

7. Consider the following conjecture:

“There are exactly three ways that you can split a rectangle into four smaller rectangles, so that all four smaller rectangles are congruent to each other.”

Show that the conjecture is false by finding a counterexample.

Now try these:

Lesson 3.6.1 additional questions — p451

Round Up

So that's *conjectures*. You'll make conjectures about all kinds of things in math — often without even thinking about it — and you might have to show whether they're *true* or not using counterexamples or careful reasoning.

Lesson 3.6.2

Expressions and Generalizations

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression,

an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

Mathematical Reasoning 2.2

Apply strategies and results from simpler problems to more complex problems.

Mathematical Reasoning 3.3

Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

What it means for you:

You'll learn how to generalize a pattern from simple examples so that you can find any given instance of that pattern.

Key words:

- generalization
- number pattern
- number sequence
- conjecture

Don't forget:

It's always a good idea to test your generalizations. With patterns, do this by checking that a generalization or formula works on a new instance.

You met specific and general conjectures in the previous Lesson.

A **generalization** is a special kind of general conjecture. It allows you to work out quickly what any instance of a pattern will be.

A Generalization Comes from a General Conjecture

A **generalization** is a way of **extending** a general conjecture.

Making a generalization means finding some kind of **expression** or **formula** that you could use for **any instance**. Generalizations can be **mathematical expressions or word descriptions**.

You might use the first three instances of a pattern to make a the generalization that would tell you how to find the **n th instance** — the n th instance could be any instance whatsoever.

Example 1

Make a generalization for the number of dots in instance n of this pattern:



Solution

Look at what happens in each instance.

Instance 1 has $1 + 1 + 1 = 3$ dots. 

Instance 2 has $1 + 2 + 2 = 5$ dots. 

Instance 3 has $1 + 3 + 3 = 7$ dots. 

We can therefore say that Instance n will have $1 + n + n = 2n + 1$ dots.

Check this is correct by testing on instance 2:

$2n + 1 = (2 \times 2) + 1 = 5$ dots, which is correct.

Test on instance 4:

$2n + 1 = (2 \times 4) + 1 = 9$ dots.

This is what you would have expected. 

Guided Practice

1. Make a generalization of the pattern below by writing an expression for the number of dots in instance n .



Use Generalizations to Solve Problems

Example 2

In the pattern below, find the number of dots in instance 10.



Solution

In Example 1 we found a generalization for this pattern.

This was that the number of dots in instance n is $2n + 1$.

Instance 10 therefore has $2n + 1 = (2 \times 10) + 1 = \mathbf{21 \text{ dots}}$.

Another way to do this is to notice that the top line has the same number of dots as the instance number, and the bottom line has one more dot than the top line. The top line of instance 10 will have 10 dots, and the bottom line will have $10 + 1 = 11$ dots. The total number of dots is $10 + 11 = \mathbf{21 \text{ dots}}$.

Don't forget:

The examples opposite ask you to find expressions for the numbers in patterns.

One way to do this is to compare each instance number with the corresponding value in the pattern.

A table can help you do this. For example:

Instance	Sum
1	1
2	4
3	9
4	16

Check it out:

It's often useful to make a table of simple cases. For example, a table for Exercise 2 would look like this:

Instance	Number
1	1
2	3
3	5
4	7

When you look at a pattern, there can be **more than one** generalization to make.

Example 3

By making a generalization about the pattern below, find the sum of the 7th line of the pattern.

$$1 = 1$$

$$1 + 3 = 4$$

$$1 + 3 + 5 = 9$$

Solution

The pattern is the **sum of consecutive odd numbers**, adding one more odd number each line. You could say that the sum of the 7th line will be the sum of the first 7 odd numbers. A generalization would be that **the sum of the n th instance is the sum of the first n odd numbers**.

You could also generalize that the sum of each of the lines is the square of the instance number. So the **n th line will sum to n^2** .

So for line 7, the sum will be $n^2 = 7^2 = \mathbf{49}$.

✓ Guided Practice

- Use a generalization to find the 50th odd number.
- Use a generalization to find the 9th term in the following pattern:

$$4, 6, 8, 10, 12, \dots$$

✓ Independent Practice

Use the dot pattern below to answer Exercises 1–3.



Instance 1



Instance 2



Instance 3

1. Generalize the pattern using words.
2. Generalize the pattern by finding an expression for the number of dots in the n th instance.
3. Draw Instance 10.
4. Maggie created a pattern in which the n th instance had $5n - 1$ dots in it. Draw the first three instances of Maggie's pattern.

Use this pattern to answer Exercises 5–6.

$$2 = 2$$

$$2 \times 2 = 4$$

$$2 \times 2 \times 2 = 8$$

5. Extend the pattern for two more lines.
6. Find a generalization and use it to find the product of the 9th line.

For Exercises 7–9 use the pattern of numbers:

$$7, 10, 13, 16, 19, \dots$$

7. Describe the pattern in words.
8. Write an expression for the n th number in the pattern.
9. Find the 30th number in the sequence.

Now try these:

Lesson 3.6.2 additional questions — p451

Use this pattern for Exercises 10–12:

$$1 = 1$$

$$3 + 5 = 8$$

$$7 + 9 + 11 = 27$$

10. Find the next line of the sequence.
11. Find an expression for the sum of the n th line.
12. Find the sum of the 10th line.

Round Up

*Generalizing is really useful in problem solving. If you're asked to find the 100th instance in a pattern, it'll take you ages to write or draw all 100 out — better to **start simple** and then **generalize**.*

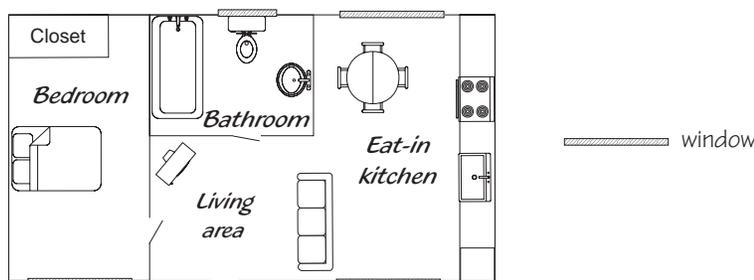
Chapter 3 Investigation

Designing a House

Before starting expensive construction, architects will make *scale drawings*, so that the size and shape of everything is clear. Often, constructions will be *complex*, rather than regular shapes.

You work for an architect company, which is designing **single-floor, one-bedroom houses**. The houses will have one bedroom, a living area/eat-in kitchen and one bathroom. They must have a floor area of **900 square foot**.

An example layout is shown here.



Part 1:

Design a house that satisfies these requirements. Be sure to include the **dimensions** of each room.

Part 2:

Calculate the area of each room. Add up the area of each room to check that it comes to 900 square feet.

Things to think about:

- You don't have to design a rectangular or square house — you could make it a **complex shape**.
- Try to make the room dimensions seem **reasonable** — a bathroom that is a 2-foot by 2-foot square wouldn't be usable.
- Be sure to include **doorways**, **windows** and other useful things, like a closet.

Extensions

- 1) Make a scale drawing of the house using a scale of 1 cm : 2 feet.
- 2) Make a scale drawing of the house using a scale of your own choosing.

Open-ended Extensions

- 1) One buyer wants the bathroom to be accessible from the bedroom and the living area. He also wants a separate kitchen. Design a house that would incorporate this concept.
- 2) A second buyer wants the rooms in the house to flow from one into the other, but to still offer privacy. Design a house that uses partial walls to separate rooms.

Round Up

When you're designing something, you'll often have certain *limitations* — like the area that the house should be. But you also have to think about how to make it *usable* — for example, you need enough space for a bed in the bedroom. But after that, you can take *preferences* into account — like where you'd prefer to put the door and the kitchen sink.

Chapter 4

Linear Functions

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Section 4.1 introduction — an exploration into: Block Patterns

In this Exploration, you'll use pattern blocks to make *patterns* that could carry on forever. These patterns all result in *different straight lines* when graphed on the coordinate plane.

Example

Pattern 1 is developed using pattern blocks. Describe the pattern and record it in a table.



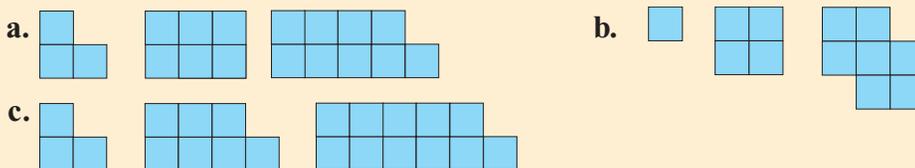
Solution

The pattern of 1, 3, 5, 7... continues on forever.  Two blocks are added each time.

Figure in pattern	1	2	3
Number of blocks	1	3	5

Exercises

1. Describe each pattern and record it in a table.



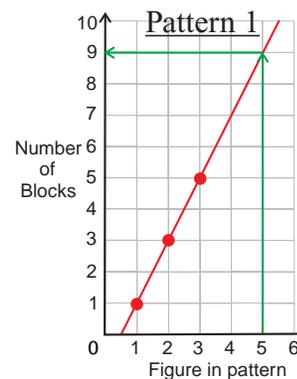
The numbers from each pattern can be **graphed on the coordinate plane**. This data can be used to make **predictions** about other figures in the pattern.

Example

Graph the data for Pattern 1 and predict the number of blocks in the fifth figure in the pattern.

Solution

Reading from the graph as shown, there will be **9 blocks** in the fifth figure.



Exercises

- Graph the data for each pattern in Exercise 1.
- Predict the number of blocks in the fifth and sixth figures of each pattern.
- Find the slope of the graph of each pattern. What do you notice about each slope?

Round Up

The figures in each pattern increase by the *same number* of blocks each time. So, when you plot the data, you get *straight line graphs*. You can use these to *predict* later figures in each pattern.

Lesson 4.1.1

Section 4.1 Graphing Equations

California Standards:

Algebra and Functions 1.5
Represent quantitative relationships graphically, and interpret the meaning of a specific part of a graph in the situation represented by the graph.

Algebra and Functions 3.3
Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same, and know that the ratio ("rise over run") is called the slope of a graph.

What it means for you:

You'll learn how to plot linear equations on a coordinate plane.

Key words:

- linear equation
- variables
- graph

Check it out:

The $y = mx + b$ equation represents a "function." A function is a rule that assigns each number to one other number. If you put a value for x into the function, you get one value for y out.

Don't forget:

Ordered pairs are often called coordinates, or coordinate pairs.

Check it out:

There are an infinite number of points on a line. So there are an infinite number of solutions to a linear equation.

Equations like $y = 3x$, $y = x + 1$, and $y = 2x + 3$ are known as *linear equations* because if you plot them on a grid, you get *straight lines*. In this Lesson you'll learn how to *plot* linear equations.

Linear Equations Have the Form $y = mx + b$

A linear equation can have **one or two variables**. The variables must be **single powers**, and if there are two variables, they must be in **separate terms**.

Linear Equations	Nonlinear Equations
$y = x - 7$	$xy = 7$
$y = 8x + 1$	$y = 2x^2 - 3$
$4y - 2x = -7$	$y = x^3 + 1$
$y = 3x$	$y = 3x^2$

A linear equation can always be written in the form below (but you might have to rearrange it first):

$y = mx + b$

y is a variable. *m* and *b* are constants, so they're numbers like 1 or 3.
x is a variable too.

The m and b values can be 0, so $y = 3x$ and $y = 4$ are linear equations too.

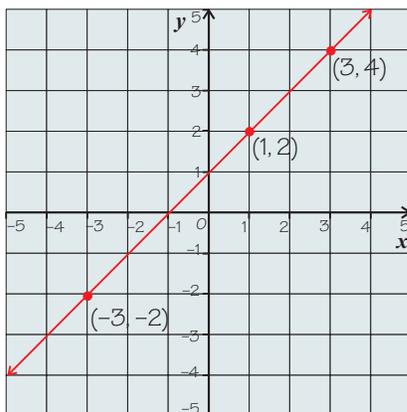
✓ Guided Practice

In Exercises 1–6, state whether the equation is a linear equation or not.

- $y = 2x - 5$
- $7y - 9x = -1$
- $y = x^2 + 4$
- $2y = 4x + 3$
- $y^3 = x^3 - 1$
- $y = x$

Every Point on the Line is a Solution to the Equation

The graph of a linear equation is always a **straight line**. Every point on the graph is an **ordered pair (x, y)** that is a **solution to the equation**.



This is the graph of the equation $y = x + 1$. The point $(1, 2)$ lies on the graph, so $x = 1$, $y = 2$ must be a solution to the equation.

You can test this by **substituting** the x - and y -values into the equation and checking that they make the equation **true**:

$$y = x + 1 \rightarrow 2 = 1 + 1$$

This makes the equation true, so $x = 1$, $y = 2$ is a solution to the equation.

✓ Guided Practice

Show that the following are solutions to the equation $y = x + 1$.

7. $x = 3, y = 4$

8. $x = -3, y = -2$

Using the graph on the previous page, explain whether the following are solutions to the equation $y = x + 1$.

9. $x = 1, y = 4$

10. $x = -4, y = -3$

Find Some Solutions to Plot a Graph

To graph a linear equation, you need to **find some ordered pairs** to plot that are **solutions** to the linear equation.

You do this by putting some **x-values** into the equation and finding their **corresponding y-values**.

Check it out:

You only really need two ordered pairs to draw a straight-line graph. But you should work out at least one more than this to make sure you haven't made any errors.

Example 1

Find the solutions to the equation $y = 2x + 1$ that have x -values of $-2, -1, 0, 1,$ and 2 .

Use these to write ordered pairs that lie on the graph of $y = 2x + 1$.

Solution

Step 1: Draw a **table** that allows you to fill in the y -values next to the corresponding x -values. Make a column to write the ordered pairs in.

x	y	Ordered Pair (x, y)
-2		
-1		
0		
1		
2		

Step 2: Substitute each x -value into the **equation**, to get the corresponding y -value. Here are a few examples:

For $x = -2$:

$$y = 2x + 1 = 2(-2) + 1 = -3$$

For $x = -1$:

$$y = 2x + 1 = 2(-1) + 1 = -1$$

Step 3: Write each set of x - and y -values as an **ordered pair (x, y)**.

x	y	Ordered Pair (x, y)
-2	-3	$(-2, -3)$
-1	-1	$(-1, -1)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	5	$(2, 5)$

✓ Guided Practice

11. Find the solutions to the equation $y = 5x - 4$ that have x -values equal to $-2, -1, 0, 1,$ and 2 . Use your solutions to write a set of ordered pairs that lie on the graph of $y = 5x - 4$.

12. Find the solutions to the equation $y = 2x - 6$ that have x -values equal to $-6, -3, 0, 3,$ and 6 . Use your solutions to write a set of ordered pairs that lie on the graph of $y = 2x - 6$.

Check it out:

If your points aren't in a straight line, you must have made a mistake — go back and check.

That's why you need more than two points — you wouldn't know if you'd made a mistake if you only had two points to join up.

Check it out:

Always label your graph with its equation.

Plot the Points and Join Them Up

You draw the graph of an equation by first plotting **ordered pairs** that represent the solutions to the equation. They should lie in a **straight line**.

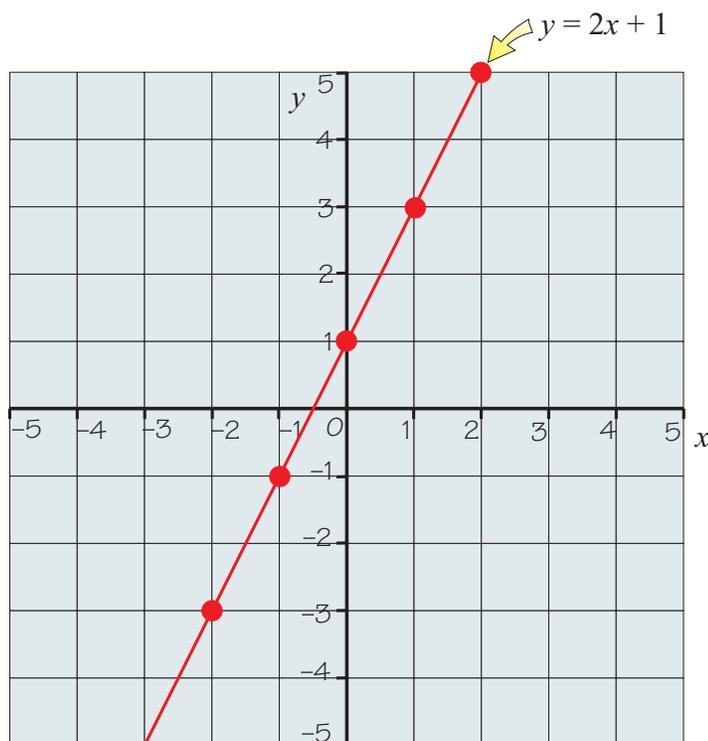
Example 2

Draw the graph of $y = 2x + 1$ by plotting the ordered pairs you found in Example 1.

Solution

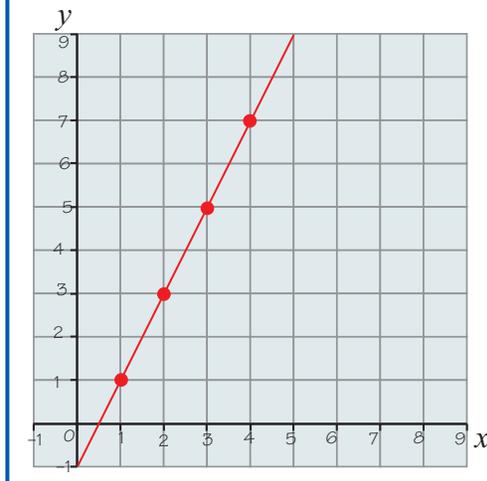
The ordered pairs that fit the equation are $(-2, -3), (-1, -1), (0, 1), (1, 3),$ and $(2, 5)$.

Plot these points and draw a straight line through them.



Example 3

Plot the ordered pairs (1, 1), (2, 3), (3, 5), and (4, 7).
Do these ordered pairs lie on a linear graph?

Solution

Plot the points on a coordinate plane.
When you join the points together,
you get a straight line.

So, the coordinates do lie on a linear graph.

(In fact, they lie on the graph of $y = 2x - 1$.)

✓ Guided Practice

- 13.** Plot the graph of the function $y = 5x - 4$.
You found some x and y pairs in Guided Practice Exercise 11.
- 14.** Plot the graph of the function $y = 2x - 6$.
You found some x and y pairs in Guided Practice Exercise 12.

✓ Independent Practice

In Exercises 1–3, use the values of x to evaluate the following equation: $y = 2x - 9$

- 1.** $x = 4$ **2.** $x = -6$ **3.** $x = -10$

4. Fill in a table for the equation $y = -5x + 3$ ready for it to be graphed on a coordinate plane. Use the x -values $-1, 0, 1,$ and 2 .

In Exercises 5–6, determine whether the set of ordered pairs lies on a linear graph.

- 5.** (0, 1), (1, 3), (2, 3), (3, 5). **6.** (1, 1), (2, 2), (3, 3), (4, 4).

7. Explain whether the point D with coordinates (4, -6) is on the line $y = -5x + 14$.

8. Construct a table to find some points on the graph of $y = 3x - 5$. Plot the values on a coordinate plane and draw the graph.

9. Construct a table to find some points on the graph of $y = \frac{2}{3}x - 6$. Plot the values on a coordinate plane and draw the graph.

Now try these:

Lesson 4.1.1 additional questions — p452

Round Up

*When you draw a graph of a linear equation you always get a **straight line**, and all the points on the graph represent solutions to the equation. It's important to understand this when you look at solving systems of equations in the next Lesson.*

Lesson 4.1.2

Systems of Linear Equations

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

Algebra and Functions 1.5

Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

What it means for you:

You'll learn what systems of linear equations are and understand how their solutions are shown by graphs.

Key words:

- system of equations
- linear equation
- solving
- intersection

In the last Lesson you graphed linear equations and saw how every point on a line is a solution to the equation of the line. In this Lesson you'll use this idea to solve a *system of equations*.

A System is a Set of Linear Equations

A system of linear equations is a **set** of two or more linear equations in the **same variables**. The equations $y = 2x + 2$ and $y = -3x - 8$ are a system of equations in the two variables x and y .

Example 1

Write a system of linear equations to represent the following statement:
“ **y is three times x and the sum of y and x is 8**”

Solution

You need to write **two equations** that **both** need to be true for the statement to be true.

The first part says, “ y is three times x ,” so **$y = 3x$** .

The second part says, “the sum of y and x is 8,” so **$y + x = 8$** .

These two equations form a **system of linear equations**.

The **solutions** to a system of equations have to satisfy all the equations at the same time. So the solution to the system of equations **$y = 3x$** and **$y + x = 8$** is **$x = 2$ and $y = 6$** . These values make **both** equations true.

✓ Guided Practice

Write systems of equations to represent the following statements.

1. x subtracted from y is 3, and y is twice x .
2. Bob buys two melons at $\$y$ each and three avocados at $\$x$ each. He is charged $\$9$ altogether. Melons cost $\$2$ more than avocados.

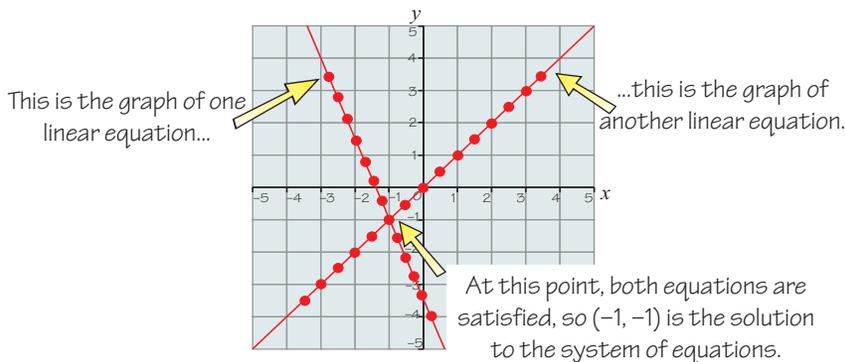
You Can Solve Systems of Equations Graphically

All points on the graph of a linear equation have **x - and y -values that make that equation true**. Points on the graph of another linear equation in a system have x - and y -values that make that equation true.

Where the graphs of two linear equations in a system **intersect**, the x - and y -values satisfy **both equations**. This intersection point is a **solution to both equations**, and so is the **solution to the system**.

Check it out:

Two straight lines that aren't parallel, or exactly the same, can only intersect each other once. So there will only ever be one solution to a system of linear equations like this. Parallel lines never cross, so there would be no solution. And if the lines are the same, there would be infinitely many solutions.



The **solution** to a system of linear equations in two variables is the **point of intersection (x, y)** of their graphs.

So you can solve a system of linear equations by **plotting the graph** of each equation and finding out where they cross.

Example 2

Solve the following system of equations by graphing:

$$y = 2x - 1$$

$$y = x - 2$$

Solution

Draw tables to find coordinates of some points on each graph.

$$y = 2x - 1$$

x	y	Ordered Pair (x, y)
2	3	$(2, 3)$
1	1	$(1, 1)$
0	-1	$(0, -1)$
-1	-3	$(-1, -3)$
-2	-5	$(-2, -5)$

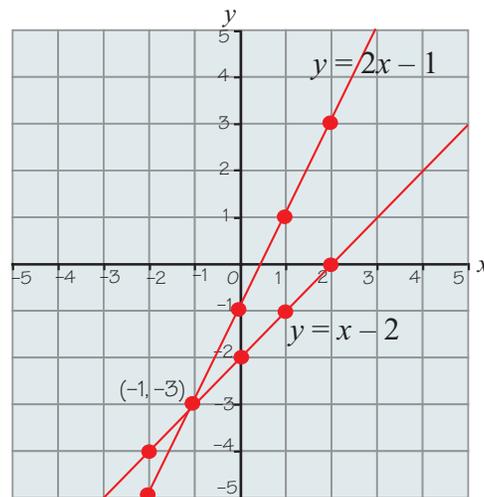
$$y = x - 2$$

x	y	Ordered Pair (x, y)
2	0	$(2, 0)$
1	-1	$(1, -1)$
0	-2	$(0, -2)$
-1	-3	$(-1, -3)$
-2	-4	$(-2, -4)$

Don't forget:

This is the same method that you learned last Lesson for plotting linear equations. If you can't remember the steps, go back and do a bit more practice on graphing equations first.

Now **plot both graphs on the same coordinate plane.**



Read off the point of intersection — it is $(-1, -3)$.

So $(-1, -3)$, or $x = -1, y = -3$, is the solution to the system of equations.

✓ Guided Practice

Solve the systems of equations in Exercises 3–4 by graphing.

3. $y = x + 2$ and $y = -2x + 5$

4. $y = x - 6$ and $y = -x + 2$

Always Check Your Solution

It's easy to make mistakes when graphing, so you should always **test the solution** you get by putting it into both equations and checking it makes them both **true**.

Example 3

Check that $(-1, -3)$ is a solution to the system of equations $y = 2x - 1$ and $y = x - 2$.

Solution

Check the solution by **substituting** the x - and y -values into both equations:

Equation 1: $y = 2x - 1 \Rightarrow -3 = 2(-1) - 1 \Rightarrow -3 = -3$ — **true**

Equation 2: $y = x - 2 \Rightarrow -3 = -1 - 2 \Rightarrow -3 = -3$ — **true**

So $(-1, -3)$ is a solution to the system of equations.

Don't forget:

The first number in an ordered pair is always the x -value — (x, y) .

✓ Guided Practice

5. Check your answer from Guided Practice Exercise 3 by substituting it back into the equations.

6. Check your answer from Guided Practice Exercise 4 by substituting it back into the equations.

✓ Independent Practice

1. Explain whether it is possible to have two solutions to a system of two linear equations.

2. Describe the situation in which there is no solution to a system of two linear equations.

3. Solve the following system of equations by graphing.

Check your solution by substituting.

$$y = x + 3$$

$$y = -x - 1$$

4. Graph the following two equations. Explain why this system of equations has no solution.

$$y = x - 2$$

$$y = x - 6$$

Now try these:

Lesson 4.1.2 additional questions — p452

Round Up

In this Lesson you learned how to write **systems of linear equations**, and how their single solution can be read from a **graph**. In grade 8 you'll **also** solve systems of linear equations **algebraically**.

Lesson 4.1.3

Slope

California Standards:

Algebra and Functions 3.3

Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same, and know that the ratio ("rise over run") is called the slope of a graph.

What it means for you:

You'll learn what the slope of a graph is and how to calculate it.

Key words:

- slope
- steepness
- ratio

Check it out:

The distance you go across a graph is known as the "run," and the distance you go up is known as the "rise." So the

ratio $\frac{\text{change in } y}{\text{change in } x}$ is often called the "rise over run."

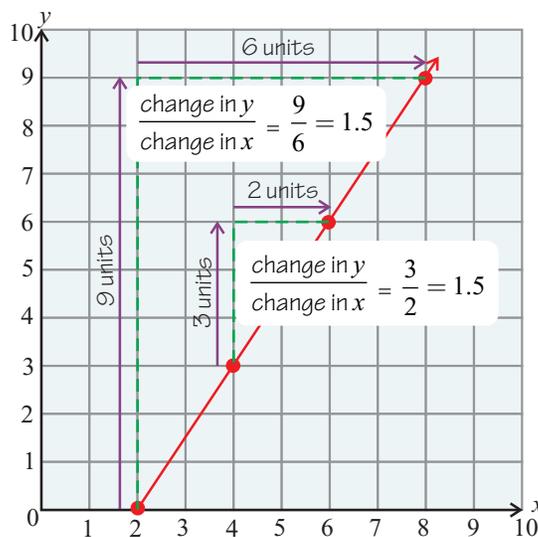
Check it out:

You can use any points on the line to calculate slope — it's normally easiest to choose points that have integer x - and y -values though.

Over the past few Lessons you've been graphing *linear equations* — which have *straight-line graphs*. Some straight-line graphs you've drawn have been *steep*, and others have been more *shallow*. There's a measure for how steep a line is — *slope*. In this Lesson you'll learn how to find the slope of a straight-line graph.

The Slope of a Line is a Ratio

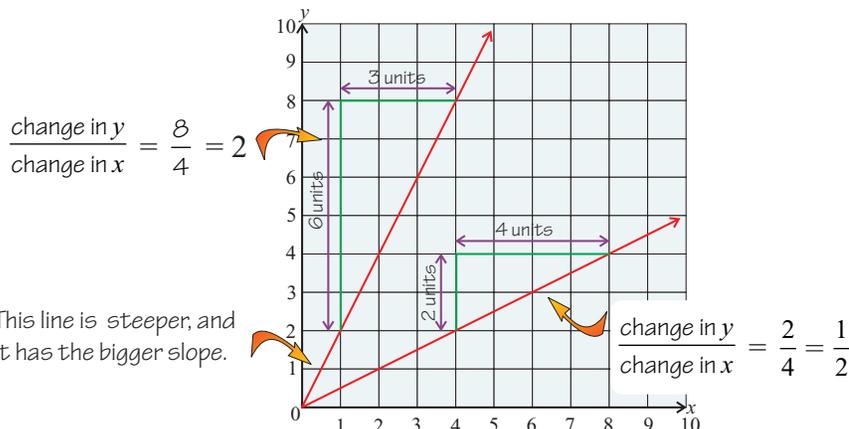
For any straight line, the ratio $\frac{\text{change in } y}{\text{change in } x}$ is always **the same** — it doesn't matter which two points you choose to measure the changes between.



This ratio, $\frac{\text{change in } y}{\text{change in } x}$, is the **slope** of the graph.

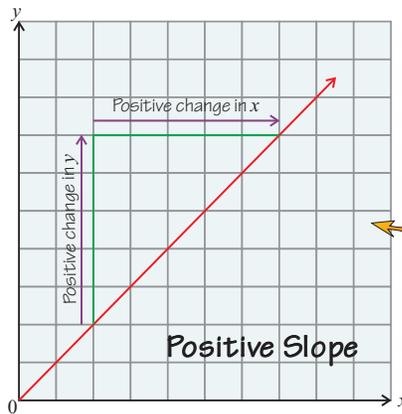
Slope is a Measure of Steepness of a Line

A larger change in y for the same change in x makes the ratio $\frac{\text{change in } y}{\text{change in } x}$ bigger, so the slope is greater.



So a slope is a measure of the **steepness** of a line — **steeper lines** have **bigger slopes**.

Slopes Can Be Positive, Negative, or Zero

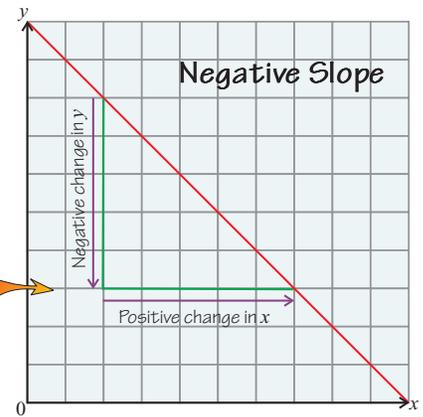


A **positive slope** is an “uphill” slope. The changes in x and y are both positive — as one increases, so does the other.

$$\frac{\text{positive change in } y}{\text{positive change in } x} = \text{positive slope}$$

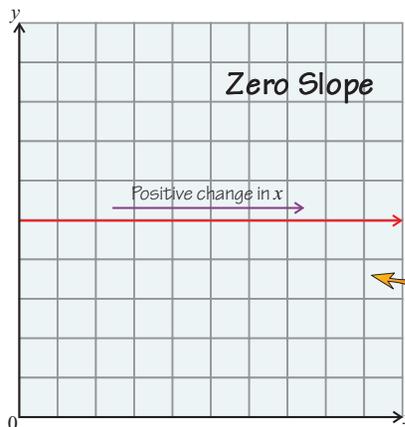
A **negative slope** is a “downhill” slope. The change in y is negative for a positive change in x . y decreases as x increases.

$$\frac{\text{negative change in } y}{\text{positive change in } x} = \text{negative slope}$$



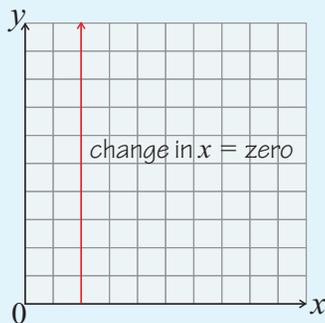
A line with **zero slope** is **horizontal**. There is no change in y .

$$\frac{0 \text{ change in } y}{\text{positive change in } x} = 0 \text{ slope}$$



Check it out:

The slope of a vertical line is undefined. There's a change of zero on the x -axis, and you can't divide by zero.



✓ Guided Practice

- Plot the points $(1, 3)$ and $(2, 5)$ on a coordinate plane. Find the slope of the line connecting the two points.
- Does the graph of $y = -x$ have a positive or negative slope? Explain your answer.

Compute Slopes from Coordinates of Two Points

Instead of counting unit squares to calculate slope, you can use the **coordinates** of any two points on a line. There's a formula for this:

For the line passing through coordinates (x_1, y_1) and (x_2, y_2) :

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Check it out:

It doesn't matter which points you use for (x_1, y_1) and (x_2, y_2) . But you have to make sure you subtract both the x - and y -values of one pair from the x - and y -values of the other pair.

Don't forget:

Subtracting a negative number is the same as adding a positive number. So $3 - (-1)$ is the same as $3 + 1$. Be careful with this when you're calculating slopes.

Don't forget:

You can always count the units on the graph to check — but be careful. The scale might not always be one square to one unit.

Check it out:

Always check your answer is reasonable. Look at the graph you're finding the slope of, and check that if it's downhill, your slope is negative, and if it's uphill, your slope is positive.

Example 1

The graph below is the graph of the equation $y = 2x + 1$. Find the slope of the line.

Solution

Start by **drawing a triangle connecting two points on the graph**.

Choose two points that are easy to read from the graph, for example:

$$(x_1, y_1) = (-1, -1)$$

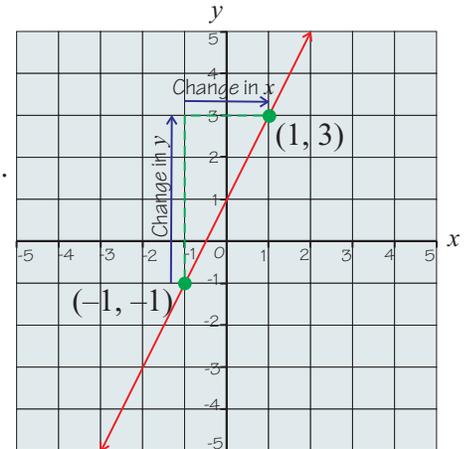
$$(x_2, y_2) = (1, 3)$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

This is the change in y .

$$= \frac{3 - (-1)}{1 - (-1)} = \frac{3 + 1}{1 + 1} = \frac{4}{2} = 2$$

This is the change in x .



So the slope of the graph is 2.

Example 2

Find the slope of the line connecting the points C $(-2, 5)$ and D $(1, -4)$.

Solution

You don't need to draw the line to calculate the slope — you are given the coordinates of two points on the line.

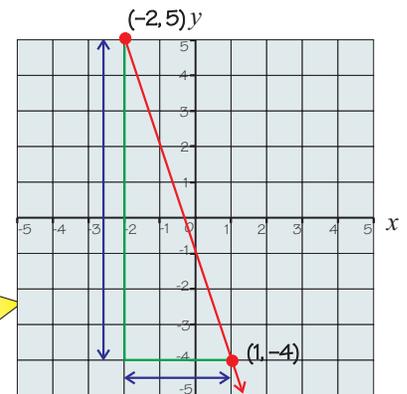
$$(x_1, y_1) = (-2, 5) \text{ and } (x_2, y_2) = (1, -4).$$

Substitute the coordinates into the formula for slope:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 5}{1 - (-2)} = \frac{-9}{3}$$

$$\text{Slope} = -3$$

If you plot these points and draw a line through them, you can see that the slope is negative (it's a "downhill" line).



Guided Practice

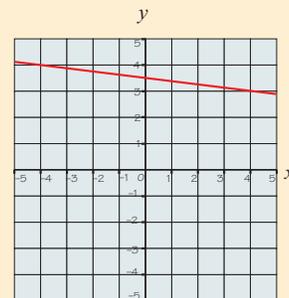
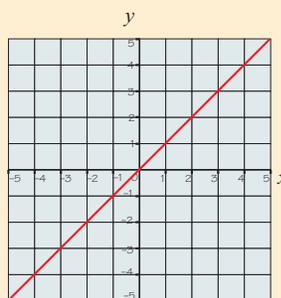
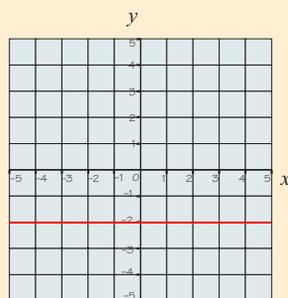
3. Plot the points $(-2, 3)$ and $(2, 5)$ on a coordinate plane.

Find the slope of the line connecting the two points.

4. Plot the graph of the equation $y = 4x - 2$ and find its slope.

Independent Practice

1. Identify whether the slope of each of the lines below is positive, negative, or zero.



On a coordinate plane, draw lines with the slopes given in Exercises 2–5.

2. 3 3. 6 4. -1 5. -4

In Exercises 6–9, find the slope of the line passing through the two points.

6. W (3, 6) and R (–2, 9) 7. Q (–5, –7) and E (–11, 0)
8. A (–12, 18) and J (–10, 6) 9. F (2, 3) and H (–4, 6)

10. The move required to get from point C to D is up six and left eight units. What is the slope of the line connecting C and D?

11. Point G with coordinates (7, 12) lies on a line with a slope of $\frac{3}{4}$. Write the coordinates of another point that lies on the same line.

12. On the coordinate plane, draw a line through the points E (–2, 5) and S (4, 1). Find the slope of this line. On the same plane, draw a line through the points P (–2, –2) and N (4, –6). Find the slope of this line. What can you say about the two lines you have drawn and their slopes?

13. Consider the statement: “The slope of a line becomes less steep if the distance you have to move along the line for a given change in y increases.” Determine whether this statement is true or not.

14. Is it possible to calculate the slope of a vertical line? Explain your answer.

Now try these:

Lesson 4.1.3 additional questions — p452

Round Up

The slope of a line is the *ratio of the change in the y -direction to the change in the x -direction* when you move between two points on the line — it’s basically a measure of how *steep* the line is. *Positive slopes* go “uphill” as you go from left to right across the page, and *negative slopes* go “downhill.” Slope is actually a *rate* — and you’ll be looking at rates over the next few Lessons.

Section 4.2 introduction — an exploration into: Pulse Rates

In this Exploration, you'll measure your *pulse rate* and convert it to several different *unit rates*.

A **rate** is a comparison of two amounts that have **different units of measure**.

For example: 100 miles in 2 hours or $\frac{100 \text{ miles}}{2 \text{ hours}}$.

A **unit rate** is a rate where the second amount is **1**.

You find a unit rate by **dividing the first amount by the second**.

Example

A car travels 100 miles in 2 hours. What is its unit rate?

Solution

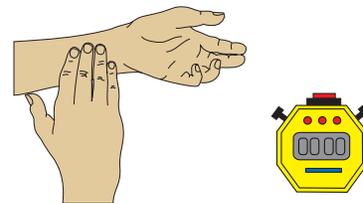
$\frac{100 \text{ miles}}{2 \text{ hours}}$ → **Unit rate = 50 miles/hour, or 50 miles per hour**

Exercises

- Write the unit rate for each.
 - 90 words in 3 minutes
 - 10 feet for 2 inches of height
 - 100 miles on 4 gallons

You'll now make some measurements involving **heart rate**. Work with a partner for this.

Find your pulse on your left wrist, using two fingers of your right hand, as shown. When you've found your pulse, your partner should start the stopclock and say "go." Count how many **pulse beats** you feel, until your partner calls "stop," after **15 seconds**. Write this number down.



Now swap, so that your partner counts his or her pulse, and you time 15 seconds for them.

Exercises

- Write your results in this form: _____ **beats in 15 seconds**.
Now change it into a **unit rate**.
- Pulse rate is usually given in beats per minute (bpm).
Calculate your unit pulse rate in beats per minute.
- Approximately how many times will your heart beat in:
 - 1 hour
 - 1 day
 - 1 week
 - 1 year (365 days in a nonleap-year)

Round Up

The *unit rate* is generally more useful than other rates — it makes it easier to *compare things*. For example, it's difficult to compare 18 beats in 15 seconds with 23 beats in 20 seconds — it's much easier to compare 72 beats per minute with 69 beats per minute.

Lesson 4.2.1

Section 4.2 Ratios and Rates

California Standards:

Measurement and
Geometry 1.3

Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

What it means for you:

You'll learn what rates are and how you can use them to compare things — such as which size product is better value.

Key words:

- rate
- ratio
- fraction
- denominator
- unit rate

Don't forget:

You can simplify ratios by dividing both numbers by a common factor. Rates can be simplified in the same way. For example — you can divide top and bottom of the rate $\frac{3 \text{ inches}}{9 \text{ years}}$ by 3 to get $\frac{1 \text{ inches}}{3 \text{ years}}$.

Check it out:

You must always write the units after a rate. The unit miles per hour could also be written as miles/hour.

Rates are used a lot in daily life. You often hear people talk about speed in miles per hour, or the cost of groceries in dollars per pound. A rate tells you how much one thing changes when something else changes by a certain amount. Imagine buying apples for \$2 per pound — the cost will increase by \$2 for every pound you buy.

Ratios are Used to Compare Two Numbers

You might remember ratios from grade 6. Ratios **compare two numbers**, and **don't have any units**. For example, the ratio of boys to girls in a class might be 5 : 6. There are three ways of expressing a ratio.

The ratio 5 : 6 could also be expressed as “5 to 6” or as the fraction $\frac{5}{6}$.

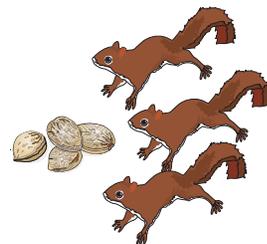
Example 1

There are four nuts between three squirrels. What is the ratio of nuts to squirrels?

Solution

There are 4 nuts to 3 squirrels so the ratio of nuts to squirrels is **4 : 3**.

This could also be written “4 to 3” or $\frac{4}{3}$.



Rates Compare Quantities with Different Units

A rate is a **special kind of ratio**, because it compares two quantities that **have different units**. For example, if you travel at 60 miles in 3 hours you would be traveling at a rate of $\frac{60 \text{ miles}}{3 \text{ hours}}$.

You'd normally write this as a **unit rate**. That's one with a denominator of 1. So $\frac{60 \text{ miles}}{3 \text{ hours}} = \frac{20 \text{ miles}}{1 \text{ hour}}$, or 20 miles per hour.

Example 2

John takes 110 steps in 2 minutes. What is his unit rate in steps per minute?

Solution

110 steps in 2 minutes means a rate of:

$$\frac{110 \text{ steps}}{2 \text{ minutes}} = \frac{55 \text{ steps}}{1 \text{ minute}} = \mathbf{55 \text{ steps per minute.}}$$

Numerator ÷ Denominator Gives a Unit Rate

Check it out:

Rates don't have to be expressed as fractions or whole numbers. Rates can also be decimals.

Dividing the numerator by the denominator of a rate gives the **unit rate**. So, if it costs 2 dollars for 3 apples, the unit rate is the price per apple,

$$\text{which is } \frac{2 \text{ dollars}}{3 \text{ apples}} = 2 \text{ dollars} \div 3 \text{ apples} = 0.66 \text{ dollars per apple.}$$

Example 3

A car goes 54 miles in 3 hours. Write this as a unit rate in miles per hour.

Solution

Divide the top by the bottom of the rate.

$$\frac{54 \text{ miles}}{3 \text{ hours}} = (54 \div 3) \text{ miles per hour} = \mathbf{18 \text{ miles per hour.}}$$

This is a unit rate because the denominator is now 1 (it's equivalent to $\frac{18}{1}$ mi/h).



Check it out:

The unit rate that's calculated here is average speed — there's more on this in Lesson 4.2.3.

Example 4

If a wheel spins 420 times in 7 minutes, what is its unit rate in revolutions per minute?

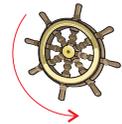
Solution

The rate is $\frac{420}{7}$ revolutions per minute.

Divide the top by the bottom of the rate.

$$(420 \div 7) \text{ revolutions per minute} = \mathbf{60 \text{ revolutions per minute.}}$$

This is a unit rate because 60 revolutions per minute has a denominator of 1 ($60 = \frac{60}{1}$).



✓ Guided Practice

In Exercises 1–3, find the unit rates.

1. \$3.60 for 3 pounds of tomatoes.
2. \$25 for 500 cell phone minutes.
3. 648 words typed in 8 minutes.
4. Joaquin buys 2 meters of fabric, which costs him \$9.50. What was the price per meter?
5. Mischa buys a \$19.98 ticket for unlimited rides at a fairground. She goes on six rides. How much did she pay per ride?

Use Unit Rates to Find the “Better Buy”

Stores often sell **different sizes of the same thing**, such as clothes detergent or fruit juice. A bigger size is often a **better buy** — meaning that you get more product for the same amount of money. But this isn’t always the case, so it’s useful to be able to work out which is the **better buy**.

You can do this by finding the **price for a single unit** of each product. The units can be ounces, liters, meters, or whatever is most sensible.

Check it out:

There are situations where the “better buy” is not actually the most sensible option. For example, a store might sell a product at a much cheaper unit price when you buy 20 — but if you only want 1 of the product, it’s silly paying the extra if you’re not going to use it all.

Example 5

A store sells two sizes of cereal.
Which is the better buy?

Solution

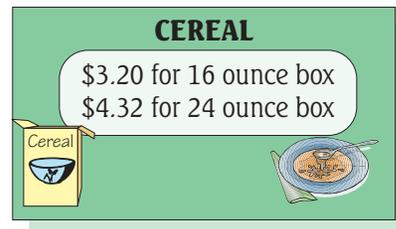
16 ounce box: Rate is $\frac{3.20 \text{ dollars}}{16 \text{ ounces}}$.

Unit rate = $(3.20 \div 16)$ dollars per ounce = **\$0.20 per ounce**

24 ounce box: Rate is $\frac{4.32 \text{ dollars}}{24 \text{ ounces}}$.

Unit rate = $(4.32 \div 24)$ dollars per ounce = **\$0.18 per ounce**

The 24 ounce box is the better buy — the price per ounce is lower.



✓ Guided Practice

6. Determine which phone company offers the better deal:

Phone Company A: \$40 for 800 minutes.

Phone Company B: \$26 for 650 minutes.

7. Determine which is the better deal on carrots: \$1.20 for 2 lb or \$2.30 for 5 lb.

✓ Independent Practice

In Exercises 1–6, write each as a unit rate.

1. \$4.50 for 6 pens 2. 100 miles in 8 h 3. 200 pages in 5 days

4. 120 miles in 2 h 5. \$400 for 10 items 6. \$36 in 6 hours

7. Peanuts are either \$1.70 per pound or \$8 for 5 pounds.
Which is the better buy?

8. Lemons sell for \$4.50 for 6, or \$10.50 for 15.
Which is the better buy?

9. “\$40 for 500 pins or \$60 for 800 pins.” Which is the better buy?

Now try these:

Lesson 4.2.1 additional questions — p453

Round Up

Rates *compare one thing to another and always have units*. A *unit rate* is a rate that has a denominator of one. In the next Lesson you’ll see how rate is related to the slope of a graph.

Lesson 4.2.2

Graphing Ratios and Rates

California Standards:

Algebra and Functions 3.4

Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle).

Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.

What it means for you:

You'll learn how to graph two quantities in a ratio or rate and understand what the slope means in this context.

Key words:

- straight-line graph
- slope
- rate

Check it out:

When one quantity in a ratio is zero, the other quantity must be zero also. So you know a graph of quantities in a ratio must go through the point (0, 0).

When you're buying apples, the price you pay increases *steadily* the more apples you buy. If you *plot a graph* of the weight of apples against the cost, you get a *straight line*. The *slope* of this line is the same as the *unit rate* — the cost per pound.

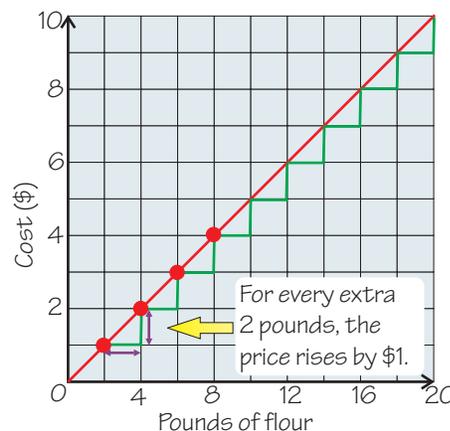
Quantities in Ratios Make Straight-Line Graphs

When you **increase** one quantity in a ratio or rate, the other quantity **increases in proportion** with it.

For example, if flour cost **\$0.50 per pound**, you know that two pounds of flour would cost \$1. This is because if you double the amount of flour, you also double the cost. In the same way, if you buy ten times as much flour, it costs ten times as much.

You can represent the cost of different amounts of flour on a **graph**:

Pounds of flour	Cost
2	$2 \times \$0.50 = \1
4	$4 \times \$0.50 = \2
6	$6 \times \$0.50 = \3
8	$8 \times \$0.50 = \4



By joining these points you get a **straight-line graph**. You get a **straight-line graph** whenever you plot quantities in a ratio or rate.

Example 1

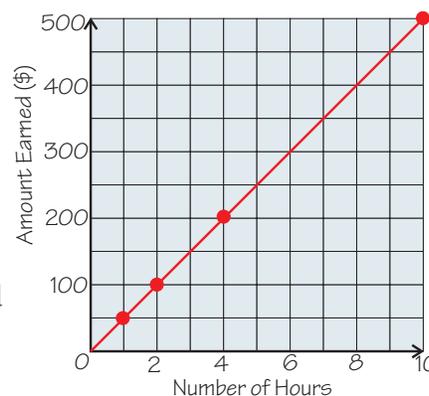
Suzi the decorator earns \$50 per hour. Plot a graph to show how the amount Suzi earns increases with the amount of time she works.

Solution

Number of Hours	Amount She Earns
1	\$50
2	$2 \times \$50 = \100
4	$4 \times \$50 = \200
10	$10 \times \$50 = \500

Step 1: You know she earns \$50 per hour. Draw a table of her earnings for different numbers of hours.

Step 2: Plot a graph from your table to show the number of hours worked against the amount she earns.



✓ Guided Practice

1. You can buy 5 kg of sand from a toy store for \$3.00. If the sand always costs the same amount per kilogram, draw a graph to represent the relationship between the cost and the mass of sand.
2. It costs \$1 to run an electricity generator for half an hour. Draw a graph to represent the relationship between cost and time.

Use the Graph to Find Unknown Values

Once you've drawn a graph, you can use it to find **unknown values**.

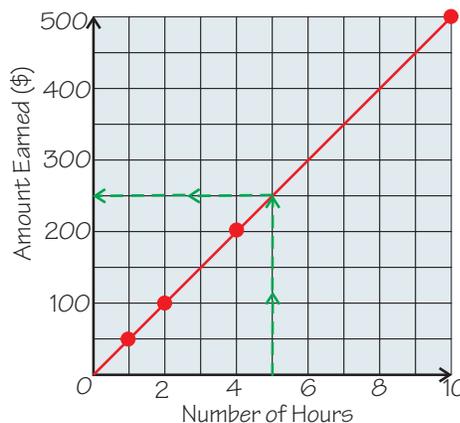
Example 2

Suzi the decorator worked for 5 hours on Monday. Use the graph in Example 1 to work out how much she earned.

Solution

She worked for 5 hours, so find 5 hours on the horizontal axis.

Go up to the line, and then across to find the amount earned for 5 hours' work.



On Monday Suzi earned \$250 for 5 hours' work.

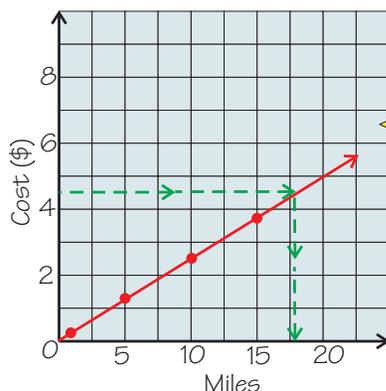
Example 3

A car rental company charges \$0.25 per mile driven. Plot a graph to show this rate and use it to find how far you could drive for \$4.50.

Solution

Draw a table that will allow you to plot a straight-line graph of miles traveled and price charged.

Miles	Cost
1	\$0.25
5	\$1.25
10	\$2.50
15	\$3.75



Plot the graph on a coordinate plane.

To find the number of miles you could drive for \$4.50, find \$4.50 on the vertical axis.

Go across to the line, and read off the corresponding **number of miles**.

You can drive 18 miles for \$4.50.

Check it out:

You could use the graph to find the cost for any distance driven — for example, 5.3 miles. In reality, the company would probably charge to the nearest mile.

✓ Guided Practice

- Rita is filling a sand box at the day camp where she works. She needs 8 kg of sand. Use your graph from Guided Practice Exercise 1 to find the approximate cost of the sand.
- Use your graph from Guided Practice Exercise 2 to find the approximate price of running the generator for 3 hours.

The Slope of the Graph Tells You the Rate

The **slope** of a graph of two quantities is the **unit rate**. It tells you **how much** the quantity on the vertical axis changes when the quantity on the horizontal axis changes by **one unit**.

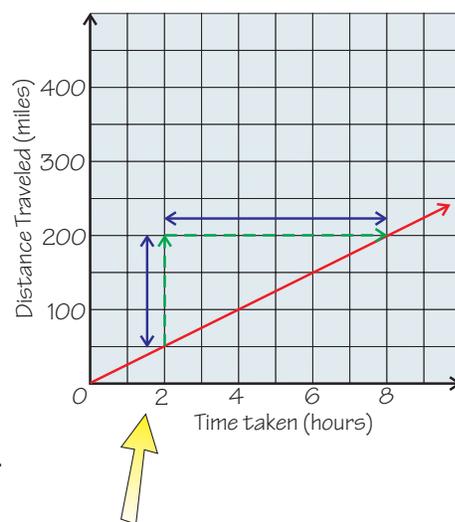
The **slope** of a straight-line graph is found using this formula:

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

Example 4

This graph shows the progress of Selina, who is traveling at a constant rate.

Use the graph to find how many miles per hour Selina is traveling at.



Solution

The slope is the **change in y** divided by the **change in x** .

On this graph, this is the **distance traveled divided by the time taken**, which is a unit rate in miles per hour.

Find **two points on the line**, and find the vertical change and the horizontal change between them by drawing a triangle onto the graph.

Change in y = **150 miles**

Change in x = **6 hours**

$$\text{So, Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{150 \text{ miles}}{6 \text{ hours}}$$

$$\begin{aligned} \text{Rate} &= (150 \div 6) \text{ miles per hour} \\ &= \mathbf{25 \text{ miles per hour}} \end{aligned}$$

Selina is traveling at 25 miles per hour.

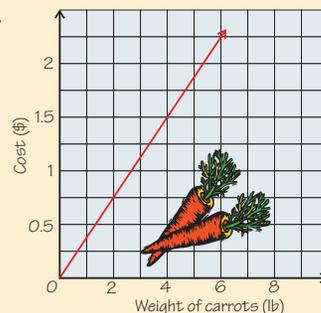
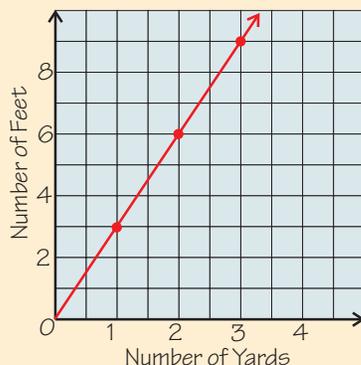
Check it out:

The slope of a distance-time graph (with time on the horizontal axis) is always speed.

✓ Guided Practice

5. The y -axis of a graph shows the cost of hiring an engineer, and the x -axis shows the number of hours you get the engineer's services for. What does the slope of the graph tell you?

6. The graph on the right shows the price of carrots in a grocery store. Use the graph to find the unit rate for the price of carrots.



7. The graph on the left shows the number of feet against the equivalent number of yards. Use the graph to find the number of feet in a yard.

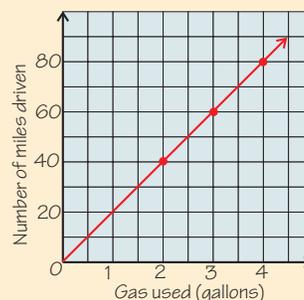
✓ Independent Practice

1. Water is dripping into an empty tank from a pipe. The water depth is increasing by a depth of 4 inches every 24 hours. Draw a graph and use it to find the depth of water in the tank after 36 hours. Use the graph to find the unit rate of water depth increase.

2. A store earns about \$100,000 over seven months. Draw a graph and use it to estimate the store's earnings for a year.

3. There are 12 inches in a foot. Draw a graph and use it to find the number of inches in 9 feet. Then use it to estimate the number of feet in 50 inches.

4. The graph on the right shows the number of miles John drives, and the number of gallons of gas he uses. Find the number of miles John's car does per gallon.



5. Plot a graph of circle diameter against circumference. Put diameter on the horizontal axis. Find the slope of the graph. What does this slope represent?

6. Plot a graph of circle radius against circumference. Put radius on the horizontal axis. Compare the slope of your graph to the slope of your graph from Exercise 5. Explain any differences.

Now try these:

Lesson 4.2.2 additional questions — p453

Don't forget:

Circumference = $\pi \times$ diameter
Diameter = $2 \times$ radius

Round Up

When you graph quantities that are always in the *same ratios* you get *straight-line graphs*. You can use these graphs to *convert from one quantity to another*. They aren't the only way to convert quantities — the next Lesson is about *conversion factors*, which are another way.

Lesson 4.2.3

Distance, Speed, and Time

California Standards:

Algebra and Functions 4.2
Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

What it means for you:

You'll learn the formula for speed, and how to use it to solve problems.

Key words:

- speed
- distance
- time
- formula

Check it out:

Gila might not have walked at a steady speed for the entire 8 hours. That's why we have to calculate **average speed**.

Check it out:

Miles per hour is often shortened to mi/h or mph. This means the same as miles \div hours, and $\frac{\text{miles}}{\text{hours}}$.

Don't forget:

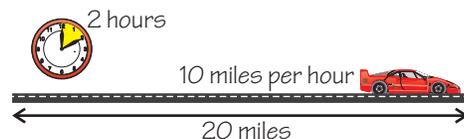
Speed can be measured in **any unit of distance per unit of time**. If you divide a distance in kilometers by a time in hours, your answer will be in kilometers per hour.

Speed is a rate — it's the distance you travel per unit of time. 55 miles per hour is the speed limit on some roads. If you drive steadily at this speed, you'll travel 55 miles every hour. There's a formula that links speed, distance, and time — and you're going to use it in this Lesson.

Speed is a Rate

Speed is a rate. It is the **distance** traveled in a certain amount of **time**.

Speed can be measured in lots of different units, such as miles per hour, meters per second, inches per minute...



The formula for speed is:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Example 1

Gila walked 6 miles in 8 hours. What was Gila's average speed?

Solution

Use the formula, and substitute in the values from the question.

$$\begin{aligned}\text{speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{6 \text{ miles}}{8 \text{ hours}} \\ &= (6 \div 8) \text{ miles per hour} \\ &= 0.75 \text{ miles per hour}\end{aligned}$$

Gila's average speed was 0.75 miles per hour.

Rearrange the Equation to Find Other Unknowns

You can **rearrange** the speed formula, and use it to find **distance** or **time**.

To change the equation $\text{speed} = \frac{\text{distance}}{\text{time}}$ into an equation that gives

distance in terms of speed and time, **multiply both sides of the equation by time**.

$$\text{speed} \times \text{time} = \frac{\text{distance} \times \cancel{\text{time}}}{\cancel{\text{time}}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

Example 2

Alyssa runs for 0.5 hours at a speed of 11 kilometers per hour. How far does she run?

Solution

Use the formula for distance, and substitute the values for speed and time.

$$\begin{aligned} \text{Distance} &= \text{speed} \times \text{time} \\ &= 11 \text{ kilometers per hour} \times 0.5 \text{ hours} \\ &= \mathbf{5.5 \text{ kilometers}} \end{aligned}$$

You can find the equation for time in terms of speed and distance in a similar way.

Example 3

Andy is planning a walk. He walks at an average speed of 3 miles per hour, and plans to cover 15 miles. How long should his walk take him?

Solution

You need to rearrange the speed formula first.

$$\text{distance} = \text{speed} \times \text{time}$$

$$\frac{\text{distance}}{\text{speed}} = \frac{\cancel{\text{speed}} \times \text{time}}{\cancel{\text{speed}}} \quad \text{Divide both sides by speed}$$

$$\mathbf{\text{time} = \frac{\text{distance}}{\text{speed}}}$$

Now you can use the formula to answer the question:

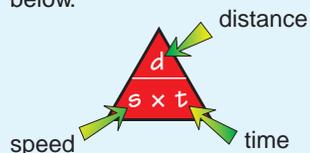
$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{15 \text{ miles}}{3 \text{ miles per hour}}$$

$$\text{time} = (15 \div 3) \text{ hours} = 5 \text{ hours}$$

Andy's walk should take him 5 hours.

Check it out:

Formula triangles help you find the formula you want. The formula triangle for speed-distance-time is shown below.



To use it, cover up the thing you want to find, and the equation is what's left.

So if you want to find distance, cover up "d" and you're left with "s × t."

If you want to find time, cover up "t" and you're left with " $\frac{d}{s}$."

**Guided Practice**

- Juan ran in a marathon that was 26 miles long. If his time was 4 hours, what was his average speed?
 - Moesha goes to school every day by bike. The journey is 6 miles long, and takes her 0.6 hours. What is her average speed?
 - Monica travels 6 miles to work at a speed of 30 miles per hour. How long does the journey take her each morning?
- Josh has been walking for 5 hours at a speed of 4 miles per hour.
- His walk is 22 miles long. How far does he have left to walk?
 - How much longer will he take if he continues at the same speed?

Some Problems Might Need More than One Step

Example 4

On a three-hour bike ride, a cyclist rode 58 miles. The first two hours were downhill, so the cyclist rode 5 miles per hour quicker than she did for the last hour.

- What was her speed for the first two hours?
- What was her speed for the last hour?

Solution

Let the cyclist's speed for the first two hours be $(x + 5)$ miles per hour. So her speed for the last hour = x miles per hour.

You need to write an **equation** using the information you're given.

Total distance = distance traveled in first two hours + distance traveled in last hour

$$58 = (x + 5) \times 2 + (x) \times 1$$

$$58 = 2x + 10 + x$$

$$58 = 3x + 10$$

$$48 = 3x \Rightarrow x = 16$$

distance = speed \times time

- The speed for the first two hours was $(x + 5) = 16 + 5 = 21$ mi/h
- So the speed for the last hour was $x = 16$ mi/h

Check it out:

This answer seems reasonable — the cyclist rode 5 mi/h faster for the first two hours. 21 mi/h and 16 mi/h fit this and sound reasonable speeds for a cyclist.

Guided Practice

6. Train A travels 20 mi/h faster than Train B. Train A takes 3 hours to go between two cities, and Train B takes 4 hours to travel the same distance. How fast does each train travel?

Independent Practice

- A mouse ran at a speed of 3 meters per second for 30 seconds. How far did it travel in this time?
- A slug crawls at 70 inches per hour. How long will it take it to crawl 630 inches?
- A shark swims at 7 miles per hour for 2 hours, and then at 9 miles per hour for 3 hours. How far does it travel altogether?
- Bike J moves at a rate of x miles per hour for 2 hours. Bike K travels at $0.5x$ miles per hour for 4 hours. Which bike travels the furthest?
- On a two-day journey, you travel 500 miles in total. On the first day you travel for 5 hours at an average speed of 60 mi/h. On the second day you travel for 4 hours. What's your average speed for these 4 hours?

Now try these:

Lesson 4.2.3 additional questions — p453

Round Up

You need to remember the formula for *speed*. If you know this, you can *rearrange* it to figure out the formulas for *distance* and *time* when you need them — so that's two less things to remember.

Lesson 4.2.4

Direct Variation

California Standards:
Algebra and Functions 4.2
Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

What it means for you:
You'll learn how to use the fact that two things are in proportion to solve problems.

Key words:

- direct variation
- proportion
- ratio
- variables
- constant of proportionality

Check it out:

You could also divide the quantities the other way around ($x \div y$) to get a different constant of proportionality.

Check it out:

Corresponding lengths in similar shapes show direct variation. If you divide a length on one shape by the corresponding length on another, you get a constant that's the same whichever length you pick — this is the scale factor. See Lesson 3.4.3.

Check it out:

You could have divided n by m to get a constant of proportionality, k , of 0.5. This means your equation is $\frac{n}{m} = k$. This rearranges to $m = \frac{n}{k}$, so when you substitute in $k = 0.5$ and $n = 8$, you still get 16.

Direct variation is when two things change in proportion to each other — this means that the ratio between the two quantities always stays the same. For example, if fencing is sold at \$15.99 per meter, then you can say that the length of fencing bought and the cost show direct variation.

Direct Variation Means Proportional Change

Two quantities show direct variation if the ratio between them is always the same.

If you have two quantities, x and y , that show **direct variation**, the ratio between them, $\frac{y}{x}$, is **always the same** — it's a constant.

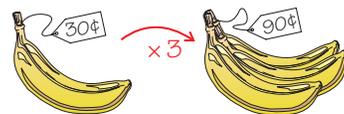
If you call this constant k , then $\frac{y}{x} = k$. This rearranges to $y = kx$.

$$y = kx$$

k is known as the “**constant of proportionality**.”

You've seen things that show direct variation before when you learned about rates.

For example, imagine a store selling bananas at a certain price per banana. The price per banana is **constant** and doesn't change no matter how many bananas you have:



What your bananas cost = price per banana \times number of bananas.

What your bananas cost and the number of bananas are the **variables**, and the price per banana is the **constant of proportionality**.

Example 1

If m and n show direct variation, and $m = 4$ when $n = 2$, find m when $n = 8$.

Solution

First find the constant of proportionality: $k = \frac{m}{n} = \frac{4}{2} = 4 \div 2 = 2$

The constant of proportionality, $k = 2$.

The formula rearranges to $m = kn$.

So substitute in the value for k and the new value for n .

$$\begin{aligned} m &= kn \\ &= 2 \times 8 = 16 \end{aligned}$$

So **$m = 16$** when $n = 8$.

Example 2

A person's earnings and the number of hours they work show direct variation. An employee earns \$600 for 40 hours' work. Find their earnings for 60 hours' work.

Solution

First write a **direct variation equation**:

$$k = \text{earnings} \div \text{number of hours worked}$$

Now **substitute in** the pair of variables you know and find k :

$$k = \$600 \div 40 \text{ hours}$$

$$k = \text{\$15 per hour.}$$

The direct variation equation rearranges to:

$$\text{Earnings} = k \times \text{number of hours worked}$$

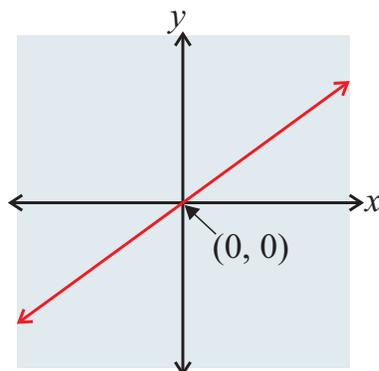
$$\begin{aligned} \text{So, earnings for 60 hours} &= \$15 \text{ per hour} \times 60 \text{ hours} \\ &= \text{\$900} \end{aligned}$$

✓ Guided Practice

- y and x show direct variation and $y = 4$ when $x = 6$. Find x when $y = 9$.
- s and t show direct variation and $s = 70$ when $t = 10$. Find s when $t = 7$.
- The cost of gas varies directly with the number of gallons you buy. If 10 gallons of gas costs \$25, what is the cost per gallon of gas? What does 18 gallons of gas cost?

Direct Variation Graphs are Straight Lines

Graphs of quantities that show **direct variation** are always **straight lines** through the **point (0, 0)**.



Check it out:

The line of a direct variation graph always crosses the origin (0, 0). If one quantity is 0, the other quantity will be 0 too.

The **slope** of this direct variation graph is equal to $\frac{y}{x}$.

As $\frac{y}{x} = k$, the slope is the same as the **constant of proportionality**, k .

Example 3

If y and x show direct variation, and $x = -1$ when $y = 2$:

- Write an equation relating x and y .
- Graph this equation.
- Find the value of y when $x = 1$.

Solution

a) Because y and x show direct variation, $\frac{y}{x} = k$.

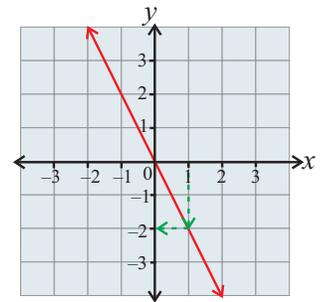
Now you need to find out the value of k : $k = \frac{y}{x} = \frac{2}{-1} = -2$

So $\frac{y}{x} = -2$, or $y = -2x$.

b) The graph of $y = -2x$ must go through $(0, 0)$.
Because $x = -1$ when $y = 2$, it must go through $(-1, 2)$.

The **slope** of the line is equal to k , and $k = -2$.

c) Reading from the graph, when $x = 1$, $y = -2$.



Check it out:

You could have found the value of y when $x = 1$ from the equation.

$y = -2x$, so if $x = 1$, then $y = -2(1) = -2$.

Guided Practice

When you add a weight to the end of a spring, the spring stretches. The amount of weight you add (w) and the distance the spring stretches (d) show direct variation (until you have added 100 pounds).

- If 30 pounds causes a stretch of 25 centimeters, write an equation relating w and d .
- Graph the direct variation. Use the graph to estimate how far the spring stretches when 50 pounds is added.

Independent Practice

In Exercises 1–2, x and y show direct variation, with $y = kx$.

- If $y = 36$ and $x = 6$, find k .
- If $y = 12$ and $k = 2$, find x .
- You can buy 6 pounds of apples for \$4.50. Given that the cost and weight show direct variation, find the cost of 5 pounds of apples.
- The graph of a line crosses the y -axis at 1. Explain whether the line could represent a direct variation.
- The stopping distance of a toy cart varies directly with its mass. A 2 kg cart stops after 30 cm. Write an equation linking the stopping distance and the mass of the cart. Graph this equation and find the stopping distance of a 3.5 kg cart.
- A graph showing direct variation is drawn on the coordinate plane. The line goes from top left to bottom right. What can you say about the constant of proportionality?

Now try these:

Lesson 4.2.4 additional questions — p454

Round Up

If two things *vary directly*, the *ratio* between them is *constant*. Most *rates* are examples of direct variation, like the *price of something per kilogram* — the price and weight stay in *proportion*.

Lesson 4.3.1

Section 4.3 Converting Measures

California Standards:

Measurement and
Geometry 1.1

Compare weights, capacities, geometric measures, times, and temperatures **within** and between **measurement systems** (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

What it means for you:

You'll learn how to convert between different units of length, weight, and capacity in the customary and metric systems.

Key words:

- customary system
- metric system
- conversion table

Don't forget:

"Capacity" is often called "volume."

Don't forget:

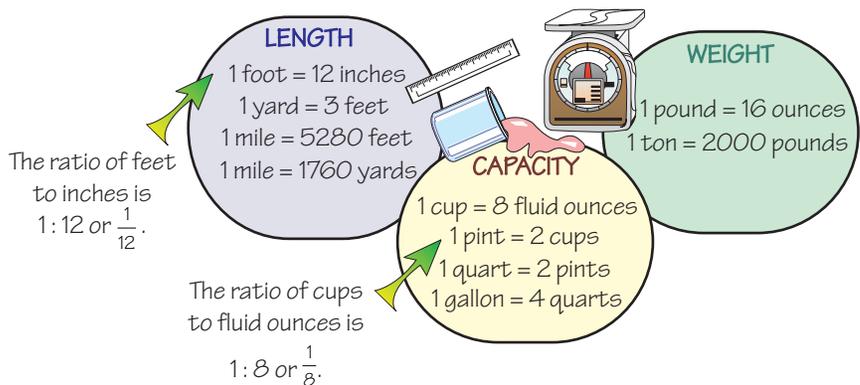
You'll often see units abbreviated — for instance, mL for milliliters. The strangest one is the pound — which is abbreviated to lb.

There are lots of circumstances where you might want to *convert from one unit to another*. For instance — say you have 2 *pounds* of flour and you want to know how many cakes you can make that each need 6 *ounces* of flour. This Lesson is about how you *convert between different units of measurement*.

The Customary System — Feet, Pounds, and Pints...

The **customary system** includes units such as **feet, pints, and pounds**.

To convert between the different units in the customary system you can use a **conversion table**. A conversion table tells you **how many of one unit is the same as another unit**.



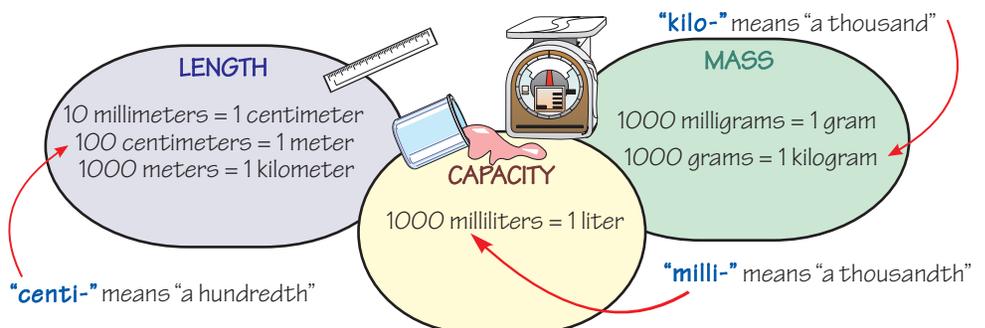
Guided Practice

In Exercises 1–6, find the ratio between the units.

- yards : feet
- quarts : pints
- tons : pounds
- ounces : pounds
- yards : miles
- quarts : gallons

Metric Units Have the Same Prefixes

The **meter, the liter, and the gram** are metric units, and the prefixes "**kilo-**," "**centi-**," and "**milli-**" are used in this system:



✓ Guided Practice

In Exercises 7–12, find the ratio between the units.

7. millimeters : meters

8. liters : milliliters

9. grams : kilograms

10. kilometers : meters

11. milliliters : liters

12. grams : milligrams

Convert Between Units by Setting Up Proportions

You might remember proportions from grade 6. They're a good way of converting between different units.

Example 1

How many yards are equivalent to 58 feet?

Solution

Step 1: The **ratio** of yards to feet is **1 : 3** or $\frac{1}{3}$.

Step 2: You want to find the number of yards in 58 feet.

So write another ratio — **the ratio of yards to feet is $x : 58$** , where x stands for the number of yards in 58 feet.

You know that the ratios 1 : 3 and $x : 58$ have to be **equivalent** — which means they **simplify to the same thing**, because there are always 3 feet in every yard.

So you can write these ratios as an equation — this is called a **proportion**.

$$\frac{1}{3} = \frac{x}{58}$$

Step 3: **Solve** the proportion for x using **cross-multiplication**.

$$1 \times 58 = 3 \times x$$

$$58 = 3x$$

$$x = 58 \div 3 = 19.333\dots = \mathbf{19.3}$$

So 58 feet is approximately equivalent to 19.3 yards.

Step 4: **Check the reasonableness of your answer.**

The conversion table tells you that there are 3 feet in every yard, so estimate:

$$20 \text{ yards} \times 3 \text{ ft per yard} = 60 \text{ feet.}$$

The estimation is close to the answer — so the answer is reasonable.

Don't forget:

To cross-multiply the following proportion:

$$\frac{a}{b} = \frac{c}{d}$$

1. Multiply both sides of the equation by b , and cancel.

$$\frac{a}{\cancel{b}} \times \cancel{b} = \frac{c}{d} \times b \Rightarrow a = \frac{c}{d} \times b$$

2. Multiply both sides of the equation by d , and cancel.

$$a \times d = \frac{c}{\cancel{d}} \times b \times \cancel{d}$$

$$\Rightarrow a \times d = c \times b$$

So if $\frac{a}{b} = \frac{c}{d}$, then $a \times d = c \times b$

Example 2

How many kilometers are equivalent to 7890 meters?

Solution

There are 1000 meters in a kilometer, so the ratio of meters to kilometers is $1000 : 1$ or $\frac{1000}{1}$.

Write a proportion where there are x kilometers in 7890 meters:

$$\frac{1000}{1} = \frac{7890}{x}$$

Cross-multiply and **solve** for x :

$$1000 \times x = 7890 \times 1$$

$$1000x = 7890$$

$$x = 7890 \div 1000 = 7.89$$

So 7.89 kilometers is equivalent to 7890 meters.

Check the reasonableness: there are 1000 meters in a kilometer, so estimate $8 \text{ km} \times 1000 \text{ m} = 8000 \text{ m}$ — **the answer is reasonable.**

Check it out:

Make sure you set up your proportion correctly. In this example, meters are on the top on both sides of the proportion, and kilometers are on the bottom.

$$\frac{1000 \text{ m}}{1 \text{ km}} = \frac{7890 \text{ m}}{x \text{ km}}$$

✓ Guided Practice

In Exercises 13–18, find the missing value.

13. 6 miles = ? feet 14. 40 tons = ? pounds 15. 18 quarts = ? pints
16. 4560 ml = ? l 17. 45 g = ? kg 18. 670 km = ? m

✓ Independent Practice

In Exercises 1–6, find the ratio between the units.

1. gallons : quarts 2. ounces : pounds 3. cups : pints
4. meters : centimeters 5. liters : milliliters 6. milligrams : grams

In Exercises 7–12, find the missing value.

7. 560 cm = ? m 8. 8.2 kg = ? g 9. 9.67 l = ? ml
10. 20 inches = ? feet 11. 5 cups = ? pints 12. 5 pounds = ? tons

13. A recipe uses 8 ounces of butter and 12 ounces of flour.

The supermarket sells butter and flour by the pound. How many pounds of butter and flour do you need for the recipe?

14. Jackie wants to drink 2 liters of water a day. She sees a bottle of water that contains 250 milliliters. How many bottles would she need to drink in order to get the full 2 liters?

Now try these:

Lesson 4.3.1 additional questions — p454

Round Up

There are two main systems of measurement — the *customary system* and the *metric system*.

You can convert between units in a system by *setting up proportions* and *solving them*.

You'll often have to convert *between* the systems too, which you'll learn about in the next Lesson.

Lesson 4.3.2

Converting Between Unit Systems

California Standards:

Measurement and Geometry 1.1

Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

What it means for you:

You'll learn how to convert from a unit in one measurement system to a unit in another measurement system. You'll also convert between temperature scales using a formula.

Key words:

- customary system
- metric system
- conversion table
- Celsius
- Fahrenheit

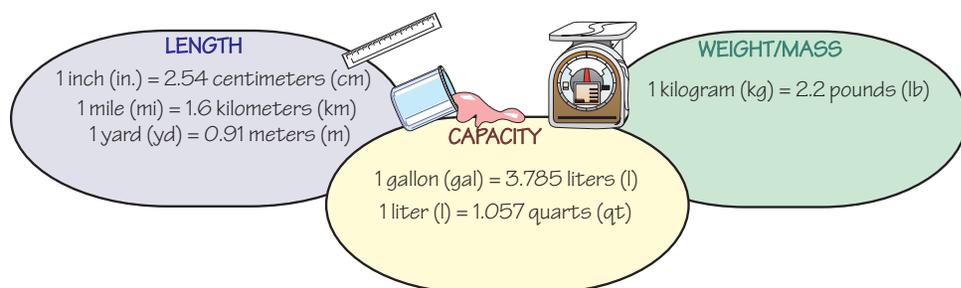
Check it out:

This is exactly the same method that you used in the previous Lesson to convert measures within either the customary or metric system.

Lots of countries use the *metric system* as their standard system of measurement — for example, distances on European road signs are often given in kilometers. So if you ever go abroad you'll find it useful to be able to *convert* the metric measures into the customary units that you're more familiar with.

Convert Between the Customary and Metric Systems

Here's a conversion table that tells you **approximately how many customary units make a metric unit**.



You can convert between customary and metric units by **setting up** and **solving proportions**.

Example 1

How many gallons are equivalent to 29 liters?

Solution

The ratio of gallons to liters is $1 : 3.785$ or $\frac{1}{3.785}$.

Write a proportion where there are x gallons in 29 liters:

$$\frac{1}{3.785} = \frac{x}{29}$$

Cross-multiply and solve for x :

$$29 \times 1 = 3.785 \times x$$

$$29 = 3.785x$$

$$x = 29 \div 3.785 = 7.661\dots$$

So there are approximately 7.66 gallons in 29 liters.

Check the reasonableness: 1 gallon is around 4 liters.

So 8 gallons is about $8 \times 4 = 32$ liters.

This estimation is close to the answer, so it is **reasonable**.

✓ Guided Practice

In Exercises 1–6, find the missing value.

1. 235 lb = ? kg
2. 9.3 mi = ? km
3. 500 cm = ? in.
4. 5.7 m = ? yd
5. 7.32 kg = ? lb
6. 76 qt = ? l

Convert Twice to Get to Units That Aren't in the Table

Only the **most common** conversions are in the conversion table. There are **lots more** you could make. For example, you might want to convert centimeters into feet, or liters into cups.

Example 2

Find how many feet are in 30 centimeters.

Solution

There's **no direct conversion** from centimeters to feet listed in the table. But there are conversions from **centimeters to inches**, and **inches to feet**.

Step 1: Convert 30 centimeters to inches.

The ratio of inches to centimeters is **1 : 2.54**. So set up a proportion and solve it to find x , the number of inches in 30 cm.

Set up a proportion...

$$\frac{1}{2.54} = \frac{x}{30}$$

$$1 \times 30 = x \times 2.54 \quad \dots \text{cross-multiply to solve}$$

$$x = 30 \div 2.54 = 11.81102\dots \approx 11.8$$

So there are approximately **11.8 inches** in 30 centimeters.

Step 2: Now convert 11.8 inches to feet.

The ratio of feet to inches is **1 : 12**. Set up a second proportion and solve it to find y , the number of feet in 11.8 inches.

Set up a proportion...

$$\frac{1}{12} = \frac{y}{11.8}$$

$$1 \times 11.8 = y \times 12 \quad \dots \text{cross-multiply to solve}$$

$$y = 11.8 \div 12 = 0.98 \approx 1$$

This means that there's approximately 1 foot in 30 centimeters.

Check it out:

For conversions like these, you need to use the metric and customary conversion tables from the last Lesson too.

✓ Guided Practice

In Exercises 7–12, find the missing value.

7. 235 kg = ? tons
8. 0.08 mi = ? cm
9. 500 cm = ? yd
10. 12.3 m = ? ft
11. 7.32 kg = ? oz
12. 0.05 qt = ? ml

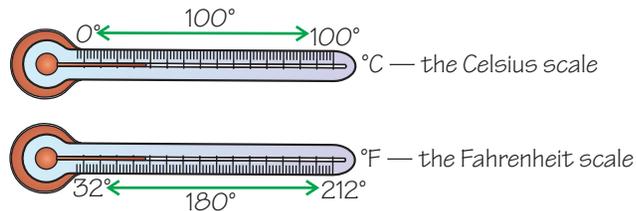
There's a Formula to Convert Temperatures

There are two common units for temperature — **degrees Fahrenheit** (°F) and **degrees Celsius** (°C).

Converting from one to the other is more complicated than other conversions because the scales don't have the same **zero point**. 0 degrees Celsius is the same as 32 degrees Fahrenheit.

Check it out:

Water freezes at 0 °C or 32 °F.
It boils at 100 °C or 212 °F.



So to convert **from degrees Celsius to degrees Fahrenheit**, you have to use this formula:

$$F = \frac{9}{5}C + 32$$

Where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

Example 3

What is 30 °C in degrees Fahrenheit?

Solution

Use the formula: $F = \frac{9}{5}C + 32$

You know C , so put this into the formula and work out F .

$$F = \frac{9}{5} \times 30 + 32 = 54 + 32 = \mathbf{86\text{ }^\circ\text{F}}$$

Don't forget:

Remember the order of operations, PEMDAS. You've got to do the $\frac{9}{5}C$ bit first and then add 32 to the result — or else you'll get the wrong answer.

Rearrange the Formula to Convert °F to °C

You can **rearrange** the formula so that you can convert **from degrees Fahrenheit to degrees Celsius**:

1. **Take 32 from both sides:** $F - 32 = \frac{9}{5}C$
2. **Multiply both sides by 5:** $5 \times (F - 32) = 9C$
3. **Divide both sides by 9:** $\frac{5}{9} \times (F - 32) = C$

$$C = \frac{5}{9}(F - 32)$$

Where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

Example 4

What is 52 °F in degrees Celsius?

Solution

Use this formula: $C = \frac{5}{9}(F - 32)$

You know F , so put this into the formula and work out C .

$$C = \frac{5}{9} \times (52 - 32) = \frac{5}{9} \times 20 = 11.11111... \approx \mathbf{11^\circ C}.$$

✓ Guided Practice

In Exercises 13–18, find the missing value.

13. $212^\circ\text{F} = ?^\circ\text{C}$

14. $0^\circ\text{C} = ?^\circ\text{F}$

15. $88^\circ\text{F} = ?^\circ\text{C}$

16. $132^\circ\text{C} = ?^\circ\text{F}$

17. $-273^\circ\text{C} = ?^\circ\text{F}$

18. $-15^\circ\text{F} = ?^\circ\text{C}$

✓ Independent Practice

In Exercises 1–3, find the missing value.

1. $128\text{ ft} = ?\text{ m}$

2. $340\text{ miles} = ?\text{ km}$

3. $75\text{ kg} = ?\text{ lb}$

4. The weight limit for an airplane carry-on is 18 kilograms.

The weight of Joe's carry-on bag is 33 pounds.

Will Joe be able to take his carry-on on the airplane?

5. A car has a 10-gallon tank for gasoline. How many liters of gasoline are needed to fill the tank?

6. Javine has set up the following proportion to convert 70 km to mi:

$$\frac{1}{1.6} = \frac{70}{x}$$

Explain whether Javine has set up the proportion correctly.

7. A car is traveling at 50 miles per hour.

How fast is this in kilometers per hour?

8. A recipe needs 180 ounces of apples.

What is this in kilograms?

In Exercises 9–11, find the missing value.

9. $45^\circ\text{C} = ?^\circ\text{F}$

10. $108^\circ\text{F} = ?^\circ\text{C}$

11. $5727^\circ\text{C} = ?^\circ\text{F}$

12. Josie has a new baby. She reads that the ideal temperature of a baby's bath is between 36°C and 38°C , but her thermometer only shows the Fahrenheit temperature scale. Advise Josie on the ideal temperature for her baby's bath in degrees Fahrenheit.

Now try these:

Lesson 4.3.2 additional questions — p454

Round Up

Conversions between *different systems* of length, mass, and capacity *don't need a formula* because they all start at the *same point* — $0\text{ kg} = 0\text{ lb}$, etc. The Fahrenheit and Celsius scales start at *different places* — $0^\circ\text{C} = 32^\circ\text{F}$, so you need to *use a formula* for these conversions.

Lesson 4.3.3

California Standards:

Algebra and Functions 4.2
Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Measurement and Geometry 1.3

Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

What it means for you:

You'll learn how to use dimensional analysis to check the units of your answers to rate problems.

Key words:

- dimensional analysis
- formulas
- canceling

Check it out:

A person-day is a unit that means the amount of work done by 1 person working for 1 day. Units separated by hyphens are products.

Dimensional Analysis

This lesson is about *dimensional analysis*. Dimensional analysis is a neat way of *checking* the units in a calculation. It shows whether or not your answer is *reasonable*.

Dimensional Analysis — Check Your Units

You can use **dimensional analysis** to check the **units** for an answer to a calculation.

For example, if you were trying to calculate a **distance** and dimensional analysis showed that the units should be **seconds**, you know something has **gone wrong**.

You can **cancel units** in the same way that you can cancel numbers. For example,

$$\frac{70 \text{ miles}}{2 \text{ hours}} \times 6 \text{ hours} = 210 \text{ miles}$$

The units on both sides of the calculation must balance — so the answer must be a distance in miles.

Example 1

Jonathan earns 10 dollars per hour.
How much does he earn for 40 hours' work?

Solution

You need to multiply Jonathan's hourly rate by the number of hours he works.

$$\text{Earnings} = 10 \text{ dollars per hour} \times 40 \text{ hours} \\ = \mathbf{400 \text{ dollars}}$$

You can use dimensional analysis to check your answer is reasonable:

$$10 \frac{\text{dollars}}{\text{hour}} \times 40 \text{ hours} = 400 \text{ dollars}$$

Example 2

It takes 12 person-days to tile a large roof. If there are three workers working on the roof, how many days will it take them to tile it?

Solution

$$\text{Number of days} = \text{total person-days} \div \text{number of persons} \\ = 12 \div 3 = \mathbf{4 \text{ days}}$$

You can use dimensional analysis to check your answer is reasonable:
 $3 \text{ persons} \times 4 \text{ days} = 12 \text{ person-days}$

Example 3

You are organizing a three-legged race. You need 2.5 feet of ribbon for every two people. You have 660 inches of ribbon. How many people can join in the race?

Solution

First convert the length of the ribbon from inches to feet. You need to set up a proportion. 12 inches = 1 foot, so:

$$\frac{12}{1} = \frac{660}{x} \quad \leftarrow x = \text{length in feet}$$

$$12x = 660 \times 1 \Rightarrow x = \mathbf{55 \text{ feet}}$$

So 660 inches is equivalent to 55 feet.

You can check this by **dimensional analysis**:

$$x = \frac{660 \cancel{\text{inches}} \times 1 \text{ foot}}{12 \cancel{\text{inches}}} = 55 \text{ feet}$$

Now divide the length of ribbon by the amount you need per person:

$$55 \text{ feet} \div \frac{2.5 \text{ feet}}{2 \text{ people}} = 55 \cancel{\text{ feet}} \times \frac{2 \text{ people}}{2.5 \cancel{\text{ feet}}} = \mathbf{44 \text{ people}}$$

✓ Guided Practice

In Exercises 1–2, find the missing unit.

$$1. \ 4 \text{ miles} \times \frac{1.6 \text{ km}}{1 \text{ mile}} = 6.4 \ ? \quad 2. \ 26.5 \text{ inches} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 67.31 \ ?$$

Checking Formulas with Dimensional Analysis

Dimensional analysis is useful for **checking whether a formula is reasonable**. The units on each side of a formula must balance.

Example 4

A formula says that speed (in meters per second) multiplied by time (in seconds) is equal to the distance traveled (in meters). Use dimensional analysis to check the reasonableness of the formula.

Solution

Write out the formula suggested and include the units.

$$\text{speed} \left(\frac{\text{m}}{\cancel{s}} \right) \times \text{time} (\cancel{s}) = \text{distance} (\text{m}?)$$

So the seconds cancel and leave units of meters.

This means that **the formula is reasonable** since the units on each side of the equation match.

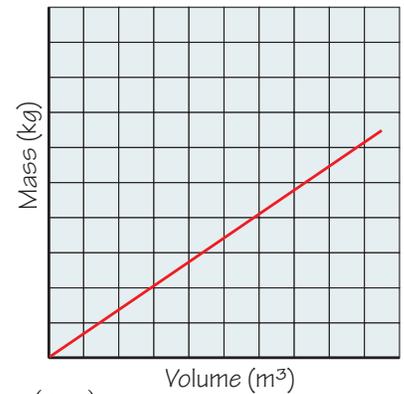
Check it out:

A unit of meters per second means “meters ÷ 1 second.” This is the number of meters traveled in 1 second. When you get a unit with the word “per” in it, you know that you can write the unit as a fraction.

Example 5

It is suggested that the slope of this graph is equal to density, which is measured in kg per m^3 .

Is this a reasonable suggestion?

**Solution**

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope} = \frac{\text{change in mass (kg)}}{\text{change in volume (m}^3\text{)}} = \text{density} \left(\frac{\text{kg}}{\text{m}^3} \right)$$

So the unit of the slope of the graph is $\left(\frac{\text{kg}}{\text{m}^3} \right)$.

This is the same as kg per m^3 .

So it's reasonable that the slope is equal to density, since it has the right units.

✓ Guided Practice

3. Find eight different units of speed, if $\text{speed} = \text{distance} \div \text{time}$.

4. The formula for acceleration is “change in speed \div time.”

Which of the following could be a unit for acceleration:

- A. $\frac{\text{in.}}{\text{h}^2}$ B. $\frac{\text{cm}^2}{\text{min}}$ C. $\frac{\text{s}}{\text{m}^2}$ D. $\frac{\text{km}}{\text{s}^2}$

✓ Independent Practice

In Exercises 1–4, find the missing unit.

1. $15 \text{ tomatoes} \times \frac{\$0.20}{1 \text{ tomato}} = 3 ?$

2. $\$56 \times \frac{1.3 \text{ lb}}{\$1} = 72.8 ?$

3. $54.3 \text{ yd} \times \frac{0.9144 \text{ m}}{1 \text{ yd}} = 49.65... ?$

4. $\$500 \times \frac{\pounds 0.53}{\$1} = 265 ?$

5. Use dimensional analysis to check that this expression is reasonable.

Use it to find the number of seconds in an hour.

$$1 \text{ hour} = 1 \text{ hour} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}}$$

6. You need to save up \$240 for a ski trip. You earn six dollars per hour babysitting. How many hours do you need to work to pay for the trip? Check your answer using dimensional analysis.

Now try these:

Lesson 4.3.3 additional questions — p455

Round Up

So *dimensional analysis* is basically making sure your units balance — it's useful for checking you've worked out what you think you have. Try to get into the habit of using it for all types of problems.

Lesson 4.3.4

Converting Between Units of Speed

California Standards:

Measurement and Geometry 1.1

Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

What it means for you:

You'll learn how to convert from one unit of speed to another unit of speed.

Key words:

- conversion
- dimensional analysis

Don't forget:

1 is the multiplicative identity. See Lesson 1.1.4.

Check it out:

If you don't end up with the units you wanted, your conversion fraction may be upside down.

Check it out:

You'll need to look back at the conversion tables in Lessons 4.3.1 and 4.3.2 for these.

There are a lot of units that can be used for *speed* — kilometers per hour, miles per hour, meters per second, inches per minute.

Speed units are all made up of a *distance unit divided by a time unit*. This makes them a bit tougher to convert than other units.

You Can Set Up Conversion Fractions Equal to 1

When you multiply something by **1**, it **doesn't change**.

Fractions with the same thing on the top and the bottom, such as $\frac{5}{5}$, are equal to 1, so whatever you multiply by them doesn't change either.

60 seconds = 1 minute, so the fraction $\frac{60 \text{ seconds}}{1 \text{ minute}}$ has the same on the top and the bottom, so it's **equal to 1** too — this is called a **conversion fraction**.

Use Conversion Fractions to Convert Units

You can use a conversion fraction equal to 1 to convert from one unit to another. Here's how a time can be converted:

Example 1

A ship takes 1.75 days to reach its destination. How many hours is this?

Solution

24 hours = 1 day **Start with a conversion equation**

$$\frac{24 \text{ hours}}{1 \text{ day}} = \frac{1 \text{ day}}{1 \text{ day}} \quad \text{Divide both sides by 1 day}$$

$$\frac{24 \text{ hours}}{1 \text{ day}} = 1 \quad \leftarrow \text{You now have a fraction that is equal to 1.}$$

So, whatever you multiply by the fraction $\frac{24 \text{ hours}}{1 \text{ day}}$ won't change.

$$1.75 \text{ days} \times \frac{24 \text{ hours}}{1 \text{ day}} = 1.75 \times 24 \text{ hours} = \mathbf{42 \text{ hours}} \quad \text{Cancel the units}$$

1.75 days is equivalent to 42 hours.

✓ Guided Practice

Convert each of the following by multiplying by a conversion fraction.

1. 6 inches to centimeters
2. 45 minutes to seconds
3. 12 miles to kilometers
4. 6 liters to quarts

Speed Units May Have Two Parts to Convert

A speed unit is always a **distance unit** divided by a **time unit**.

If you want to change both of these parts, you need to do **two separate conversions**. For instance, if you were converting **centimeters per minute** to **inches per second**, you might do the following conversions:

centimeters per minute \Rightarrow inches per minute \Rightarrow inches per second

Check it out:

You could also convert centimeters per minute to **centimeters per second**, before finally converting this to inches per second.

Example 2

A train travels 1.2 miles per minute.

What is the speed of the train in kilometers per hour?

Solution

Break this down into **two stages** —

Stage 1: Convert from miles per minute to miles per hour.

First, you have to write a conversion fraction:

$$60 \text{ minutes} = 1 \text{ hour}$$

Start with a conversion equation

$$\frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{1 \text{ hour}}{1 \text{ hour}}$$

Divide both sides by 1 hour

$$\frac{60 \text{ minutes}}{1 \text{ hour}} = 1$$

You've now got a fraction equal to 1

So, whatever you multiply by the fraction $\frac{60 \text{ minutes}}{1 \text{ hour}}$ won't change.

$$\frac{1.2 \text{ miles}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = (1.2 \times 60) \frac{\text{miles}}{\text{hour}} \quad \text{Cancel the units}$$

$$= \mathbf{72 \text{ miles per hour}}$$

Stage 2: Convert from miles per hour to kilometers per hour.

Write another conversion fraction:

$$1 \text{ mile} = 1.6 \text{ kilometers}$$

Start with a conversion equation

$$\frac{1 \text{ mile}}{1 \text{ mile}} = \frac{1.6 \text{ kilometers}}{1 \text{ mile}}$$

Divide both sides by 1 mile

$$1 = \frac{1.6 \text{ kilometers}}{1 \text{ mile}}$$

You've now got a fraction equal to 1

So, whatever you multiply by the fraction $\frac{1.6 \text{ kilometers}}{1 \text{ mile}}$ won't change.

$$\frac{72 \text{ miles}}{1 \text{ hour}} \times \frac{1.6 \text{ kilometers}}{1 \text{ mile}} = (72 \times 1.6) \frac{\text{kilometers}}{\text{hour}} \quad \text{Cancel the units}$$

$$\approx \mathbf{115 \text{ kilometers per hour}}$$

Check it out:

Dimensional analysis shows that the product should have units of miles per hour — that's the unit we are aiming for at this stage.

Check it out:

Using dimensional analysis here shows that the product should have units of kilometers per hour — that's the unit we want.

✓ Guided Practice

- Convert 18 miles per hour into kilometers per hour.
- Which is faster — 56 miles per hour or 83 kilometers per hour?
- Convert 14 inches per minute into the unit feet per second.
- Which is faster — 22 centimeters per minute or 500 inches per hour?

✓ Independent Practice

Write conversion fractions from the equations given below.

- 3 feet = 1 yard
- 1 kilometer = 1000 meters
- 3600 seconds = 1 hour

Convert the following by multiplying by a conversion fraction.

- 3 feet into inches
- 4.5 hours into seconds
- 36 miles into kilometers
- Jonah and Ken were both running separate races. Jonah ran 100 meters in 12 seconds, and Ken ran 100 yards in 0.18 minutes. Who ran faster?
- A snail managed to crawl 50 centimeters in 14 minutes. A slug crawled 40 inches in 0.5 minutes. Which was faster?

Convert the following speeds into the units given.

- 25.6 kilometers per hour into miles per hour
- 36 inches per hour into centimeters per minute
- 7 feet per minute into yards per hour
- A boat travels 20 miles in 4 hours. How fast is this in kilometers per hour?
- A car travels 0.6 miles in two minutes.
- How fast is this in kilometers per hour?
- How fast is this in meters per hour?

Now try these:

Lesson 4.3.4 additional questions — p455

Round Up

Speed units have two parts — a *distance part* and a *time part*. That's why you often have to do *two separate conversions* to convert a speed into different units. Multiplying by a *conversion fraction* is a useful way of converting any units. But it's real important that you always check your work using *dimensional analysis*.

Lesson 4.4.1

Section 4.4 Linear Inequalities

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

Algebra and Functions 4.1

Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

What it means for you:

You'll solve inequalities by addition and subtraction using the same methods that you used for solving equations.

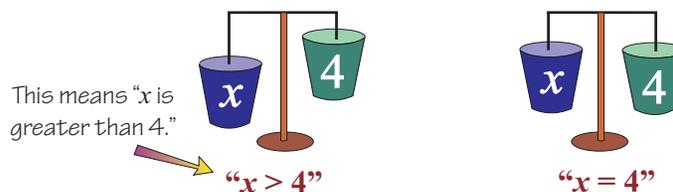
Key words:

- inequality
- addition
- subtraction

In Chapter 1 you learned what *inequalities* were and how to write them. In this Lesson, you'll *review* some of the things you've already seen, and learn how to *solve inequalities* using the same kind of method that you used to solve equations.

Inequalities Have an Infinite Number of Solutions

Inequalities have **more than one solution**. The inequality $x > 4$ tells you that x could take **any value greater than 4**, whereas the equation $x = 4$ tells you x can **only** take the value of 4.



Remember the four **inequality symbols** you learned in Chapter 1:

The Inequality Symbols

">" means "greater than," "more than," or "over"

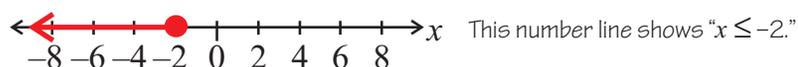
"<" means "less than" or "under"

"≥" means "greater than or equal to," "minimum," or "at least"

"≤" means "less than or equal to," "maximum," or "at most"

Show Solutions to an Inequality on a Number Line

The solution to an inequality with one variable can be shown on a **number line**. A **ray** is drawn **in the direction of all the numbers in the solution set**. So for $x > 4$, the ray should go through all numbers greater than 4.



An **open circle** ○ means the number **is not** included in the solution; a **closed circle** ● means the number **is** included in the solution.

Don't forget:

You've done problems like this before in Section 1.3. To write inequalities that are given in words, look for the key words given in the table of inequality symbols on the previous page. Then think about which way around the expression should go.

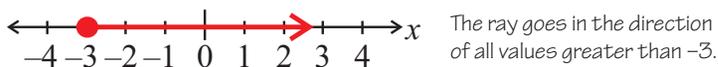
Example 1

Write and plot an inequality to say that y must be a minimum of -3 .

Solution

The word “**minimum**” tells you that $y \geq -3$.

You need to use a **closed circle**, because -3 is included in the solution set.



Guided Practice

In Exercises 1–4, write the inequality in words.

1. $z < 7$

2. $y > -10$

3. $x \leq -1$

4. $n \geq 89$

In Exercises 5–8, plot the inequality on a number line.

5. $k > 8$

6. $j \geq 2.5$

7. $a \leq -4$

8. $d < -50$

9. To go on Ride A, children must be at least 1 m tall. Write this as an inequality, and plot the inequality on a number line.

Don't forget:

You wrote systems of equations in Lesson 4.1.2. Systems of inequalities are similar, but are likely to have a range of solutions.

You Can Write Systems of Inequalities

A system of inequalities is a **set** of two or more inequalities in the **same variables**. The inequalities $x > 2$ and $x - 1 \leq 6$ make a system of inequalities in the variable x .

The **solutions** to a system of inequalities have to satisfy **all** the inequalities at the same time.

So if x is an **integer**, the solution set of the system of inequalities $x > 2$ and $x - 1 \leq 6$, must be $\{3, 4, 5, 6, 7\}$. These values make **both** inequalities true.

Example 2

Write a system of inequalities to represent the following statement: “**3 times y is greater than 5**, and **2 plus y is less than or equal to 7**.”

Solution

You need to write **two inequalities** that **both** need to be true for the statement to be true.

The first part says “3 times y is greater than 5,” so $3y > 5$.

The second part says “2 plus y is less than or equal to 7,” so $2 + y \leq 7$.

These two equations form a **system of inequalities**.

✓ Guided Practice

10. d is an integer, and $d > -1$, and $d \leq 4$.

What values of d would make this system of inequalities true?

In Exercises 11–12, write a system of inequalities to represent each statement:

11. z is less than 0, and the sum of z and 4 is greater than -12 .

12. A third of p is less than or equal to 0, and the product of p and -3 is less than 30.

Solve Inequalities by Reversing Their Operations

To solve an inequality you need to get the variable **by itself** on one side — you do this by “undoing” the operations that are done to it. This means **doing the “opposite.”**

So, if a variable has a number **subtracted** from it, you undo this by **adding** the same number to it. Remember — you have to do exactly the same to **each side** of the inequality.

$$\begin{array}{r} +6 \left(\right. \\ x - 6 > 16 \\ \left. \right) \\ x > 16 + 6 \\ x > 22 \end{array} \quad \begin{array}{r} \left. \right) +6 \\ \end{array}$$

Check it out:

When you're solving inequalities by addition and subtraction, treat the expression like an equation, but with the inequality sign in the middle.

Check it out:

You might see this solution set written as $\{y : y \leq 14\}$ — it means the same.

Example 3

Solve the inequality $y + 7 \leq 21$.

Solution

$$y + 7 - 7 \leq 21 - 7$$

$$y \leq 14$$

Subtract 7 from each side of the inequality

Simplify

You might get **word problems** that ask you to solve inequalities.

Example 4

A number increased by 3 is at most 9. Write and solve this inequality.

Solution

“A number increased by 3” means $x + 3$.

“at most 9” means “less than or equal to 9” so ≤ 9 .

This means the inequality is $x + 3 \leq 9$.

Now solve the inequality by subtracting 3 from each side:

$$x + 3 - 3 \leq 9 - 3$$

$$x \leq 6$$

Example 5

An elevator has a weight of 1250 pounds already in it. If the maximum load for the elevator is 2500 pounds, write and solve an inequality to find the amount of additional load that can be put in the elevator safely.

Solution

“maximum” load of 2500 pounds means ≤ 2500 pounds.

1250 pounds plus the additional load that can be added, x , must be less than or equal to 2500 pounds.

$$1250 + x \leq 2500$$

$$x + 1250 - 1250 \leq 2500 - 1250 \quad \text{Take 1250 from both sides}$$

$$x \leq 1250$$

The load that can be added is a maximum of 1250 pounds.

Guided Practice

In Exercises 13–20, solve the inequality for the unknown.

13. $z + 5 < 17$ 14. $y - 10 > -10$ 15. $2 + x \leq -1$ 16. $p + 45 \leq 76$

17. $h - 6 > 3$ 18. $-6 + h > 3$ 19. $14 + x \geq 12$ 20. $1 + y < 1$

21. A number decreased by 17 is at least 16. Write and solve an inequality to find the number.

22. Sophia must complete at least 40 hours of training to qualify. She has already completed 32 hours of training. Write an inequality and solve it to find the remaining hours of training she must complete.

Independent Practice

In Exercises 1–4, plot the inequality on a number line.

1. $x > 3$ 2. $t \leq 14$ 3. $n < 1$ 4. $z \geq -2$

5. An elevator has a safe maximum load of 2750 pounds. Write an inequality that shows the safe load for this elevator.

6. Write a system of inequalities to represent this statement: 4 plus f is greater than 14, and the product of f and 6 is less than 14.

In Exercises 7–14, solve the inequality for the unknown.

7. $p - 6 > 10$ 8. $z + 12 < 1$ 9. $c + 1 \leq -6$ 10. $13 + d \geq 12$

11. $x + 7 \geq -7$ 12. $-12 + y > 6$ 13. $f - 100 \leq -2$ 14. $g + 130 < 12$

15. A number, y , increased by 12 is larger than 12. Write and solve an inequality to find the solution set. Plot the solution on a number line.

16. The area of Portia’s yard is 32 ft². The area of Gene’s yard is at least 4 ft² larger than Portia’s. Write and solve an inequality for the area of Gene’s yard.

Now try these:

Lesson 4.4.1 additional questions — p455

Round Up

Solving *inequalities* that involve *addition and subtraction* is exactly like solving equations. In the next Lesson you’ll solve inequalities involving *multiplication and division* — this has an *important difference*.

Lesson 4.4.2

More on Linear Inequalities

California Standards:

Algebra and Functions 4.1

Solve two-step **linear** equations and **inequalities in one variable over the rational numbers**, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

What it means for you:

You'll get more practice at solving linear inequalities and learn how to multiply and divide inequalities by negative numbers.

Key words:

- inequality
- multiplication
- division
- inequality symbol

Check it out:

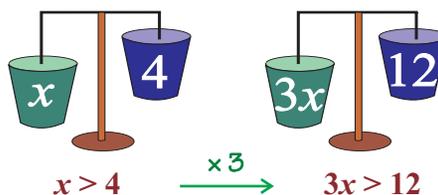
You can't do this if you're multiplying or dividing by a negative number. There's a different rule for negative numbers, which you'll come to next.

So far you've set up and solved *linear inequalities* that use *addition and subtraction*. The next step is to use *multiplication and division*. This is a bit trickier, because you need to remember to *swap the inequality symbol* when you multiply or divide by a *negative number*.

Multiplying and Dividing by Positive Numbers

The **rules** for multiplying and dividing inequalities by positive numbers are **the same** as for multiplying and dividing equations. The **inequality symbol doesn't change**.

The main thing to remember is to **always do the same thing to both sides**.



Example 1

Solve the inequality $4x < 32$.

Solution

$$\frac{4x}{4} < \frac{32}{4} \quad \text{Divide both sides of the inequality by 4}$$
$$x < 8$$

Example 2

Solve the inequality $\frac{x}{9} \geq 45$.

Solution

$$\frac{x}{9} \times 9 \geq 45 \times 9 \quad \text{Multiply both sides of the equation by 9}$$
$$x \geq 405$$

✓ Guided Practice

In Exercises 1–4, solve the inequality for the unknown.

1. $5c > 12.5$
2. $3p \geq 63$
3. $\frac{f}{12} < 9$
4. $\frac{g}{60} \leq -3$

Multiplying and Dividing by Negative Numbers

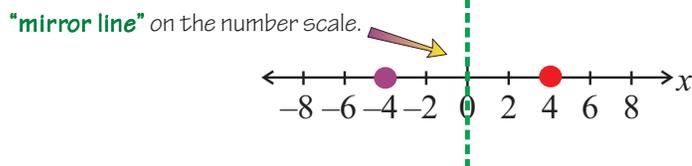
You know that a number and its opposite are the **same distance** away from zero on a number line. So 4 and -4 are both 4 units from zero.

When you multiply a number by -1 you are effectively “**reflecting**” the number about **zero on the number line**.

For example, $4 \times -1 = -4$ and $-4 \times -1 = 4$.

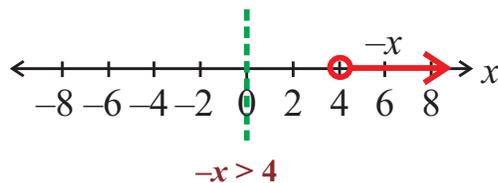
Check it out:

You could think of the negative scale as a reflection of the positive scale about zero on the number line.



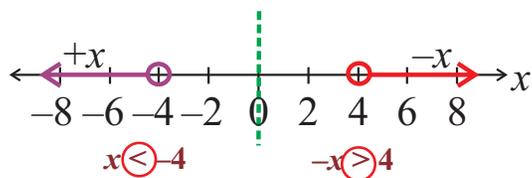
You can use this idea to understand what happens when you **multiply or divide an inequality by -1** on the number line.

For example, consider the inequality $-x > 4$. This is saying that some number, $-x$, can be anywhere in the region **greater than 4** on the number line.



You want to solve the inequality to find x , so you need to **divide both sides of the inequality by -1** .

Reflect the inequality about the **origin** of the number line to see what the solution looks like.



The inequality symbol is reflected too.

So if $-x$ is **greater than** 4, then x is **less than** -4 .

This “reflection” idea works for all inequalities, so there’s a rule:

When you multiply or divide by a negative number, always reverse the sign of the inequality.

Check it out:

If you're finding all this stuff really tricky, you can sometimes **avoid** multiplying and dividing by a negative altogether.

Take the inequality $-4x < 32$.

Add $+4x$ to both sides:

$$-4x + 4x < 32 + 4x.$$

So, $0 < 32 + 4x$

Now take 32 from both sides:

$$-32 < 32 - 32 + 4x$$

$$-32 < 4x$$

Now divide by 4:

$$-8 < x$$

This is your answer.

It's the same as $x > -8$.

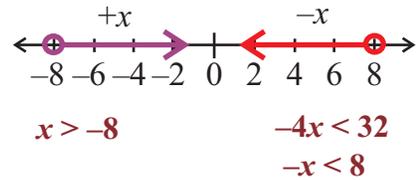
Example 3

Solve the inequality $-4x < 32$.

Solution

Divide both sides of the equation by -4 . As you're dividing by a negative number, **reverse the symbol of inequality**.

$$\begin{array}{l} -4x < 32 \\ + -4 \quad \downarrow \\ \frac{-4x}{-4} > \frac{32}{-4} \\ \quad \quad \quad \downarrow \\ x > -8 \end{array}$$



Example 4

Solve the inequality $\frac{x}{-6} \leq 3$.

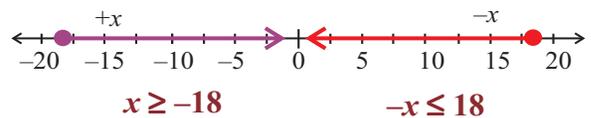
Solution

You need to **multiply** both sides of the equation by -6 . As you're multiplying by a negative number, **reverse the symbol of inequality**.

$$\frac{x}{-6} \leq 3$$

$$x \geq 3 \times -6$$

$$x \geq -18$$



Guided Practice

In Exercises 5–12, solve the inequality for the unknown.

5. $-4h > 24$ 6. $-10y \geq -20$ 7. $\frac{r}{-5} < 25$ 8. $\frac{z}{-32} \leq -1$

9. $\frac{x}{-2} > -6$ 10. $-100x < -25$ 11. $\frac{k}{-7} \leq 3$ 12. $-17d > 34$

Independent Practice

In Exercises 1–8, solve the inequality for the unknown.

1. $2g > 4$ 2. $56b < 112$ 3. $-5h < -15$ 4. $-8x > 12$

5. $2k \geq -6$ 6. $-13p \leq 39$ 7. $\frac{x}{-4} \leq 1.5$ 8. $\frac{y}{-3} \geq 3.3$

9. Write and solve the inequality that says that twice the negative of a number is no more than 14.

Now try these:

Lesson 4.4.2 additional questions — p456

Round Up

So that's how to solve inequalities that involve multiplication and division. You've got to remember to switch the direction of the inequality symbol each time you multiply or divide by a negative number.

Lesson 4.4.3

Solving Two-Step Inequalities

California Standards:

Algebra and Functions 4.1

Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

What it means for you:

You'll learn how to solve two-step linear inequalities.

Key words:

- inequality
- system of inequalities

Don't forget:

You learned how to set up two-step inequalities in Section 1.3. Now you're going to solve them too.

Don't forget:

You've used PEMDAS to remember the order of operations when evaluating expressions. Don't be tempted to use this when solving equations and inequalities. The calculation is often simpler if you add and subtract first so that you only have one term on each side of the inequality before multiplying and dividing.

So far in this Section you've learned how to solve *one-step linear inequalities*, and why you have to *reverse the inequality* whenever you multiply or divide by a *negative number*. Two-step inequalities follow the same rules, but you need to do *two steps* to solve them.

Two-Step Inequalities Have Two Different Operations

A two-step inequality contains **two different operations**. So you need to do **two steps** to **solve** the inequality.

$$2 \times x + 12 > 10$$

first operation   second operation

You need to get the variable by itself on one side of the inequality, so you must undo whatever has been done to it. It's usually best to undo **additions and subtractions first**, and **multiplications and divisions second**. That way, you only have to multiply or divide one term.

Example 1

Solve the inequality $2x + 12 > 10$.

Solution

- First **subtract** 12 from both sides of the inequality:

$$\begin{aligned} 2x + 12 - 12 &> 10 - 12 \\ 2x &> -2 \end{aligned}$$

- Then **divide** both sides by 2:

$$\begin{aligned} 2x \div 2 &> -2 \div 2 \\ x &> -1 \end{aligned}$$

Don't forget to **reverse the sign** when you multiply or divide by a **negative**.

Example 2

Solve the inequality $\frac{x}{-4} - 2 < 14$.

Solution

- First **add** 2 to both sides of the inequality:

$$\frac{x}{-4} < 16$$

- Then **multiply** both sides by -4 , remembering to reverse the sign:

$$x > -64$$

Don't forget:

You're reversing the sign of the inequality because you're dividing both sides by a negative number. Remember you need to reverse the sign whenever you multiply or divide by a negative number.

Example 3

Solve the inequality $-5x - 2 > 103$.

Solution

- First **add** 2 to both sides of the inequality:

$$-5x - 2 + 2 \leq 103 + 2$$

$$-5x \leq 105$$

- Then **divide** both sides by -5 , remembering to reverse the sign:

$$x \geq 105 \div -5$$

$$x \geq -21$$

✓ Guided Practice

In Exercises 1–6, solve the inequality for the unknown.

1. $4c - 2 > 6$

2. $-6z - 14 < -36$

3. $3x + 3 \leq -18$

4. $\frac{z}{-32} + 18 \geq 2$

5. $\frac{r}{-5} - 12 > -6$

6. $-12g + 4 < 12$

Solve Real Problems with Inequalities

There are lots of **real-life problems** that involve inequalities. The key is in interpreting the question and coming up with a **sensible answer** in the **context of the question**.

Example 4

Two students decide to go to a restaurant for lunch. They order two drinks at \$2 each, then realize they only have a maximum of \$20 to spend between them.

If they want one meal each, what is the maximum price they can spend on each meal? Assume their meals cost the same amount.

Solution

First you have to **write this as an inequality**.

Call the price of each meal x . They want two equally priced meals, which is $2x$. The price of the meals plus the two drinks they have already bought must be no more than \$20.

So, $2x + 4 \leq 20$. This is your inequality.

Now you have to solve the inequality to find x , the price of each meal.

$$2x + 4 \leq 20$$

Subtract 4 from both sides

$$2x \leq 16$$

Divide both sides by 2

$$x \leq 8$$

So the maximum price of each meal is \$8.

Don't forget:

You've got to interpret your answer in the context of the question here. Look back at what x represents and what units you need to use in your answer.

Example 5

Joaquin goes to a fair. He buys an unlimited ticket that costs \$30 and allows him entry to all the rides that normally cost \$4 each. The ticket also gives him one go on the coconut shy, which normally costs \$2. How many rides does Joaquin need to go on in order to have made buying the unlimited ticket worthwhile?

Solution

First you have to **write this as an inequality**.

Call the number of rides that Joaquin goes on x . So the amount that Joaquin would normally spend on the rides is $\$4 \times x$, or $4x$.

For buying the ticket to have been worthwhile, the total value of the rides plus the value of the go on the coconut shy must be at least the price of the unlimited ticket.

So, $4x + 2 \geq 30$.

Now **solve the inequality** to find the number of rides, x .

$$4x \geq 28 \quad \text{Subtract 2 from both sides}$$

$$x \geq 7 \quad \text{Divide both sides by 4}$$

Joaquin needs to go on at least 7 rides in order to get his money's worth.

✓ Guided Practice

7. Anne-Marie is saving up to buy a concert ticket by babysitting for \$5 an hour. Anne-Marie owes \$15 to her mother already, and the concert ticket costs \$25. How many hours does she need to work in order to be able to buy the ticket and pay her mother? Show how you reached your solution using an inequality.

✓ Independent Practice

In Exercises 1–6, solve the inequality for the unknown.

1. $4x - 3 > 5$

2. $7x + 12 < 19$

3. $-6x - 6 > 6$

4. $0.5g + 3 \geq 6$

5. $\frac{x}{-5} - 5 < 15$

6. $\frac{x}{8} + 14 \leq -2$

7. Juan runs a salsa class on a Wednesday night. Entrance to his class is \$3 each. If the venue costs \$50 and the music equipment costs \$10 to hire, what is the minimum number of people needed to attend the class in order for Juan to make his money back on the night?

Don't forget:

Remember that multiplying by 0.5 is the same as dividing by 2. If in doubt, convert 0.5 into a fraction first.

Now try these:

Lesson 4.4.3 additional questions — p456

Round Up

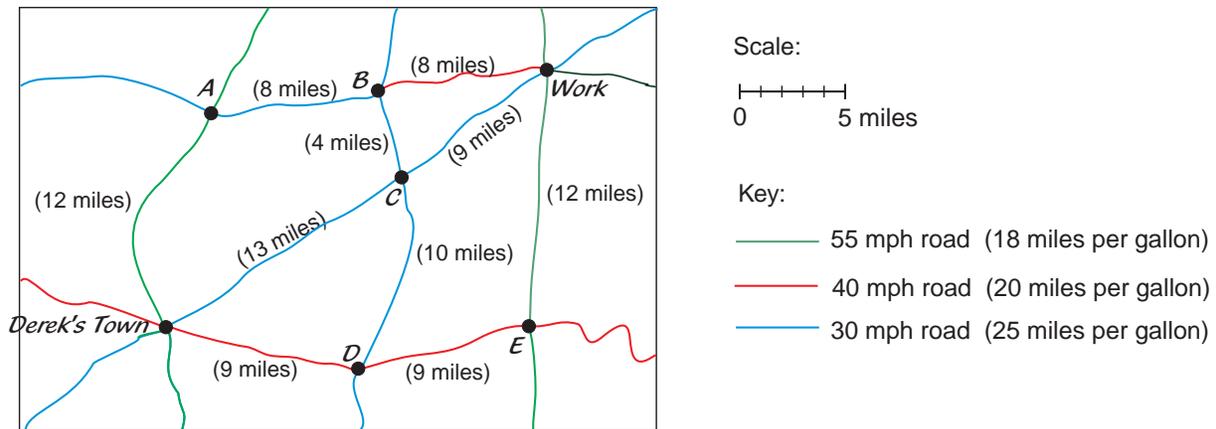
The trick with real-life inequality problems is understanding what the question is telling you. Try to break the question down into parts, and work out what each part means in math terms.

Chapter 4 Investigation

Choosing a Route

Rates compare one quantity to another. There are rates involved in driving to places — roads have speed limits in miles per hour, and when driving at a steady speed, you travel a particular number of miles per gallon of gas. “Per” just means “for each.”

Derek is starting a new job next week in a different town. He wants to get to work **quickly**, but also doesn't want to put a lot of miles on his car. He looks at a map to examine all of the possible routes.



Find the **shortest** and the **quickest** routes that Derek could take. Decide which route you think is the **best overall**. Explain your reasoning.

Things to think about:

The best route should be a **balance** between time and distance. A route that is 15 miles long and takes 15 minutes might be less desirable than a route that is 10 miles long and takes 20 minutes. Compare different scenarios before picking the best route.

Extensions

- 1) Derek is concerned about the effect of burning gas on climate change. Determine **how many gallons of gas** are used driving the route you have chosen. Are there any other routes that would **reduce** the amount of gas he uses?
- 2) Determine the **minimum** yearly cost of Derek's commute. Assume he drives to work and back home again on 250 days each year, and that the average price for a gallon of gas is \$2.80.

Open-ended Extensions

- 1) Using a map of your town or city, examine different routes from your school to a major town landmark. Which do you think is the best route? Consider factors such as distance, speed limit and time of travel.
- 2) Convert the scale, distances, and speed limits on this map to the metric system.

Round Up

*Real-life decisions are often not straightforward. There's often **no perfect right answer** — you have to decide what's most **important** to you, or find the **best compromise**.*

Chapter 5

Powers

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Lesson 5.1.1

Section 5.1 Multiplying with Powers

California Standards:

Number Sense 2.1

Understand negative whole-number exponents. **Multiply** and divide **expressions involving exponents with a common base**.

Number Sense 2.3

Multiply, divide, and **simplify rational numbers by using exponent rules**.

Algebra and Functions 2.1

Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. **Simplify and evaluate expressions that include exponents**.

What it means for you:

You'll see how to use a rule that makes multiplying powers easier.

Key words:

- power
- base
- exponent

Check it out:

The variables a , m , and n could stand for any numbers at all here. Whatever numbers they are, the equation will still be true.

When you have to **multiply powers together**, like $5^3 \cdot 5^4$, there's a rule you can use to **simplify the calculation**. But it only works with powers that have the **same base**.

A Power is a Repeated Multiplication

In Chapter 2 you saw that a power is a **product** that results from **repeatedly multiplying** a number by itself.

You can write a power in **base and exponent form**.

The base is the number that is being repeated as a factor in the multiplication.



The exponent tells you how many times the base is repeated as a factor in the multiplication.

For example $7 \cdot 7 = 7^2$, and $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$.

Guided Practice

Write the expressions in Exercises 1–4 in base and exponent form.

1. $3 \cdot 3$

2. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

3. $k \cdot k \cdot k \cdot k$

4. $(-5) \cdot (-5) \cdot (-5) \cdot (-5)$

The Multiplication of Powers Rule

Look at the multiplication $2^2 \cdot 2^4$. When you write it out it looks like this:

$$\underbrace{2 \cdot 2}_{2 \text{ factors}} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4 \text{ factors}} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{6 \text{ factors}}$$

The solution is also a **repeated multiplication**. 2 is repeated as a factor six times, so it's the same as writing 2^6 .

That means you can write $2^2 \cdot 2^4 = 2^6$.

In the multiplication expression the two exponents are 2 and 4, and in the solution the exponent is 6. If you **add** the exponents of the original powers together you get the exponent of the solution — the base stays the same.

Multiplication of Powers Rule:

When you are multiplying two powers with the same base, add their exponents to give you the exponent of the answer.

$$a^m \cdot a^n = a^{(m+n)}$$

Only Use the Rule If the Bases are the Same

It's important to remember that this rule **only** works with powers that have the same base. You **can't** use it on two powers with different bases.

- You **could** use the rule to simplify $5^3 \cdot 5^4$ as the bases are the same.
- You **couldn't** use it to simplify $3^5 \cdot 4^5$ because the bases are different.

Example 1

What is $3^2 \cdot 3^6$? Give your answer in base and exponent form.

Solution

You could write the multiplication out in full

$$\begin{aligned} & 3^2 \cdot 3^6 \\ &= (3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \\ &= 3^8 \end{aligned}$$

3 is repeated as a factor 8 times in the multiplication.

But these two powers have the same base. So you can use the multiplication of powers rule.

$$\begin{aligned} & 3^2 \cdot 3^6 \\ &= 3^{(2+6)} = 3^8 \end{aligned}$$

Don't forget:

$(-5)^2$ is not the same as $-(5^2)$.
 $(-5)^2 = (-5) \cdot (-5) = 25$
 $-(5^2) = -(5 \cdot 5) = -25$

Example 2

What is $(-5)^{12} \cdot (-5)^{14}$? Give your answer in base and exponent form.

Solution

The powers have the same base. Use the multiplication of powers rule.

$$\begin{aligned} & (-5)^{12} \cdot (-5)^{14} \\ &= (-5)^{(12+14)} = (-5)^{26} \end{aligned}$$

(This is the same as 5^{26} , since the exponent is even.)

Guided Practice

Evaluate the expressions in Exercises 5–12. Use the multiplication of powers rule and give your answers in base and exponent form.

5. $2^2 \cdot 2^2$

6. $9^{10} \cdot 9^8$

7. $6^{104} \cdot 6^{62}$

8. $(-7)^7 \cdot (-7)^3$

9. $10^8 \cdot 10^1$

10. $5^6 \cdot 5$

11. $k^8 \cdot k^5$

12. $(-t)^{14} \cdot (-t)^{17}$

Don't forget:

Any number to the power 1 is just the number itself. So an expression like $2^5 \cdot 2$ can also be written as $2^5 \cdot 2^1$.

Multiplication of Powers Can Help with Mental Math

Sometimes changing numbers into base and exponent form and using the multiplication of powers rule can make **mental math** easier.

Example 3

What is $16 \cdot 8$?

Solution

16 and 8 are both powers of 2, so you can rewrite the problem in base and exponent form:

$$16 \cdot 8 = 2^4 \cdot 2^3.$$

Now use the multiplication of powers rule:

$$2^4 \cdot 2^3 = 2^{(4+3)} = 2^7.$$

$$2^7 = 128. \text{ So } 16 \cdot 8 = \mathbf{128}.$$

Check it out:

Write yourself out a table showing the first few powers of all the numbers up to ten. Use it to help with problems like Example 3. You'll begin to remember some of the more common powers — then you'll know them when you see them again.

If you knew lots of powers of 2, this could make your mental math faster.

✓ Guided Practice

Evaluate the expressions in Exercises 13–16 using the multiplication of powers rule.

13. $9 \cdot 27$

14. $10 \cdot 100,000$

15. $4 \cdot 64$

16. $125 \cdot 25$

✓ Independent Practice

Write the expressions in Exercises 1–4 in base and exponent form.

1. $5 \cdot 5$

2. $17 \cdot 17 \cdot 17 \cdot 17 \cdot 17 \cdot 17$

3. $q \cdot q \cdot q \cdot q \cdot q$

4. $-y \cdot -y \cdot -y$

5. Can you use the multiplication of powers rule to evaluate $8^3 \cdot 9^3$? Explain your answer.

Evaluate the expressions in Exercises 6–13. Use the multiplication of powers rule and give your answers in base and exponent form.

6. $5^7 \cdot 5^7$

7. $11^{26} \cdot 11^9$

8. $(-15)^3 \cdot (-15)^5$

9. $(-23)^{11} \cdot (-23)^{17}$

10. $9^5 \cdot 9$

11. $h^5 \cdot h^{10}$

12. $(-b)^9 \cdot (-b)^{11}$

13. $a^x \cdot a^y$

14. A piece of land is 2^6 feet wide and 2^7 feet long. What is the area of the piece of land? Give your answer as a power in base and exponent form. Then evaluate the power.

15. Evaluate 81 times 27 by converting the numbers to powers of three.

Now try these:

Lesson 5.1.1 additional questions — p457

Round Up

Using the *multiplication of powers* rule makes multiplying powers with the *same base* much easier. Just *add* the exponents together, and you'll get the exponent that goes with the answer.

Lesson 5.1.2

Dividing with Powers

California Standards:

Number Sense 2.1

Understand negative whole-number exponents. Multiply and **divide expressions involving exponents with a common base**.

Number Sense 2.3

Multiply, **divide**, and **simplify rational numbers by using exponent rules**.

What it means for you:

You'll learn about a rule that makes dividing powers easier.

Key words:

- power
- base
- exponent

Don't forget:

You can rewrite any division problem as a fraction.

$x \div y$ is the same as $\frac{x}{y}$.

So $x^n \div y^m$ is the same as

$$\frac{x^n}{y^m}.$$

Check it out:

The variables m and n could stand for any numbers, and a could stand for any number except zero.

Whatever numbers they are, the equation will still be true.

In the last Lesson you saw how you can use the multiplication of powers rule to help **simplify** expressions with powers in them. There's a similar rule to use when you're **dividing** powers with the **same base**.

The Division of Powers Rule

Look at the division $2^7 \div 2^4$.

If you write it out as a **fraction** it looks like this: $\frac{2^7}{2^4}$

Now write out the powers in **expanded form**: $\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2}$

If you do the same thing to both the top and bottom of the fraction you don't change its value — you create an **equivalent fraction**. So you can cancel four twos from the numerator with four twos from the denominator.

$$\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = 2 \cdot 2 \cdot 2 = 2^3$$

Now you can say that $2^7 \div 2^4 = 2^3$.

In the division expression the two exponents are 7 and 4, and in the solution the exponent is 3.

When you **subtract** the exponent of the denominator from the exponent of the numerator, you get the exponent of the solution. The base stays the same.

This is called the **division of powers rule**:

Division of powers rule:

When you are dividing two powers with the same base, subtract the second exponent from the first to give you the exponent of the answer.

$$a^m \div a^n = a^{(m-n)}$$

This Rule Only Works for the Same Bases

Just like with the multiplication of powers rule, it's important to remember that this rule **only** works with powers that have the same base. You **can't** use it on two powers with different bases.

- You **could** use the rule to simplify $5^3 \div 5^4$ as the bases are the same.
- You **couldn't** use it to simplify $3^5 \div 4^5$ because the bases are different.

Don't forget:

If you do the same thing to the top and bottom of a fraction its value stays the same. This means that if you have the same factor in the multiplication expressions on the top and bottom of a fraction you can cancel them with each other. It's the same as dividing the numerator and denominator by the same amount.

For example: $\frac{x \cdot y}{x \cdot z} = \frac{y}{z}$

In Example 1 you can cancel three fourteens from the top with three fourteens from the bottom.

Example 1

What is $14^6 \div 14^3$? Give your answer in base and exponent form.

Solution

You could write the division out in full

$$\begin{aligned} 14^6 \div 14^3 &= \frac{14 \cdot 14 \cdot 14 \cdot 14 \cdot 14 \cdot 14}{14 \cdot 14 \cdot 14} \\ &= \frac{\cancel{14} \cdot \cancel{14} \cdot \cancel{14} \cdot 14 \cdot 14 \cdot 14}{\cancel{14} \cdot \cancel{14} \cdot \cancel{14}} = 14^3 \end{aligned}$$

But these two powers have the same base. So you can use the division of powers rule.

$$\begin{aligned} 14^6 \div 14^3 &= 14^{(6-3)} = 14^3 \end{aligned}$$

Example 2

What is $(-5)^{18} \div (-5)^{10}$? Give your answer in base and exponent form.

Solution

The powers have the same base. Use the division of powers rule.

$$\begin{aligned} (-5)^{18} \div (-5)^{10} &= -5^{(18-10)} = (-5)^8 \text{ or } 5^8 \end{aligned}$$

Check it out:

The order that you do the subtraction in is very important. For example:

$$5^5 \div 5^3 = 5^2 = 25$$

$$5^3 \div 5^5 = 5^{-2} = \frac{1}{25} = 0.04$$

The answers are different. You'll see more about what a negative power means in Section 5.2.

When you're using the division of powers rule you must **always** remember to subtract the exponent of the **second power** from the exponent of the **first power** — never the other way around.

The commutative property doesn't apply to subtraction problems. If you change the order of the numbers you'll get a **different** answer.

Guided Practice

Evaluate the expressions in Exercises 1–8. Use the division of powers rule and give your answer in base and exponent form.

1. $6^9 \div 6^4$

2. $15^{25} \div 15^{10}$

3. $4^{206} \div 4^{54}$

4. $(-3)^7 \div (-3)^2$

5. $27^4 \div 27^1$

6. $7^5 \div 7$

7. $d^{10} \div d^7$

8. $(-w)^{14} \div (-w)^{-17}$

Division of Powers Can Help with Mental Math

Like the multiplication of powers rule the **division of powers rule** can come in handy when you're doing **mental math**. Convert the numbers into **base and exponent form** and use the rule to **simplify** the problem.

Example 3

What is $1024 \div 64$?

Solution

1024 and 64 are both powers of 4. So you can rewrite the problem in base and exponent form:

$$1024 \div 64 = 4^5 \div 4^3.$$

Now use the division of powers rule:

$$4^5 \div 4^3 = 4^{(5-3)} = 4^2.$$

$$4^2 = 16. \text{ So } 1024 \div 64 = \mathbf{16}.$$

Check it out:

343 and 49 are both powers of 7.

512 and 32 are both powers of 2.

Guided Practice

Evaluate the expressions in Exercises 9–12.

9. $1024 \div 16$

10. $100,000 \div 100$

11. $343 \div 49$

12. $512 \div 32$

Independent Practice

1. Evaluate $7^6 \div 7^4$ by writing it out in full as a fraction and canceling the numerator with the denominator. Check your answer using the division of powers rule.

Evaluate the expressions in Exercises 2–9. Use the division of powers rule and give your answer in base and exponent form.

2. $3^6 \div 3^2$

3. $23^{42} \div 23^{23}$

4. $(-8)^{20} \div (-8)^9$

5. $(-41)^{112} \div (-41)^{52}$

6. $4^8 \div 4$

7. $z^7 \div z^3$

8. $(-p)^{17} \div (-p)$

9. $g^a \div g^b$

10. A research lab produces 10^7 placebos (sugar pills) for a medical experiment. It distributes the placebos evenly among 10^3 bottles. How many placebos are in each bottle? Give your answer as a power in base and exponent form, then evaluate the power.

11. Evaluate 1296 divided by 216 by converting the numbers to powers of six.

12. What is half of 2^n ?

Now try these:

Lesson 5.1.2 additional questions — p457

Round Up

When you have two powers with the **same base** you can divide one by the other using the **division of powers rule**. Just **subtract** the exponent of the second power from the exponent of the first power, and you'll get the exponent that goes with the answer.

Lesson 5.1.3

Fractions with Powers

California Standards:

Number Sense 2.1

Understand negative whole-number exponents. **Multiply and divide expressions involving exponents with a common base.**

Number Sense 2.3

Multiply, divide, and simplify rational numbers by using exponent rules.

What it means for you:

You'll see how to use the rules you saw in Lessons 5.1.1 and 5.1.2 to simplify expressions with fractions in.

Key words:

- power
- exponent
- base

Don't forget:

When you raise a fraction to a power it's the same as raising its numerator and denominator separately to the same power. For example:

$$\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3}$$

For a reminder see Section 2.5.

The *multiplication and division of powers rules* still work if the bases are fractions. But you have to remember that to *raise a fraction to a power* you must raise the *numerator and denominator separately to the same power* — you saw this in Chapter 2.

The Rules Apply to Fractions Too

When the **bases** of powers are **fractions**, the multiplication and division of powers rules still apply, just as they would for whole numbers.

For example:

$$\left(\frac{2}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{(6+4)} = \left(\frac{2}{3}\right)^{10} \quad \text{and} \quad \left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{(6-4)} = \left(\frac{2}{3}\right)^2$$

For the rules to work the bases must be **exactly the same** fractions.

✓ Guided Practice

Simplify the expressions in Exercises 1–4.

Give your answers in base and exponent form.

1. $\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^2$

2. $\left(\frac{1}{3}\right)^7 \div \left(\frac{1}{3}\right)^5$

3. $\left(-\frac{2}{5}\right)^{15} \cdot \left(-\frac{2}{5}\right)^{23}$

4. $\left(\frac{a}{b}\right)^{10} \div \left(\frac{a}{b}\right)^7$

Simplifying Fraction Expressions with Different Bases

If you have an expression with **different** fractions raised to powers, apply the powers to the **numerators** and **denominators** of the fractions **separately**.

Then use the rules to **simplify** the expression.

Look at this expression.

$$\left(\frac{5}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^4$$

1) You can't use the multiplication of powers rule to simplify the expression as it is, because the bases are two different fractions.

$$\text{fraction 1} \left(\frac{5}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^4 \text{ fraction 2}$$

2) So write out the fractions with the numerators and denominators raised separately to the powers.

$$\left(\frac{5}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^4 = \frac{5^2}{3^2} \cdot \frac{2^4}{3^4}$$

3) When you multiply two fractions together, you multiply their **numerators** and their **denominators**.

$$\frac{5^2}{3^2} \cdot \frac{2^4}{3^4} = \frac{5^2 \cdot 2^4}{3^2 \cdot 3^4}$$

4) Now you can apply the **multiplication of powers** rule to the denominator of the fraction.

$$\frac{5^2 \cdot 2^4}{3^2 \cdot 3^4} = \frac{5^2 \cdot 2^4}{3^6}$$

5) The powers in the numerator have different bases. You can't simplify the fraction further without evaluating the exponents, so leave the answer in base and exponent form.

$$\frac{5^2 \cdot 2^4}{3^2 \cdot 3^4} = \frac{5^2 \cdot 2^4}{3^6}$$

Don't forget:

Dividing by a fraction is the same as multiplying by its reciprocal.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To divide one fraction by another fraction, flip the second fraction upside down and multiply. There's more on this in Lesson 2.3.4.

Check it out:

The number 1 to any power is always 1. That's because $1 \cdot 1 = 1$, $1 \cdot 1 \cdot 1 = 1$, etc. So $1^3 = 1$, and $1^3 \cdot 3^4 = 3^4$.

Example 1

Simplify the expression $\left(\frac{1}{2}\right)^3 \div \left(\frac{2}{3}\right)^4$ as far as possible, leaving your answer in base and exponent form.

Solution

$$\begin{aligned} & \left(\frac{1}{2}\right)^3 \div \left(\frac{2}{3}\right)^4 \\ &= \frac{1^3}{2^3} \div \frac{2^4}{3^4} && \text{Apply the exponents to the numerators and denominators separately} \\ &= \frac{1^3}{2^3} \cdot \frac{3^4}{2^4} && \text{Multiply by the reciprocal of the second fraction} \\ &= \frac{1^3 \cdot 3^4}{2^3 \cdot 2^4} && \text{The two powers on the bottom of the fraction have the same base. You can use the multiplication of powers rule.} \\ &= \frac{3^4}{2^7} && \text{Multiply the numerators and denominators} \end{aligned}$$

Guided Practice

Simplify the expressions in Exercises 5–10.

Give your answers in base and exponent form.

5. $\left(\frac{5}{2}\right)^3 \cdot \left(\frac{3}{2}\right)^2$

6. $\left(\frac{2}{9}\right)^2 \cdot \left(\frac{5}{9}\right)^{10}$

7. $\left(\frac{1}{4}\right)^{55} \cdot \left(\frac{3}{4}\right)^{72}$

8. $\left(\frac{1}{2}\right)^4 \div \left(\frac{2}{7}\right)^8$

9. $\left(\frac{1}{3}\right)^{15} \div \left(\frac{3}{2}\right)^{10}$

10. $\left(\frac{3}{2}\right)^3 \div \left(\frac{2}{3}\right)^2$

You Can Use Powers to Simplify Fraction Expressions

Sometimes converting numbers into **base and exponent** form can help you to **simplify** an expression that has **fractions** in it.

For example: $\left(\frac{2}{3}\right)^3 \cdot \frac{1}{9}$

You can expand the $\left(\frac{2}{3}\right)^3$ to $\frac{2^3}{3^3}$. At first this doesn't seem to help because the other fraction doesn't contain any powers with a base of 2 or 3.

But 9 is a power of 3: $9 = 3^2$. If you change the 9 in the fraction into 3^2 then the expression becomes:

$$\left(\frac{2}{3}\right)^3 \cdot \frac{1}{9} = \frac{2^3}{3^3} \cdot \frac{1}{3^2}$$

Now you can use the multiplication of powers rule to simplify the denominator.

$$\frac{2^3}{3^3} \cdot \frac{1}{3^2} = \frac{2^3 \cdot 1}{3^3 \cdot 3^2} = \frac{2^3}{3^5}$$

Example 2

Simplify $\left(\frac{4}{3}\right)^2 \div \frac{81}{64}$. Leave your answer in base and exponent form.

Solution

$$\left(\frac{4}{3}\right)^2 \div \frac{81}{64}$$

$$= \frac{4^2}{3^2} \div \frac{81}{64}$$

$$= \frac{4^2}{3^2} \cdot \frac{64}{81}$$

$$= \frac{4^2 \cdot 4^3}{3^2 \cdot 3^4}$$

$$= \frac{4^2 \cdot 4^3}{3^2 \cdot 3^4} = \frac{4^5}{3^6}$$

Apply the exponent to the numerator and denominator separately

Multiply by the reciprocal of the second fraction

Convert the numbers in the second fraction into powers

Multiply the numerators and denominators

Check it out:

1296 is a power of 6.
343 is a power of 7.
81 is a power of 3.
625 is a power of 5.
32 is a power of 2.

Guided Practice

Simplify the expressions in Exercises 11–16.

11. $\left(\frac{1}{2}\right)^{15} \cdot \frac{1}{4}$

12. $\left(\frac{4}{3}\right)^5 \cdot \frac{2}{81}$

13. $\frac{64}{1296} \cdot \left(\frac{4}{6}\right)^2$

14. $\left(\frac{2}{3}\right)^2 \div \frac{5}{16}$

15. $\left(\frac{1}{7}\right)^8 \div \frac{343}{15}$

16. $\frac{625}{32} \div \left(\frac{2}{5}\right)^{12}$

Independent Practice

Simplify the expressions in Exercises 1–4.

1. $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^3$

2. $\left(\frac{3}{5}\right)^{10} \div \left(\frac{3}{5}\right)^7$

3. $\left(-\frac{1}{2}\right)^{19} \div \left(-\frac{1}{2}\right)^9$

4. $\left(\frac{x}{y}\right)^a \cdot \left(\frac{x}{y}\right)^b$

Simplify the expressions in Exercises 5–10.

5. $\left(\frac{3}{4}\right)^2 \cdot \left(\frac{7}{4}\right)^3$

6. $\left(\frac{5}{9}\right)^{15} \cdot \left(\frac{10}{9}\right)^{23}$

7. $\left(\frac{2}{x}\right)^4 \cdot \left(\frac{3}{x}\right)^6$

8. $\left(\frac{7}{5}\right)^3 \div \left(\frac{5}{3}\right)^4$

9. $\left(\frac{4}{11}\right)^{15} \div \left(\frac{11}{6}\right)^{16}$

10. $\left(\frac{4}{r}\right)^{10} \div \left(\frac{r}{16}\right)^4$

Simplify the expressions in Exercises 11–16.

11. $\left(\frac{1}{3}\right)^5 \cdot \frac{7}{27}$

12. $\left(\frac{5}{6}\right)^4 \cdot \frac{125}{10}$

13. $\frac{9}{512} \cdot \left(\frac{5}{8}\right)^7$

14. $\left(\frac{2}{9}\right)^2 \div \frac{81}{5}$

15. $\left(\frac{4}{3}\right)^{10} \div \frac{47}{256}$

16. $\frac{1000}{49} \div \left(\frac{7}{10}\right)^5$

Now try these:

Lesson 5.1.3 additional questions — p457

Check it out:

27 is a power of 3.
512 is a power of 8 (and 2).
256 is a power of 4 (and 2).
125 is a power of 5.
1000 is a power of 10.

Round Up

If you can spot powers of simple numbers you'll be able to *recognize* when you can *simplify expressions* using the *multiplication and division of powers rules*. And that's a useful thing to be able to do in math — whether the bases are whole numbers or fractions.

Lesson 5.2.1

Section 5.2 Negative and Zero Exponents

California Standards:

Number Sense 2.1

Understand negative whole-number exponents. Multiply and divide expressions using exponents with a common base.

Algebra and Functions 2.1

Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

What it means for you:

You'll learn what zero and negative powers mean, and simplify expressions involving them.

Key words:

- base
- exponent
- power

Don't forget:

The expression " $a \neq 0$ " means a is not equal to zero.

Don't forget:

When you are dividing numbers with the same base, you can subtract the exponents.

Check it out:

a^0 doesn't change the value of whatever it's multiplied by. That's because it's equal to 1 — the multiplicative identity.

Up to now you've worked with only *positive* whole-number exponents. These show the number of times a base is *multiplied*. As you've seen, they follow certain rules and patterns.

The effects of *negative* and *zero* exponents are trickier to picture. But you *can* make sense of them because they follow the *same rules and patterns* as positive exponents.

Any Number Raised to the Power 0 is 1

Any number that has an exponent of 0 is equal to 1.

So, $2^0 = 1$, $3^0 = 1$, $10^0 = 1$, $\left(\frac{1}{2}\right)^0 = 1$.

For any number $a \neq 0$, $a^0 = 1$

You can show this using the **division of powers rule**.

If you start with 1000, and keep **dividing by 10**, you get this pattern:

$1000 = 10^3$	Now divide by 10: $10^3 \div 10^1 = 10^{(3-1)} = 10^2$
$100 = 10^2$	
$10 = 10^1$	Now divide by 10: $10^2 \div 10^1 = 10^{(2-1)} = 10^1$
$1 = 10^0$	Now divide by 10: $10^1 \div 10^1 = 10^{(1-1)} = 10^0$

The most important row is the second to last one, shown in **red**.

When you **divide 10 by 10**, you have $10^1 \div 10^1 = 10^{(1-1)} = 10^0$.

You also know that 10 divided by 10 is 1. So you can see that **$10^0 = 1$** .

This pattern works for any base.

For instance, $6^1 \div 6^1 = 6^{(1-1)} = 6^0$, and 6 divided by 6 is 1. So **$6^0 = 1$** .

You can use the fact that any number to the power 0 is 1 to simplify expressions.

Example 1

Simplify $3^4 \times 3^0$. Leave your answer in base and exponent form.

Solution

$$3^4 \times 3^0 = 3^4 \times 1 = 3^4$$

You can use the multiplication of powers rule to show this is right:

$$3^4 \times 3^0 = 3^{(4+0)} = 3^4 \quad \text{Add the exponents of the powers}$$

You can see that being multiplied by 3^0 didn't change 3^4 .

✓ Guided Practice

Evaluate the following.

- | | | |
|---------------------|-------------------------|----------------------------------|
| 1. 4^0 | 2. x^0 ($x \neq 0$) | 3. $11^0 + 12^0$ |
| 4. $(7 + 6)^0$ | 5. $4^3 \div 4^3$ | 6. $y^2 \div y^2$ ($y \neq 0$) |
| 7. $3^2 \times 3^0$ | 8. $2^4 \times 2^0$ | 9. $a^8 \div a^0$ ($a \neq 0$) |

You Can Justify Negative Exponents in the Same Way

By continuing the pattern from the previous page you can begin to understand the meaning of **negative exponents**.

Carry on dividing each power of 10 by 10:

1000	= 10^3	Now divide by 10: $10^3 \div 10^1 = 10^{(3-1)} = 10^2$
100	= 10^2	
10	= 10^1	Now divide by 10: $10^2 \div 10^1 = 10^{(2-1)} = 10^1$
1	= 10^0	
		Now divide by 10: $10^1 \div 10^1 = 10^{(1-1)} = 10^0$
		Now divide by 10: $10^0 \div 10^1 = 10^{(0-1)} = 10^{-1}$
$\frac{1}{10}$	= 10^{-1}	
		Now divide by 10: $10^{-1} \div 10^1 = 10^{(-1-1)} = 10^{-2}$
$\frac{1}{100}$	= 10^{-2}	
		Now divide by 10: $10^{-2} \div 10^1 = 10^{(-2-1)} = 10^{-3}$
$\frac{1}{1000}$	= 10^{-3}	

Look at the last rows, shown in red, to see the pattern:

One-tenth, which is $\frac{1}{10}$, can be rewritten as $\frac{1}{10^1} = 10^{-1}$.

One-hundredth, which is $\frac{1}{100}$, can be rewritten as $\frac{1}{10^2} = 10^{-2}$.

One-thousandth, which is $\frac{1}{1000}$, can be rewritten as $\frac{1}{10^3} = 10^{-3}$.

This works with any number, not just with 10.

For example:

$$6^0 = 1 \quad 6^0 \div 6^1 = 6^{-1} \text{ and } 1 \div 6 = \frac{1}{6}, \text{ so } 6^{-1} = \frac{1}{6}.$$

$$6^{-1} \div 6^1 = 6^{-2} \text{ and } \frac{1}{6} \div 6 = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = \frac{1}{6^2}, \text{ so } 6^{-2} = \frac{1}{6^2}.$$

This pattern illustrates the **general definition for negative exponents**.

For any number $a \neq 0$, $a^{-n} = \frac{1}{a^n}$

Check it out:

Two numbers that multiply together to give 1 are multiplicative inverses.

$6^{-1} = 1 \div 6^1$, so $6^1 \times 6^{-1} = 1$, meaning 6^{-1} is the multiplicative inverse or reciprocal of 6^1 .

But also, $6^{-1} \times \frac{1}{6^{-1}} = \frac{6^{-1}}{6^{-1}} = 1$.

So, $\frac{1}{6^{-1}}$ is the multiplicative inverse of 6^{-1} too.

You can multiply 6^{-1} by either 6^1 or $\frac{1}{6^{-1}}$ to get 1, so $6^1 = \frac{1}{6^{-1}}$.

Example 2

Rewrite 5^{-3} without a negative exponent.

Solution

$$5^{-3} = \frac{1}{5^3} \text{ or } \frac{1}{125} \quad \text{Using the definition of negative exponents}$$

Example 3

Rewrite $\frac{1}{7^5}$ using a negative exponent.

Solution

$$\frac{1}{7^5} = 7^{-5} \quad \text{Using the definition of negative exponents}$$

Guided Practice

Rewrite each of the following without a negative exponent.

10. 7^{-3}

11. 5^{-m}

12. x^{-2} ($x \neq 0$)

Rewrite each of the following using a negative exponent.

13. $\frac{1}{3^3}$

14. $\frac{1}{6^4}$

15. $\frac{1}{q \times q \times q}$ ($q \neq 0$)

Independent Practice

Evaluate the expressions in Exercises 1–3.

1. 8702^0

2. g^0 ($g \neq 0$)

3. $2^0 - 3^0$

Write the expressions in Exercises 4–6 without negative exponents.

4. 45^{-1}

5. x^{-6} ($x \neq 0$)

6. $y^{-3} - z^{-3}$ ($y \neq 0, z \neq 0$)

Write the expressions in Exercises 7–9 using negative exponents.

7. $\frac{1}{8^2}$

8. $\frac{1}{r^6}$ ($r \neq 0$)

9. $\frac{1}{(p+q)^v}$ ($p+q \neq 0$)

In Exercises 10–12, simplify the expression given.

10. $5^4 \times 5^0$

11. $c^5 \times c^0$ ($c \neq 0$)

12. $f^3 \div f^0$ ($f \neq 0$)

13. The number of bacteria in a petri dish doubles every hour.

The numbers of bacteria after each hour are 1, 2, 4, 8, 16, ...

Rewrite these numbers as powers of 2.

14. Rewrite the numbers 1, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ as powers of 2.

Now try these:

Lesson 5.2.1 additional questions — p458

Round Up

So remember — *any number (except 0) to the power of 0 is equal to 1. This is useful when you're simplifying expressions and equations. Later in this Section, you'll see how negative powers are used in scientific notation for writing very small numbers efficiently.*

Lesson 5.2.2

Using Negative Exponents

California Standards:

Number Sense 2.1

Understand negative whole-number exponents. **Multiply and divide expressions using exponents with a common base.**

Algebra and Functions 2.1

Interpret positive whole-number powers as repeated multiplication and **negative whole-number powers** as repeated division or **multiplication by the multiplicative inverse.** **Simplify and evaluate expressions that include exponents.**

What it means for you:

You'll multiply and divide expressions that share the same base. This includes expressions containing negative exponents.

Key words:

- base
- denominator
- exponent
- numerator
- power

Don't forget:

As with regular addition and subtraction of integers, you need to be very careful with your positive and negative signs.

- Adding a negative can be thought of as a subtraction. For example, $7 + (-3) = 7 - 3$.
- Subtracting a number is the same as adding its opposite. For example, the subtraction $6 - (-2)$ can be rewritten as $6 + 2$.

Don't forget:

10 can be rewritten as 10^1 .

Negative exponents might seem a bit tricky at first. But the rules for positive exponents work with negative exponents in exactly the same way. This Lesson gives you plenty of practice at using the rules with negative exponents.

Simplifying Expressions with Integer Exponents

Both the **multiplication of powers rule** ($a^m \times a^n = a^{m+n}$) and the **division of powers rule** ($a^m \div a^n = a^{m-n}$) work with **any** rational exponents — it doesn't matter if they are positive or negative.

The examples below apply the multiplication and division of powers rules to numbers with **negative** exponents.

Example 1

Simplify $5^{-4} \times 5^{-3}$.

Solution

The bases are the same, so the multiplication of powers rule can be applied.

$$\begin{aligned} 5^{-4} \times 5^{-3} &= 5^{(-4+(-3))} && \text{Use the multiplication of powers rule} \\ &= 5^{(-4-3)} \\ &= \mathbf{5^{-7}} \end{aligned}$$

Example 2

Simplify $7^6 \div 7^{-2}$.

Solution

The bases are the same, so the division of powers rule can be applied.

$$\begin{aligned} 7^6 \div 7^{-2} &= 7^{(6-(-2))} && \text{Use the division of powers rule} \\ &= 7^{(6+2)} \\ &= \mathbf{7^8} \end{aligned}$$



Guided Practice

Simplify the expressions in Exercises 1–8.

1. $5^8 \times 5^{-2}$

2. $6^{-7} \times 6^3$

3. $10^{-4} \times 10$

4. $7^{-3} \times 7^{-9}$

5. $7^5 \div 7^{-4}$

6. $11^{-3} \div 11^7$

7. $10 \div 10^6$

8. $2^{-7} \div 2^{-5}$

You Can Turn Negative Exponents into Positive Ones

This is a different way to tackle the problems on the previous page. You can **decide** which method you prefer to use.

Some people like to convert negative exponents into **positive exponents** before doing any problems involving them.

So when they see a number like 4^{-6} , they rewrite it as $\frac{1}{4^6}$.

The examples on the previous page are repeated below using this method.

Check it out:

Another way to write x^{-n}

$$\text{is } \left(\frac{1}{x}\right)^n.$$

Example 3

Simplify $5^{-4} \times 5^{-3}$.

Solution

$$\begin{aligned} 5^{-4} \times 5^{-3} &= \frac{1}{5^4} \times \frac{1}{5^3} \\ &= \frac{1 \times 1}{5^4 \times 5^3} \\ &= \frac{1}{5^{(4+3)}} \\ &= \frac{1}{5^7} \text{ or } 5^{-7} \end{aligned}$$

First convert to positive exponents

Multiply the fractions

Use the multiplication of powers rule

Example 4

Simplify $7^6 \div 7^{-2}$.

Solution

$$\begin{aligned} 7^6 \div 7^{-2} &= 7^6 \div \frac{1}{7^2} \\ &= 7^6 \times \frac{7^2}{1} \\ &= 7^6 \times 7^2 \\ &= 7^{(6+2)} \\ &= 7^8 \end{aligned}$$

First convert to positive exponents

To divide by $\frac{1}{7^2}$, multiply by its reciprocal

Use the multiplication of powers rule

Don't forget:

To divide by a fraction, you multiply by its reciprocal. You find the reciprocal by flipping the fraction.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Guided Practice

Simplify the expressions in Exercises 9–16 by first converting any negative exponents to positive exponents.

9. $6^3 \times 6^{-9}$

10. $4^2 \times 4^{-5}$

11. $7^8 \times 7^{-4}$

12. $12^{-7} \times 12^{-2}$

13. $3^{-4} \div 3$

14. $2^3 \div 2^{-6}$

15. $10^5 \div 10^{-6}$

16. $11^{-8} \div 11^{-3}$

Getting Rid of Negative Exponents in Fractions

You might have to deal with fractions that have **negative exponents** in the numerator and denominator, like $\frac{2^{-4}}{3^{-7}}$. It's useful to be able to change them into fractions with only **positive exponents** — because it's a simpler form.

A number with a **negative exponent** in the numerator is equivalent to the same number with a **positive exponent** in the denominator $\Rightarrow 2^{-4} = \frac{2^{-4}}{1} = \frac{1}{2^4}$.

A number with a **negative exponent** in the denominator is equivalent to the same number with a **positive exponent** in the numerator $\Rightarrow \frac{1}{3^{-7}} = \frac{3^7}{1} = 3^7$.

So: 2^{-4} gets moved from the numerator to the denominator, where it is written as 2^4 . $\Rightarrow \frac{2^{-4}}{3^{-7}} = \frac{3^7}{2^4}$ $\leftarrow 3^{-7}$ moves from the denominator and becomes 3^7 in the numerator.

Don't forget:

You could also do this sort of calculation by converting the negative exponents into positive exponents before multiplying.

$$\frac{7^3}{8^{-4}} \times \frac{7^{-6}}{8^2} = \frac{8^4 \times 7^3}{1} \times \frac{1}{8^2 \times 7^6}$$

$$\frac{8^4 \times 7^3}{1} \times \frac{1}{8^2 \times 7^6} = \frac{8^4 \times 7^3}{8^2 \times 7^6}$$

$$\frac{8^4 \times 7^3}{8^2 \times 7^6} = 8^{(4-2)} \times 7^{(3-6)}$$

$$8^{(4-2)} \times 7^{(3-6)} = 8^2 \times 7^{-3} \text{ or } \frac{8^2}{7^3}$$

Example 5

Simplify $\frac{7^3}{8^{-4}} \times \frac{7^{-6}}{8^2}$.

Solution

$$\begin{aligned} \frac{7^3}{8^{-4}} \times \frac{7^{-6}}{8^2} &= \frac{7^3 \times 7^{-6}}{8^{-4} \times 8^2} && \text{Multiply the fractions} \\ &= \frac{7^{(3+(-6))}}{8^{(-4+2)}} && \text{Use the multiplication of powers rule} \\ &= \frac{7^{-3}}{8^{-2}} = \frac{8^2}{7^3} && \text{Convert to positive exponents} \end{aligned}$$

Multiply the fractions

Use the multiplication of powers rule

Convert to positive exponents

Guided Practice

Rewrite the expressions in Exercises 17–20 without negative exponents.

17. $\frac{3^{-2}}{8^{-3}}$

18. $\frac{2^4}{3^{-6}}$

19. $2^4 \times 5^{-3}$

20. $\frac{11^{-3}}{7^2}$

Independent Practice

Simplify the expressions in Exercises 1–3.

1. $10^4 \div 10^{-3}$

2. $5^{-2} \times 5^5$

3. $7^{-3} \div 7^{-9}$

Rewrite the expressions in Exercises 4–6 using only positive exponents.

4. $\frac{5^{-2}}{2^{-7}}$

5. $\frac{6^3}{11^{-2}}$

6. $7^3 \times 4^{-9}$

Multiply the fractions in Exercises 7–8 and write the answers using only positive exponents.

7. $\frac{4^{-5}}{6^7} \times \frac{6^{-2}}{4^{-3}}$

8. $\frac{2^{-5}}{11^3} \times \frac{2^{-4}}{11^{-7}}$

Now try these:

Lesson 5.2.2 additional questions — p458

Round Up

After this Lesson you should be comfortable with **multiplying** and **dividing** expressions with **negative exponents**. Remember — you can only use these rules if the bases are **equal**.

Lesson 5.2.3

Scientific Notation

California Standards:

Number Sense 1.1

Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.

What it means for you:

You'll see how you can use powers of 10 to make very big or very small numbers easier to work with.

Key words:

- scientific notation
- numeric form
- power
- decimal
- base
- exponent

Don't forget:

For more about writing large numbers in scientific notation, see Lesson 2.5.3.

Check it out:

You'll come across lots of very large and very small numbers written using scientific notation outside of math. For example, large numbers like distances between planets or populations of countries, and small numbers like the length of a molecule or the weight of a speck of dust.

Scientific notation is a handy way of writing very large and very small numbers. Earlier in the book, you practiced using powers of ten to write out large numbers. In this Lesson, you'll get a reminder of how to do that. Then you'll see that with negative powers, you can do the same thing for very small numbers.

You Can Use Powers of 10 to Write Large Numbers

In Chapter 2 you saw how to write **large** numbers as a product of two factors using **scientific notation**.

The first factor is a number that is at least 1 but less than 10.

The second factor is a power of ten. The exponent tells you how many places to move the decimal point to get the number.

$$1,200,000 = 1.2 \times 10^6$$

Example 1

The planet Saturn is about 880,000,000 miles away from the Sun. Write this number in scientific notation.

Solution

$$880,000,000 = 8.8 \times 100,000,000$$

Split the number into the appropriate factors.

$$= 8.8 \times 10^8 \text{ miles}$$

Write the power of ten in base and exponent form.

Guided Practice

Write the numbers in Exercises 1–6 in scientific notation.

1. 487,000,000,000

2. 6000

3. 93,840,000

4. -1,630,000,000,000

5. 28,410,000,000,000

6. -3,854,000,000

You Can Write Small Numbers in Scientific Notation

Scientific notation is also a useful way to write very **small numbers**. A number like 0.0000054 can be rewritten as a **division**.

$$0.0000054 = 5.4 \div 1,000,000$$

Using powers of 10 you can write this as

$$0.0000054 = 5.4 \div 10^6$$

And remember that $1 \div 10^6 = \frac{1}{10^6} = 10^{-6}$, so you can write

$$0.0000054 = 5.4 \times 10^{-6}$$

5.4×10^{-6} is 0.0000054 written in scientific notation.

Example 2

A red blood cell has a diameter of 0.000007 m. Write this number in scientific notation.

Solution

$$0.000007 = 7 \div 1,000,000$$

Split the number into a decimal and a power of ten.

$$= 7 \div 10^6$$

Write the power of ten in base and exponent form.

$$= 7 \times 10^{-6} \text{ m}$$

Change division by a positive power to multiplication by a negative power.

Check it out:

Remember to include the units — they'll be the same as in the original number.

Guided Practice

Write the numbers in Exercises 7–12 in scientific notation.

7. 0.000419

8. 0.000000000015

9. 0.00000007

10. 0.000030024

11. 0.00008946

12. 0.00000004645

You Can Convert Numbers from Scientific Notation

Sometimes you might need to take a number that's in scientific notation, and write it as an ordinary number.

When you **multiply by 10**, the decimal point moves one place to the **right**.

When you **divide by 10**, the decimal point moves one place to the **left**.

You can use these facts to **convert** a number from scientific notation back to **numeric form**.

Check it out:

Numeric form means the number written out in full.

Example 3

Write 3.0×10^{11} in numeric form.

Solution

“ 3.0×10^{11} ” means “multiply 3.0 by 10, 11 times.”

To multiply 3.0 by 10^{11} , all you need to do is move the decimal point **11 places** to the **right**. It might help to write out the 3.0 with extra 0s — then you can see how the decimal point is moving.

$$\begin{aligned}
 &3.0 \times 10^{11} \\
 &= 3.\underbrace{00000000000}_{11 \text{ places}} \times 10^{11} \\
 &= \mathbf{300,000,000,000}
 \end{aligned}$$

The green line shows the decimal point moving 11 places to the right.

Lesson 5.2.4

Comparing Numbers in Scientific Notation

California Standards:

Number Sense 1.1

Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.

What it means for you:

You'll learn how to tell which is the larger of any two numbers written in scientific notation.

Key words:

- scientific notation
- exponent
- coefficient
- power

Don't forget:

The expression " $a > b$ " means " a is greater than b ."

When you look at two numbers, you can usually tell straightaway which is *larger*. If the two numbers are in *scientific notation*, you might need to think a bit harder. But once you know what *part* of the number to look at *first*, it becomes much more straightforward.

Look at the Exponent First, Then the Other Factor

The problem with comparing numbers written in scientific notation is that each number has two parts to look at.

There's the **number between 1 and 10**... 2.89×10^6

... and there's the **power of 10**. 2.89×10^6

The **first** thing to look at is the **power**. The number with the **greater** exponent is the **larger** number.

Example 1

Which of the following numbers is larger?

$$4.23 \times 10^8 \quad \text{or} \quad 7.91 \times 10^6$$

Solution

The **exponent** in 4.23×10^8 is **8**.

The **exponent** in 7.91×10^6 is **6**.

$8 > 6$, so **4.23×10^8 is the larger number.**

If two numbers have the **same power of 10**, then you need to look at the **other factor** — the number between 1 and 10.

The number with the **greater** factor is the **larger** number.

Example 2

Which of the following numbers is larger?

$$4.23 \times 10^8 \quad \text{or} \quad 7.91 \times 10^8$$

Solution

The **exponents** of the power of 10 in these two numbers are the same, so you need to compare the **other factors**.

$7.91 > 4.23$, so **7.91×10^8 is the larger number.**

✓ Guided Practice

In Exercises 1–6, say which of each pair of numbers is greater.

1. 9.1×10^6 , 8.2×10^9 2. 4.61×10^5 , 1.05×10^{10}

3. 3.21×10^3 , 6.8×10^3 4. 8.4×10^8 , 5.75×10^7

5. 6.033×10^{12} , 2.46×10^{12} 6. 2.6×10^4 , 2.09×10^4

In Exercises 7 and 8, write out each set of numbers in order from smallest to largest:

7. 8.34×10^{10} 7.1×10^9 5.71×10^{10} 9.64×10^9

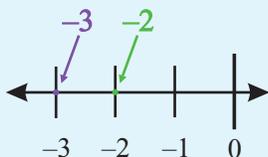
8. 3.8×10^5 3.09×10^6 3.41×10^5 4.12×10^5

Check it out:

It might help to think about where the negative numbers would be on the number line. A number that is further to the right is always greater.

For example:

$$-3 < -2 \text{ or } -2 > -3$$



Be Careful with Negative Exponents

You need to take care when you compare numbers in scientific notation that have **negative powers**.

The number with the **greater** exponent is still **larger** — but remember that negative numbers can look like they're getting bigger, when they're actually getting smaller.

Example 3

Which of the following numbers is larger?

$$4.23 \times 10^{-5} \text{ or } 7.91 \times 10^{-7}$$

Solution

The **exponent** in 4.23×10^{-5} is **-5**.

The **exponent** in 7.91×10^{-7} is **-7**.

$-5 > -7$, so **4.23×10^{-5} is the larger number.**

If the negative exponents are the **same**, then the number with the greater **coefficient** is still larger.

Example 4

Which of the following numbers is larger?

$$4.23 \times 10^{-9} \text{ or } 7.91 \times 10^{-9}$$

Solution

The **exponents** in these two numbers are the same, so you need to compare the **coefficients**.

$7.91 > 4.23$, so **7.91×10^{-9} is the larger number.**

✓ Guided Practice

In Exercises 9–14, say which of each pair of numbers is greater.

9. 1.4×10^{-4} , 2.3×10^{-6} 10. 5.0×10^{-6} , 4.8×10^{-6}
 11. 7.42×10^{-33} , 3.89×10^{-23} 12. 1.57×10^{-4} , 9.31×10^{-5}
 13. 6.04×10^{-86} , 6.2×10^{-86} 14. 9.99×10^{-40} , 1.45×10^{-17}

In Exercises 15 and 16, write out each set of numbers in order from smallest to largest:

15. 4.97×10^{-8} 4.52×10^{-7} 3.08×10^{-8} 3.18×10^{-7}
 16. 6.4×10^{-15} 6.04×10^{-13} 6.44×10^{-13} 6.14×10^{-15}

✓ Independent Practice

In Exercises 1–8, say which of each pair of numbers is greater.

1. 4.25×10^{18} , 3.85×10^{19} 2. 9.16×10^{-12} , 6.4×10^{-10}
 3. 2.051×10^7 , 1.19×10^4 4. 8.04×10^{-9} , 7.96×10^{-9}
 5. 5.22×10^{45} , 7.01×10^{45} 6. 6.861×10^{-22} , 4.0×10^{-21}
 7. 7.89×10^{11} , 7.9×10^{11} 8. 3.642×10^{-30} , 1.886×10^{-28}

This table shows the mass of one atom for five chemical elements. Use it to answer Exercises 9–11.

9. Which is the heaviest element?
 10. Which element is lighter, silver or titanium?
 11. List all five elements in order from lightest to heaviest.

Element	Mass of atom (kg)
Titanium	7.95×10^{-26}
Lead	3.44×10^{-25}
Silver	1.79×10^{-25}
Lithium	1.15×10^{-26}
Hydrogen	1.674×10^{-27}

This table shows the approximate distance from Earth of six stars. Use the table to answer Exercises 12–16.

12. Which of these stars is nearest to Earth?
 13. Which of these stars is furthest from Earth?
 14. How many of these stars are nearer than Castor A?
 15. Which is closer to Earth, Peacock or Sirius?
 16. List all six stars in order from nearest to furthest.

Name of Star	Approx. Distance from Earth (miles)
Bellatrix	1.4×10^{15}
Sirius	5.04×10^{13}
Barnard's Star	3.50×10^{13}
Castor A	3.1×10^{14}
Peacock	1.1×10^{15}
Deneb	1.9×10^{16}

Now try these:

Lesson 5.2.4 additional questions — p459

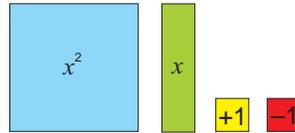
Round Up

If you have two numbers written in *scientific notation* and you want to know which is *larger*, look at the two *parts* of the numbers. First *compare their exponents*, and then, if necessary, *compare their other factors*. And don't forget to watch out for *negative exponents*.

Section 5.3 introduction — an exploration into: Monomials

A *monomial* is a term that is a constant, a variable or a combination of both. In this Exploration, you'll use *Algebra Tiles* to multiply and divide monomials.

The tiles you'll use are shown here:



When you **multiply** two numbers, the product is the same as the **area** of the rectangle that has the two numbers as the **length and width**.

$$2 \times 3 = \text{Area} = 6$$

Monomial multiplication works in exactly the same way.

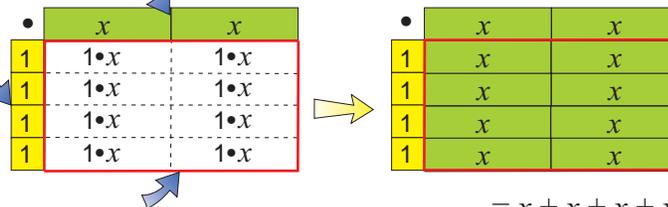
Example

Multiply the expression $4 \cdot 2x$ using algebra tiles.

Solution

Put one factor on the top ($2x$ here)

...and the other down the side (4 here)



The product ($4 \cdot 2x$) is the area inside the red box.

$$= x + x + x + x + x + x + x + x = 8x$$

You can use algebra tiles to **divide monomials**. You are given the **area** of the rectangle of tiles and one of the side lengths. The goal is to find the tiles that match the **other side length**.

Example

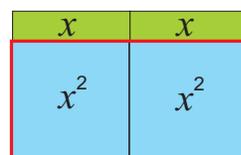
Divide the expression $2x^2 \div 2x$ using algebra tiles.

Solution

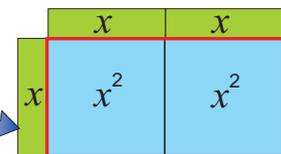
$2x^2$ is the area, and $2x$ is the length of one side.

The tiles that fit into the other side represent the quotient.

x fits on the other side. So $2x^2 \div 2x = x$



The thing you're dividing by goes on one side of the area rectangle.



x fits on this side.

Exercises

1. Use algebra tiles to simplify the expressions.

a. $2 \cdot 3x$

b. $2x \cdot 2x$

c. $3 \cdot 3x$

d. $x \cdot 4$

e. $3x^2 \div x$

f. $5x \div 5$

g. $4x^2 \div 2x$

h. $2x^2 \div x$

Round Up

You know that the *area* of a rectangle is the *length multiplied by the width*. Well, if the *length and width* are *monomials*, then it's just the same — the *area* of the rectangle is their *product*.

Lesson 5.3.1

Section 5.3 Multiplying Monomials

California Standards:

Algebra and Functions 1.4

Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

Algebra and Functions 2.2

Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

What it means for you:

You'll learn how to multiply together expressions that have only one term.

Key words:

- monomial
- coefficient
- constant

Don't forget:

The whole numbers are zero and all the positive integers: 0, 1, 2, 3, 4...

Don't forget:

$-9a^3$ means $-9 \times a^3$.

Check it out:

A monomial that is just a number is called a **constant**, since its value can't change (unlike the value of a monomial that contains a variable).

The monomials 2, 6, 3.75, and $\frac{1}{2}$ are all constants.

In Section 5.1 you saw how to do multiplications like $y \times y^2$, or $x^2 \times x^3$. Now you need to know how to multiply together terms that contain numbers or more than one variable, like $2x^2 \times 3x^3$, or $2xy \times y^2$. Things like $2x^2$ and $2xy$ are called **monomials** — and this Lesson is going to show you how to multiply them.

A Monomial Has Only One Term

A **monomial** is a type of expression that has only **one term** — meaning it has **no additions or subtractions**.

A monomial can include **numbers, fractions, and variables raised to whole number powers**, but they can only be **multiplied together**.

8 , y , y^3 , $\frac{y}{2}$, and $5xy^2$ are monomials — expressions with just one term and only whole number powers.

$2 + y$, $a^2 + 4$, and $y^4 - 9$ aren't monomials — they all have more than one term.

x^{-1} , y^{-3} , and $m^{0.5}$ aren't monomials — they contain powers that aren't whole numbers.

In a Monomial the Number is the Coefficient

The number that the variable is multiplied by is called the **coefficient**.

- In the expression $5y$ the coefficient is **5**.
- In the expression $\frac{y}{2}$ the coefficient is $\frac{1}{2}$.
- In the expression x the coefficient is **1**. Multiplying by 1 **doesn't change** the value of a number, so writing $1x$ is **the same** as writing x .

Example 1

Which of the following expressions are monomials?
For those that are monomials, what is the coefficient?

- a) $2x$, b) z^2 , c) $a + b$, d) $5n^{-3}$, e) $\frac{y^4}{2}$

Solution

- a) **$2x$ is a monomial.** The coefficient is **2**.
b) **z^2 is a monomial.** The coefficient is **1**. The coefficient is 1 because $z^2 = 1 \times z^2$.
c) **$a + b$ is not a monomial.** Two terms have been **added** together.
d) **$5n^{-3}$ is not a monomial.** -3 is not a **whole number power** because it is **negative**.
e) **$\frac{y^4}{2}$ is a monomial.** The coefficient is $\frac{1}{2}$ (since $\frac{y^4}{2} = \frac{1}{2}y^4$).

✓ Guided Practice

In Exercises 1–8, state whether or not the expression is a monomial.

- | | |
|--------------|--------------------|
| 1. $3x^4$ | 2. $3x^{-4}$ |
| 3. $3 + x^4$ | 4. x^{-4} |
| 5. $3xy^4$ | 6. $3x + y^4$ |
| 7. $x^4 - 3$ | 8. $\frac{x^4}{3}$ |

State the coefficient of each of the monomials in Exercises 9–12.

- | | |
|---------------------|--------------|
| 9. $11x^3$ | 10. $14xy^4$ |
| 11. $\frac{z^2}{8}$ | 12. $-9a^3$ |

Don't forget:

Multiply powers together by adding the exponents:

$$x^a \times x^b = x^{a+b}$$

For example, $x^3 \times x^4 = x^7$.

This is the multiplication of powers rule. You saw it in Lesson 5.1.1.

Don't forget:

This method works because of the **commutative** and **associative** properties of multiplication (which say that things can be multiplied together in any order and you get the same results — see Lesson 1.1.5).

When you work out $4x^3y$ multiplied by $6x^4y^4$, you are really doing this:

$$4 \cdot x^3 \cdot y \cdot 6 \cdot x^4 \cdot y^4.$$

By using the method in Example 2, you are just multiplying them in the most convenient order — first the numbers, then the powers of x , and then the powers of y .

Treat Each Variable Separately When Multiplying

To multiply monomials you deal with the **coefficients** and **each different variable** separately. You'll often need to use the **multiplication of powers** rule too.

Example 2

Multiply together $4x^3y$ and $6x^4y^4$.

Solution

Multiply the coefficients and each different variable separately.

- Multiply the coefficients: $4 \times 6 = 24$
- Multiply the powers of x together: $x^3 \cdot x^4 = x^7$
- Multiply the powers of y together: $y \cdot y^4 = y^5$

Now **multiply** all these results together to form your final answer.

So $4x^3y$ multiplied by $6x^4y^4$ gives $24x^7y^5$.

Sometimes, only one of the expressions contains a particular variable.

Example 3

Find $2ab^5 \cdot 4a^2c$.

Solution

Multiply the coefficients and each different variable separately.

- Multiply the coefficients: $2 \times 4 = 8$
- Multiply the powers of a together: $a \cdot a^2 = a^3$
- Multiply the powers of b together (only one monomial contains b so you just include that power of b in your answer): b^5
- Multiply the powers of c together (again, only one monomial contains c so just include that power of c in your answer): c

Now **multiply** all these results together to form your final answer.

So $2ab^5 \cdot 4a^2c = 8a^3b^5c$.

There's a quick way to work these out — **multiply the coefficients** and **add the exponents** of each different variable.

Example 4Find $6x^5y^6 \cdot 2xy^2z$.**Solution**

$$6x^5y^6 \cdot 2xy^2z = (6 \times 2) \cdot x^{5+1} \cdot y^{6+2} \cdot z = 12x^6y^8z$$

As always, be extra careful if there are **negative numbers** or **fractions**.**Example 5**Find $-12p^2qr^4$ multiplied by $\frac{3}{4}prs^3$.**Solution**

$$-12p^2qr^4 \cdot \frac{3}{4}prs^3 = \left(-12 \times \frac{3}{4}\right) p^{2+1} q r^{4+1} s^3 = -9p^3qr^5s^3$$

Don't forget:

When you are multiplying a fraction by an integer, just multiply the numerator of the fraction by the integer, and then simplify it if you can.

For example:

$$3 \cdot \frac{1}{6} = \frac{3 \cdot 1}{6} = \frac{3}{6} = \frac{1}{2}$$

Check it out:

You can multiply three (or more) monomials together in stages — first multiply two monomials together, and then multiply the result by the third.

Now try these:

Lesson 5.3.1 additional questions — p459

Don't forget:

To “square” something means to multiply it by itself.

Guided Practice

Find the results of each multiplication in Exercises 13–22.

13. $x^2 \cdot x^5$

14. $5y^8 \cdot 3y^2$

15. $2z^2 \cdot x^4$

16. $-2a \cdot 3a$

17. $3a^2b^3c^2 \cdot a^4bc^4$

18. $12a^3b^3c^3d^3 \cdot 3ab^2c^3d^4$

19. $5xy^4 \cdot x^2z$

20. $0.5x^2 \cdot 3y^2z$

21. $a^2b^4c^6$ multiplied by $a^2b^4c^6$

22. $\left(-\frac{2}{3}p^2qr\right) \cdot \left(\frac{3}{4}pq^5r^{10}s^2\right)$

Write down the answers to Exercises 23–26.

23. $xy^2 \cdot x^2y \cdot xy$

24. $ab^2 \cdot a^3b^4c^5 \cdot a^6b^7c^8$

25. $pqr^2 \cdot pr \cdot q^2r$

26. $mn^3 \cdot a^2b^7 \cdot abmn$

Independent Practice

Which of the expressions in Exercises 1–6 are monomials?

1. $2bc$

2. $12a + 2$

3. xy^3

4. $3x^{-4}$

5. $-4x^3$

6. $a^2b^3c^4$

State the coefficient in each of the monomials in Exercises 7–12.

7. $5x$

8. $8a^2b^3$

9. p^2

10. $-3y^6$

11. $0.3d^2$

12. $3.142r^2$

Calculate the coefficient of each product in Exercises 13–14.

13. $5x$ multiplied by $2y$

14. $-10a$ multiplied by $0.5b^3$

Calculate each product in Exercises 15–20.

15. $5x \cdot 4x$

16. $2xy \cdot 8x^2$

17. $3a^2b^2 \cdot 6ab^2c$

18. $-12xy^3 \cdot 7xz$

19. $\frac{1}{2}p^2q^3 \cdot \frac{2}{3}p^3q^2$

20. $2f^{33}g^{11} \cdot 8f^{71}g^{12}$

Square each monomial in Exercises 21–23.

21. x^2

22. $3y^3$

23. $4a^2b^2$

Round Up

So that's how you multiply monomials — you deal with the **numbers** first, and then each of the **variables in turn**, and then **multiply** the results together. You'll get a lot of practice with this skill because you need to use it all the time in math.

Lesson 5.3.2

Dividing Monomials

California Standards:

Algebra and Functions 1.4

Use algebraic terminology

(e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

Algebra and Functions 2.2

Multiply and **divide monomials**; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

What it means for you:

You'll learn how to divide expressions that only have one term.

Key words:

- monomial
- coefficient

Don't forget:

Any number or variable to the power 1 is just itself. For example, $2^1 = 2$, $9^1 = 9$, $a^1 = a$, $x^1 = x$, and so on. This means you can rewrite $2x^2y$ as $2x^2y^1$.

You saw how to multiply monomials in the previous Lesson. The next step is to learn how to **divide monomials** — and that's what this Lesson is all about.

Divide Monomials by Subtracting Exponents

Dividing monomials works in a very similar way to multiplying them.

You deal with coefficients and each different variable **in turn**.

But when dividing monomials, you **subtract variables' exponents** rather than add them. This is because you're using the **division of powers rule**.

Example 1

Find $8a^8 \div 4a^6$.

Solution

Treat the coefficients and the variable separately.

- Divide the coefficients: $8 \div 4 = 2$
- Divide the powers of a using the division of powers rule:

$$a^8 \div a^6 = a^{8-6} = a^2$$

Now **multiply** these results together to form your final answer.

$$\text{So } 8a^8 \div 4a^6 = 2a^2.$$

Notice how you **divide** the coefficients and the variables, but then you **multiply** all the results together at the end.

So in the previous example, you **divided** the coefficients to get **2**, and you **divided** the powers of a to get a^2 — but then you **multiplied** these together to get the final answer of $2a^2$.

Example 2

Find $10x^7y^4 \div 2x^2y$.

Solution

Treat the coefficients and each different variable separately.

- Divide the coefficients: $10 \div 2 = 5$
- Divide the powers of x using the division of powers rule: $x^7 \div x^2 = x^5$
- Divide the powers of y using the division of powers rule: $y^4 \div y = y^3$

Now **multiply** all these results together to form your final answer.

$$\text{So } 10x^7y^4 \div 2x^2y = 5x^5y^3.$$

You have to be very careful to get the **signs** of your **exponents** correct — especially if a variable only appears in the second monomial. It helps to remember that $x^0 = 1$ for **any** value of x . Example 3 shows why this is useful.

Example 3

Find $12a^4b^5 \div 6a^2c^4$.

Solution

First, rewrite the division — making sure that **all** the variables appear in **both** monomials. Do this using the fact that $b^0 = 1$ and $c^0 = 1$.

So you have to find $12a^4b^5c^0 \div 6a^2b^0c^4$.

Now divide the coefficients and each variable in turn in the normal way.

- Divide the coefficients: $12 \div 6 = 2$
- Divide the powers of a by subtracting the exponents: $a^4 \div a^2 = a^{4-2} = a^2$
- Divide the powers of b by subtracting the exponents: $b^5 \div b^0 = b^{5-0} = b^5$
- Divide the powers of c in exactly the same way: $c^0 \div c^4 = c^{0-4} = c^{-4}$

Then **multiply** all these results together to give your final answer.

So $12a^4b^5 \div 6a^2c^4 = 2a^2b^5c^{-4}$.

Check it out:

You can't change the order of division calculations.

That means you must always subtract the exponents in the second monomial from those in the first, regardless of which is bigger. This might result in a negative exponent. If you need a reminder on negative exponents, see Section 5.2.

Guided Practice

Find the results of each division in Exercises 1–8.

- $3x^4 \div x$
- $10a^6 \div 5a^3$
- $6x^5y^4 \div 2x^2y^2$
- $16p^{10}q^7r^2 \div 2p^4q^5$
- $-12m \div 2m^2n^5$
- $2xy^4 \div 5x^4y^5z^2$
- $0.5xyz \div 2m^3n^8$
- $\frac{2}{3}p^2q^3r^2 \div \frac{4}{9}pq^2r^4s^6$

Dividing Monomials Doesn't Always Give a Monomial

When you **multiply** monomials, you **always** get another **monomial**.

However, when you **divide** one monomial by another the result **isn't** always a monomial. You may end up with an answer that contains a **negative exponent** — which is **not** a monomial. That's what happened in Example 3, above.

Example 4

Find $4x^2y \div 8x^3y$.

Solution

Treat the coefficients and the variables in turn, as usual.

$4 \div 8 = 0.5$, while $x^2 \div x^3 = x^{-1}$, and $y \div y = 1$.

So $4x^2y \div 8x^3y = 0.5x^{-1}$.

But the exponent of x **isn't a whole number**, so this **isn't** a monomial.

Don't forget:

Monomials only involve numbers and whole number powers of variables. The whole numbers are 0, 1, 2, 3, 4...

So $0.5x^{-1}$ isn't a monomial.

✓ Guided Practice

Find the result of each of the divisions in Exercises 9–14.
State whether each result is a monomial.

9. $7x^5 \div x^2$

10. $8y^5 \div 2y^7$

11. $9a^2b^3 \div 3a^2b^3$

12. $18p^8q^5 \div 6p^8q^6$

13. $-6m^{11}n^8 \div 3m^2n^5$

14. $2x^5y^6z^7 \div 5x^2y^3z^4$

Divide Coefficients and Subtract Exponents

In the previous Lesson, you saw that there was a quick way to multiply monomials — you multiplied the coefficients and added exponents.

You can do a similar thing when you divide monomials.

But this time you **divide coefficients** and **subtract exponents**.

Example 5

Find $6x^5y^6 \div 2xy^2z$.

Solution

It's best to rewrite this so that **both** monomials contain **all** the different variables. Use the fact that $z^0 = 1$.

So you need to find $6x^5y^6z^0 \div 2xy^2z$.

$$6x^5y^6z^0 \div 2x^1y^2z^1 = (6 \div 2) \cdot x^{5-1} \cdot y^{6-2} \cdot z^{0-1} = 3x^4y^4z^{-1}$$

Don't forget:

A variable z can be rewritten as z^1 .

✓ Guided Practice

Find the results of each division in Exercises 15–18.

15. $12a^4 \div 4a^2$

16. $100b^6c^2 \div 25a^2b^4c$

17. $26p^3q^5 \div 4p^2q^2r^2$

18. $169m^{100}n^9 \div 13q^5$

✓ Independent Practice

Evaluate the divisions in Exercises 1–6.

1. $x^8 \div x^3$

2. $35 \div 5y^{-4}$

3. $11y^4 \div y^6$

4. $6y^2 \div 3x^4$

5. $-4a^3 \div 0.5ab^2$

6. $\frac{1}{2}m^2n^5 \div \frac{4}{9}mn^7$

7. Which of these expressions have the same value: $(1 \div a^n)$, $(a^0 \div a^n)$, a^{-n} ?

8. What is $5x^3$ divided by $-5x^5$?

Find the quotients in Exercises 9–11.

9. $20x^2y^6 \div 5x^8y^2$

10. $11ab^5 \div 121bc^4$

11. $\frac{w^7z^7}{8} \div w^2z^4$

Now try these:

Lesson 5.3.2 additional questions — p459

Round Up

*Dividing monomials isn't really any harder than multiplying them. But you do have to remember that you **subtract** exponents when dividing, which means that you could end up with **negative exponents**. You'll use these ideas in later Lessons, so make sure you remember the rules.*

Lesson 5.3.3

Powers of Monomials

California Standards:

Algebra and Functions 1.4

Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

Algebra and Functions 2.2

Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

What it means for you:

You'll see how to raise simple expressions to powers.

Key words:

- monomial
- coefficient
- power

Don't forget:

The associative law of multiplication says that you can group things in any way when multiplying and get the same result:

$$a \times (b \times c) = (a \times b) \times c$$

The commutative law of multiplication says that you can multiply things in any order and get the same result:

$$a \times b = b \times a$$

Combining these properties, you can show that you can multiply any number of things together in any order and get the same result.

So you don't need parentheses in expressions containing only multiplications. For example:

$$a \times ((b \times c) \times d) = a \times b \times c \times d$$

Earlier in the book you found out about *powers*. They show repeated multiplication — for example, $2 \times 2 \times 2 = 2^3$. This Lesson is all about how to raise *monomial expressions* to powers.

Powers Can Be Raised to Other Powers

You might see expressions in which a **power** is **raised to another power**.

For example, you can write the expression $2^4 \times 2^4 \times 2^4$ as $(2^4)^3$.

You can **add the powers** using the multiplication of powers rule to find the result. So $(2^4)^3 = 2^4 \times 2^4 \times 2^4 = 2^{4+4+4} = 2^{12}$.

This gives exactly the same result as **multiplying the exponents** in $(2^4)^3$ together. So you can write $(2^4)^3 = 2^{4 \times 3} = 2^{12}$.

You can write this more generally as:

$$(a^m)^n = a^{m \cdot n}$$

This is called the **power of a power rule**.

Example 1

By writing the expression as a multiplication, show that $(y^3)^2 = y^6$.

Solution

You've got to show that the power of a power rule works for $(y^3)^2$.

As always, work out the parentheses first: $y^3 = y \cdot y \cdot y$

$$\text{So } (y^3)^2 = (y \cdot y \cdot y) \cdot (y \cdot y \cdot y)$$

But you can remove the parentheses here, because it doesn't matter how you group things in multiplications.

$$\text{Therefore } (y^3)^2 = y \cdot y \cdot y \cdot y \cdot y \cdot y = y^6.$$

Example 2

Write $(4^3)^6$ as a power of 4.

Solution

Use the power of a power rule — multiply the powers.

$$(4^3)^6 = 4^{3 \times 6} = 4^{18}$$

Example 3

Simplify: a) $(x^5)^8$ b) $(x^5)^{-8}$

Solution

a) Multiply the powers together. $(x^5)^8 = x^{5 \times 8} = x^{40}$

b) The rule also works with negative powers. $(x^5)^{-8} = x^{5 \times (-8)} = x^{-40}$

✓ Guided Practice

Write the expressions in Exercises 1–9 using a single power.

1. $(2^3)^2$

2. $(3^5)^4$

3. $(7^{99})^{10}$

4. $(x^4)^8$

5. $(a^8)^{-101}$

6. $(r^p)^q$

7. $(5^5)^5$

8. $(s^{10})^{-10}$

9. $(a^{2m})^n$

Use the Same Rule to Find Powers of Monomials

All **monomials** can be raised to powers — even really complicated ones.

Just like with a power of a power, you can **simplify** this kind of expression by remembering that a power means **repeated multiplication**.

Example 4

Simplify $(3xy)^4$.

Solution

Everything inside the parentheses is raised to the power of 4.

$$(3xy)^4 = (3xy) \cdot (3xy) \cdot (3xy) \cdot (3xy)$$

You can simplify this by removing the parentheses.

$$(3xy)^4 = 3 \cdot x \cdot y \cdot 3 \cdot x \cdot y \cdot 3 \cdot x \cdot y \cdot 3 \cdot x \cdot y$$

Now you can rearrange this multiplication using the associative and commutative properties of multiplication.

$$\begin{aligned}(3xy)^4 &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \\ &= 3^4 \cdot x^4 \cdot y^4 \\ &= \mathbf{81x^4y^4}\end{aligned}$$

You can see in Example 4 that **each part** of the original monomial is **raised to the 4th power** in the result.

To raise any monomial to a power, use the following rule.

Raising a monomial to a power

To take a monomial to the n th power, find the n th power of each part of the monomial, and multiply the results.

When you're raising **monomials** to **powers**, you'll often need to use the **power of a power rule**.

Example 5

Simplify $(5b^3)^2$.

Solution

$$\begin{aligned}(5b^3)^2 &= 5^2 \cdot (b^3)^2 \\ &= 5^2 \cdot b^6 \\ &= \mathbf{25b^6}\end{aligned}$$

Don't forget:

$$(b^3)^2 = b^{3 \times 2} = b^6$$

There could be **any number** of variables in the monomial, but you always do exactly the same thing — raise each individual part of the monomial to the power outside the parentheses.

Example 6

Simplify $(2x^2yz^3)^5$.

Solution

$$\begin{aligned}(2x^2yz^3)^5 &= 2^5 \cdot (x^2)^5 \cdot y^5 \cdot (z^3)^5 \\ &= 2^5 \cdot x^{10} \cdot y^5 \cdot z^{15} = \mathbf{32x^{10}y^5z^{15}}\end{aligned}$$

The method stays the same if the expression contains negative exponents.

Example 7

Simplify $(a^2b)^{-4}$.

Solution

$$(a^2b)^{-4} = a^{2 \times (-4)} \cdot b^{-4} = \mathbf{a^{-8}b^{-4}}$$

✓ Guided Practice

Simplify the powers of monomials in Exercises 10–18.

10. $(3x^3)^2$

11. $(2x^2)^4$

12. $(x^2y)^3$

13. $(2a^4b^2)^4$

14. $(2p^4q^2r^3)^5$

15. $(2^p q^r)^s$

16. $\left(\frac{1}{2}ab\right)^2$

17. $\left(\frac{2}{3}x^2y\right)^3$

18. $(0.5p^3qr^4)^4$

✓ Independent Practice

1. Use the multiplication of powers rule to simplify $a^m \cdot a^m \cdot a^m$.

2. Simplify $(7^2)^8$ by writing it in the form 7^a .

Simplify each of the expressions in Exercises 3–10.

3. $(y^3)^4$

4. $(5x)^2$

5. $(p^2)^q$

6. $(8x^5)^2$

7. $(10x^7y^3)^4$

8. $(z^9)^{-3}$

9. $(5a^4b^3)^2$

10. $(x^6y^{11})^{-2}$

11. Show that $(6^5)^3 = (6^3)^5$.

12. Show that $(a^m)^n = (a^n)^m$.

13. What number is equal to $((2^2)^2)^2$?

14. What is $((a^m)^n)^p$?

15. A circular cross-section of an atom has a radius of 10^{-10} meters. Find the area of the cross-section.

Now try these:

Lesson 5.3.3 additional questions — p460

Don't forget:

The formula you use to find the area of a circle is πr^2 .

Round Up

So that's how you raise a *monomial* to a *power* — you just raise all the *individual parts* to the same *power*. You're likely to need to use the *power of powers rule* for this, so make sure you know it.

Lesson 5.3.4

Square Roots of Monomials

California Standards:

Algebra and Functions 2.2

Multiply and divide monomials; extend the process of taking powers, and **extracting roots to monomials when the latter results in a monomial with an integer exponent.**

What it means for you:

You'll learn how to find the square root of a whole expression.

Key words:

- monomial
- coefficient
- absolute value
- exponent
- square root
- perfect square

Don't forget:

Absolute value is covered in Section 2.2.

Check it out:

The **negative** square root of $\sqrt{x^2}$ is $-|x|$.

Taking the **square root** of a **monomial** is like the reverse of raising a monomial to the power of two. There's just one extra complication that you need to be aware of.

\sqrt{x} Means the Positive Square Root of x

A **square root** of a number is a factor that can be **multiplied by itself** to give the number. All **positive** numbers have **one positive** and **one negative** square root. For example, 6 and -6 are both square roots of 36 because $6 \cdot 6 = 36$ and $-6 \cdot -6 = 36$.

The **square root symbol**, $\sqrt{\quad}$, always means the **positive** square root. If you are finding the **negative** square root, you must put a **minus sign** in front of the square root symbol.

So $\sqrt{4} = 2$ and the negative square root of 4 is $-\sqrt{4} = -2$.

Example 1

Find: a) the square roots of 81 b) $\sqrt{81}$ c) $-\sqrt{81}$

Solution

a) The square roots of 81 are **9 and -9** .

b) $\sqrt{81} = 9$ c) $-\sqrt{81} = -9$

✓ Guided Practice

1. What are the square roots of 9?

2. What are the square roots of 25?

Evaluate the expressions in Exercises 3–6.

3. $\sqrt{1}$ 4. $-\sqrt{100}$ 5. $-\sqrt{196}$ 6. $\sqrt{9}$

Use Absolute Value to Give the Positive Square Root

The square roots of x^2 are x and $-x$. But if you're asked to find $\sqrt{x^2}$, only the **positive square root** is correct.

x could be any value — positive **or** negative. So if you write $\sqrt{x^2} = x$, and it turns out that $x = -2$, then you **haven't** given the positive square root.

To get around this, you can write that $\sqrt{x^2} = |x|$ (the **absolute value** of x). This way, you know you've given the **positive answer**.

Example 2Find $\sqrt{z^2}$.**Solution**

The square roots of z^2 are z and $-z$. You only want the positive value though. But without knowing anything about z , you can't say which of z or $-z$ is positive.

But you do know that $|z|$ (the **absolute value** of z) is **positive**.

So $\sqrt{z^2} = |z|$.

Divide Exponents by Two to Find the Square Root

The square roots of z^6 are z^3 and $-z^3$.

That's because $z^3 \cdot z^3 = z^6$ and $-z^3 \cdot -z^3 = z^6$.

$\sqrt{z^6}$ means just the **positive square root** of z^6 — so $\sqrt{z^6} = |z^3|$.

The absolute value signs are important because you can't say whether z^3 or $-z^3$ is positive — but you know that $|z^3|$ is definitely positive.

For instance, if z is -2 , then $z^3 = -2 \cdot -2 \cdot -2 = -8$,

but $|z^3| = |-2 \cdot -2 \cdot -2| = |-8| = 8$.

It's a bit different if the square root has an **even exponent**.

For example, $\sqrt{z^8} = z^4$. You **don't** need absolute value signs here because z^4 is **always positive** — it doesn't matter if z is positive or negative.

Again, say $z = -2$:

$z^4 = -2 \cdot -2 \cdot -2 \cdot -2 = (-2 \cdot -2) \cdot (-2 \cdot -2) = 4 \cdot 4 = 16$.

So to find the positive square root of a variable:

1. Divide the exponent by **two**.
2. Put **absolute value** signs around any expression with an **odd** exponent.

Example 3Find a) $\sqrt{z^4}$, b) $\sqrt{y^{10}}$.**Solution**

a) Divide the exponent by 2:

$$\sqrt{z^4} = z^{4 \div 2} = z^2 \quad \leftarrow \text{You don't need to include absolute value signs because the exponent, 2, is even.}$$

b) Divide the exponent by 2:

$$\sqrt{y^{10}} = |y^5| \quad \leftarrow \text{You do need to include absolute value signs here because the exponent, 5, is odd.}$$

Don't forget:

The multiplication of powers rule says that when you multiply two powers with the same base, you can add their exponents to give you the exponent of the answer.

$$a^m \cdot a^n = a^{m+n}$$

Don't forget:

$$z^4 \cdot z^4 = z^{(4+4)} = z^8$$

Don't forget:

Negative numbers with odd exponents are always negative.
Negative numbers with even exponents are always positive.

✓ Guided Practice

Evaluate the square roots in Exercises 7–12.

7. $\sqrt{t^2}$

8. $\sqrt{p^4}$

9. $\sqrt{r^{12}}$

10. $\sqrt{q^{14}}$

11. $\sqrt{s^{18}}$

12. $\sqrt{w^{30}}$

Check it out:

Raising a monomial to a power and finding a monomial's square root are so similar because finding a square root is raising to a power — the power of $\frac{1}{2}$.

Taking the Square Root of a Monomial

Finding the square root of a **monomial** is very similar to raising a monomial to a **power**. You find the square root of each “individual part” of the monomial.

Example 4

Find $\sqrt{9x^2}$.

Solution

First you need to find the positive square root of 9 and the positive square root of x^2 . Then multiply the results together.

The positive square root of 9 is **3**.

The positive square root of x^2 is $|x|$.

So $\sqrt{9x^2} = 3|x|$ or $3x$.

The method is the same even if the monomial has **many parts**.

Example 5

Find $\sqrt{36a^2b^4}$.

Solution

Find the square root of each part, since $\sqrt{36a^2b^4} = \sqrt{36} \cdot \sqrt{a^2} \cdot \sqrt{b^4}$.

$\sqrt{36} = 6$, $\sqrt{a^2} = |a|$, and $\sqrt{b^4} = b^2$.

So $\sqrt{36a^2b^4} = 6|a|b^2$

Check it out:

$6|a|b^2$ could be written $6|ab^2|$, or $|6ab^2|$ — they are exactly the same. The important thing is that the expression is definitely positive.

✓ Guided Practice

Find the square roots in Exercises 13–21.

13. $\sqrt{4x^2}$

14. $\sqrt{16r^2}$

15. $\sqrt{36s^4}$

16. $\sqrt{100p^8}$

17. $\sqrt{64x^2y^4}$

18. $\sqrt{25m^2n^6}$

19. $\sqrt{121m^2n^6p^2}$

20. $\sqrt{x^{10}y^{12}z^{14}}$

21. $\sqrt{400p^{122}q^{246}r^{38}}$

The Coefficient Might Not Always Be a Perfect Square

Don't forget:

A perfect square is the square of an integer.

Don't forget:

Irrational numbers can't be expressed as a fraction. They go on forever and have no repeating pattern — so there's no way to write them down exactly.

Every positive number has a square root. But if the number isn't a perfect square, then its square root will be a **decimal** — it may even be **irrational**. If you do get an irrational number, you should leave the **square root sign** in your answer.

Example 6

Find $\sqrt{15b^6}$.

Solution

$$\begin{aligned}\sqrt{15b^6} &= \sqrt{15} \cdot \sqrt{b^6} \\ &= \sqrt{15} |b^3|\end{aligned}$$

15 is not a perfect square — so keep the square root sign in your answer.

Guided Practice

Find the square roots of the expressions in Exercises 22–25.

22. $\sqrt{3x^2}$ 23. $\sqrt{7x^2y^6}$ 24. $\sqrt{22s^{10}t^{14}}$ 25. $\sqrt{55b^2c^{78}}$

Independent Practice

1. What are the square roots of 49?
2. What is $\sqrt{49}$?
3. Explain why your answers to Exercises 1 and 2 were different.
4. Stevie wrote this equation: $\sqrt{a^2} = a$, where a is an integer. Explain why Stevie's equation is incorrect. Write a correct version.

In Exercises 5–12, simplify the expressions.

5. $\sqrt{x^6}$ 6. $\sqrt{4x^2}$ 7. $\sqrt{9p^2q^{18}}$
8. $\sqrt{81r^4s^{22}}$ 9. $\sqrt{19x^2}$ 10. $\sqrt{2x^2y^2}$
11. $\sqrt{5x^2y^8}$ 12. $\sqrt{169p^{16}q^8}$

13. Suppose you know that $\sqrt{q} = p$, and neither p nor q equals zero. Which of p and q are positive? Explain your answer.

Now try these:

Lesson 5.3.4 additional questions — p460

Round Up

*Don't forget — the square root sign means the **positive square root**. The trickiest thing about finding the **positive square root** is remembering to make sure that your answer is definitely **positive**. You need to remember to put **absolute value bars** around any variables with odd exponents.*

Section 5.4 introduction — an exploration into: The Pendulum

There are many *real-life situations* that can be modeled with *graphs*. In this Exploration, you'll be making pendulums of different lengths and recording the *time* they take to swing back and forth a certain number of times. You'll see that the graph you get isn't a *linear* (straight line) graph — it's a *curve* (or a non-linear graph).

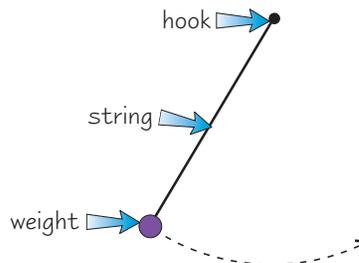
You should work with a partner to complete this Exploration. You'll make each pendulum by tying a weight to a piece of string — then you'll need to find a fixed hook to attach it to.

You need to make **four pendulums of different lengths** — 25 cm long, 50 cm long, 75 cm long, and 100 cm long.

You'll use a **stopwatch** to time how long each pendulum takes to complete **ten swings**. In one full swing, the pendulum moves from one side, to the other, and back again to its starting position. To make it a fair test, pull the weight out by the same amount each time.

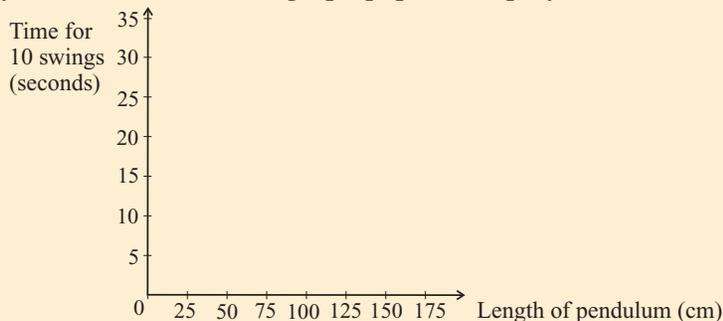
Record the times in a copy of this table.

Length (cm)	Time (s) for 10 swings
25	
50	
75	
100	



Exercises

- Copy the axes below onto graph paper. Graph your results.



- Do the points lie on a straight line or a curve?
Connect the points with an appropriate line or curve.
- Use your graph to predict the amount of time it will take for a 150 cm pendulum to swing back and forth ten times.
- Use your graph to predict the length of a pendulum which takes 17 seconds to swing back and forth ten times.

Round Up

Some things *don't* have a linear (straight-line) relationship. So when you plot them on a graph the points don't lie in a straight line. They sometimes lie in a *smooth curve* — so you mustn't try to join them with a straight line. There's lots about non-straight line graphs later in this Section.

Lesson 5.4.1

Section 5.4 Graphing $y = nx^2$

California Standards:

Algebra and Functions 3.1

Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

Mathematical Reasoning 2.5

Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

What it means for you:

You'll learn how to plot graphs of equations with squared variables in them.

Key words:

- parabola
- vertex

Check it out:

The scales on the x - and y -axes are different, so be careful when you plot points or read off values.

Check it out:

The graph of $y = x^2$ is symmetrical about the y -axis. This means the y -axis is a "mirror line" for the graph.

Think about the monomial x^2 . You can put any number in place of x and work out the result — different values of x give different results. The results you get form a **pattern**. And the best way to see the pattern is on a **graph**.

The Graph of $y = x^2$ is a Parabola

You can find out what the **graph** of $y = x^2$ looks like by **plotting** points.

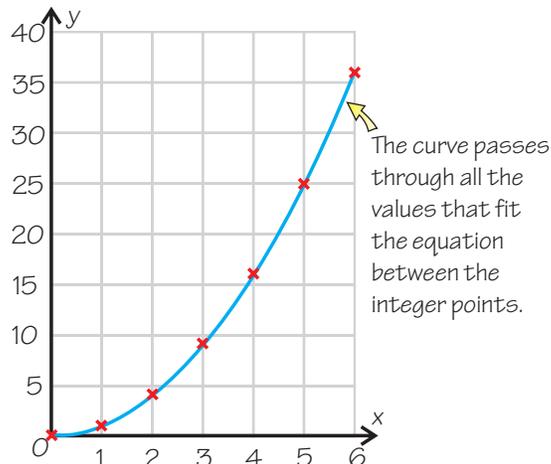
Example 1

Plot the graph of $y = x^2$ for values of x between 0 and 6.

Solution

The best thing to do first is to make a **table** for the integer values of x like the one below. Then you can **plot points** on a set of axes using the x - and y -values as **coordinates**, and join the points with a smooth curve.

x	$y (= x^2)$
0	0
1	1
2	4
3	9
4	16
5	25
6	36



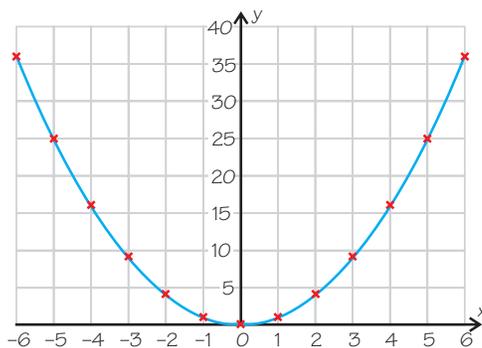
To see what happens for **negative** values of x , you can extend the table.

Example 2

Plot the graph of $y = x^2$ for values of x between -6 and 6 .

Solution

The table of values and the curve look like this:



x	$y (= x^2)$
-6	36
-5	25
-4	16
-3	9
-2	4
-1	1

This kind of curve is called a **parabola**.

Check it out:

There are two ways to check if a point is on a graph. You can either find the point on the coordinate plane and see if it lies on the graph. Or you can put the x - and y -coordinates into the equation for the graph and see if the equation is true. For instance, to test whether $(2, 3)$ lies on the graph $y = x^2$, put $x = 2$ and $y = 3$ into the equation:
 $y = x^2$
 $3 \neq 2^2$
So the point $(2, 3)$ doesn't lie on the graph.

Don't forget:

Always join points on graphs using a smooth curve.

Check it out:

Equations like $y = x^2$ represent functions. A function is a rule that assigns each number to one other number. If you put a value for x into a function, you get one value for y out.

Check it out:

If $x^2 = 12$, then
 $x = \sqrt{12}$ or $x = -\sqrt{12}$.

Check it out:

The square roots of 12 are actually 3.464 and -3.464 (to 3 decimal places).

Guided Practice

1. Which of the following points are on the graph of $y = x^2$?
 $(1, 1)$, $(-1, 1)$, $(-2, -4)$, $(2, 4)$, $(3, 9)$, $(-4, -16)$, $(5, 25)$, $(6, -36)$

In Exercises 2–5, calculate the y -coordinate of the point on the graph of $y = x^2$ whose x -coordinate is shown.

2. 6 3. -10 4. -2.5 5. $\frac{1}{3}$

In Exercises 6–9, calculate the two possible x -coordinates of the points on the graph of $y = x^2$ whose y -coordinate is shown.

6. 16 7. 25 8. 49 9. 30

You Can Use a Graph to Solve an Equation

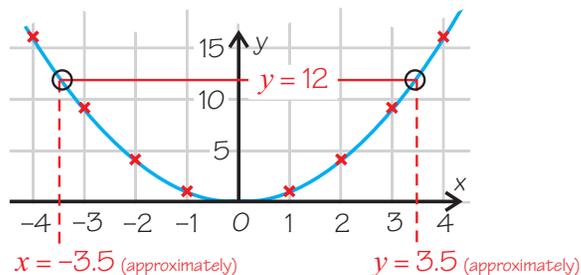
Graphs can be useful if you need to **solve an equation**. Using them means you don't have to do any tricky calculations — and they often show you **how many solutions** the equation has. The downside is that it can be impossible to get an **exact answer** by reading off a graph.

Example 3

Using the graph of $y = x^2$ in Example 2, solve $x^2 = 12$.

Solution

Since the graph shows $y = x^2$, you need to find where $y = 12$. Then you can find the corresponding value (or values) of x .



You can see that there are **two** different values of x that correspond to $y = 12$, at approximately $x = 3.5$ and $x = -3.5$.

This is because 12 has two square roots — a positive one (3.5) and a negative one (-3.5). Or you can look at that another way, and say that the numbers 3.5 and -3.5 can both be squared to give 12 (approximately).

Guided Practice

Use the graph of $y = x^2$ to solve the equations in Exercises 10–13.

10. $x^2 = 16$ 11. $x^2 = 25$
12. $x^2 = 10$ 13. $x^2 = 30$

The Graph of $y = nx^2$ is Also a Parabola

The graph of $y = x^2$ is $y = nx^2$ where $n = 1$. It has the U shape of a parabola. Other values of n give graphs that look very similar.

Example 4

Plot the graphs of the following equations for values of x between -5 and 5 .

a) $y = 2x^2$ b) $y = 3x^2$ c) $y = 4x^2$ d) $y = \frac{1}{2}x^2$

Solution

All these equations are of the form $y = nx^2$, for different values of n (2 then 3 then 4 then $\frac{1}{2}$).

The best place to start is with a table of values, just like before.

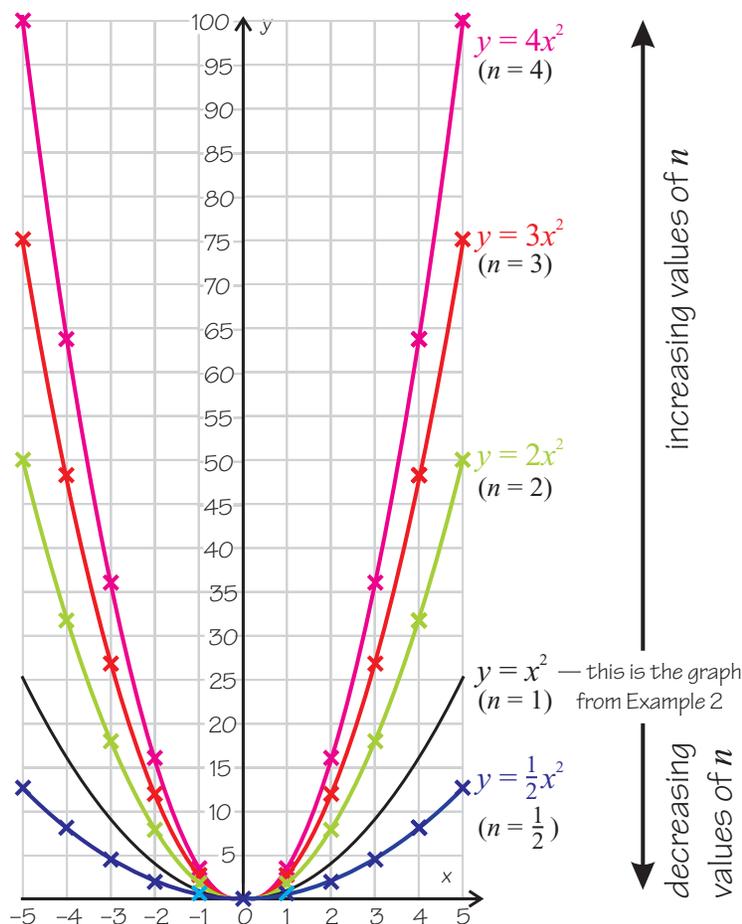
The table on the right shows values for parts a)–d).

x	$2x^2$	$3x^2$	$4x^2$	$\frac{1}{2}x^2$
0	0	0	0	0
1 and -1	2	3	4	0.5
2 and -2	8	12	16	2
3 and -3	18	27	36	4.5
4 and -4	32	48	64	8
5 and -5	50	75	100	12.5

Check it out:

Notice how $x = -1$ and $x = 1$ give the same value of $y = nx^2$. The same goes for any pair of positive and negative numbers with the same absolute value.

You then need to plot the **y-values** in each colored column against the **x-values** in the first column.



Check it out:

Values of n greater than 1 give parabolas steeper (or “narrower”) than $y = x^2$.

Values of n between 0 and 1 give parabolas less steep (or “wider”) than $y = x^2$.

Notice how all the graphs are “u-shaped” parabolas. And all the graphs have their **vertex** (the lowest point) at the same place, the **origin**.

In fact, this is a general rule — if n is positive, the graph of $y = nx^2$ will always be a “u-shaped” parabola with its vertex at the origin.

Also, the **greater** the value of n , the **steeper** the parabola will be.

In Example 4, the graph of $y = 4x^2$ had the steepest parabola, while the graph of $y = \frac{1}{2}x^2$ was the least steep.

✓ Guided Practice

For Exercises 14–17, draw on the same axes the graph of each of the given equations.

14. $y = 5x^2$ 15. $y = \frac{1}{4}x^2$ 16. $y = 10x^2$ 17. $y = \frac{1}{10}x^2$

In Exercises 18–23, use the graphs from Example 4 to solve the given equations.

18. $2x^2 = 20$ 19. $3x^2 = 25$ 20. $4x^2 = 15$
21. $\frac{1}{2}x^2 = 10$ 22. $3x^2 = 70$ 23. $2x^2 = 42$

✓ Independent Practice

Using a table of values, plot the graphs of the equations in Exercises 1–3 for values of x between -4 and 4 .

1. $y = 1.5x^2$ 2. $y = 5x^2$ 3. $y = \frac{1}{3}x^2$

On the same set of axes as you used for Exercises 1–3, sketch the approximate graphs of the equations in Exercises 4–6.

4. $y = 2.5x^2$ 5. $y = 6x^2$ 6. $y = \frac{2}{3}x^2$

7. If s is the length of a square’s sides, then a formula for its area, A , is $A = s^2$. Plot a graph of A against s , for values of s up to 10.

8. On a graph of $y = x^2$, what is the y -coordinate when $x = 10^3$?

For Exercises 9–12, find the y -coordinate of the point on the graph of $y = x^2$ for each given value of x .

9. $x = 10^{-1}$ 10. $x = 10^{-4}$ 11. $x = \frac{2}{3}$ 12. $x = \frac{8}{5}$

For Exercises 13–15, find the x -coordinates of the point on the $y = x^2$ graph for each given value of y .

13. $y = 10^2$ 14. $y = 10^{-6}$ 15. $y = 2^8$

Now try these:

Lesson 5.4.1 additional questions — p460

Don't forget:

For Exercise 7, make sure all your values of s make sense as the length of a square’s sides.

Don't forget:

To take a square root of a power, you need to divide the exponent by 2. See Lesson 5.3.4 for more information.

Round Up

In this Lesson you’ve looked at graphs of the form $y = nx^2$, where n is positive. The basic message is that these graphs are all *u-shaped*. And the **greater** the value of n , the **narrower** and **steeper** the parabola is. Remember that, because in the next Lesson you’re going to look at graphs of the same form where n is **negative**.

Lesson 5.4.2

More Graphs of $y = nx^2$

California Standards:

Algebra and Functions 3.1

Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

What it means for you:

You'll learn more about how to plot graphs of equations with squared variables in them, and how to use the graphs to solve equations.

Key words:

- graph
- vertex

Don't forget:

$y = -x^2$ is just $y = nx^2$ with $n = -1$.

In the last Lesson you saw a lot of bucket-shaped graphs. These were all graphs of equations of the form $y = nx^2$, where n was positive. The obvious next thing to think about is what happens when n is negative.

The Graph of $y = nx^2$ is Still a Parabola if n is Negative

By plotting points, you can draw the graph of $y = -x^2$.

Example 1

Plot the graph of $y = -x^2$ for values of x from -5 to 5 .

Solution

As always, first make a table of values, then plot the points.

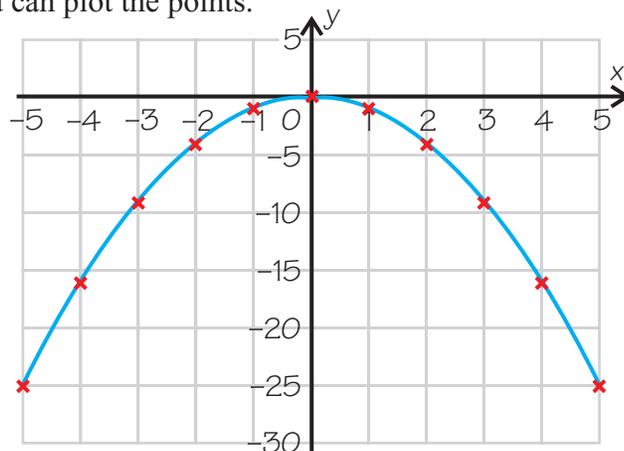
This time, the table of values is drawn horizontally, but it shows exactly the same information.

x	-5	-4	-3	-2	-1	0
x^2	25	16	9	4	1	0
$y = -x^2$	-25	-16	-9	-4	-1	0

You don't need a table for $x = 1, 2, 3, 4,$ and 5 , as it will contain the same values of y as above. However, if you find it easier to have all the values of x listed separately, then make a bigger table including the values below.

x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$y = -x^2$	0	-1	-4	-9	-16	-25

Now you can plot the points.



The graph of $y = -x^2$ is also a **parabola**.

But instead of being “u-shaped,” it’s “upside down u-shaped.”

Nearly everything from the last Lesson about $y = nx^2$ for positive values of n also applies for negative values of n . However, for negative values of n , the graphs are **below** the x -axis.

Example 2

Plot the graphs of the following equations for values of x between -4 and 4 .

a) $y = -2x^2$

b) $y = -3x^2$

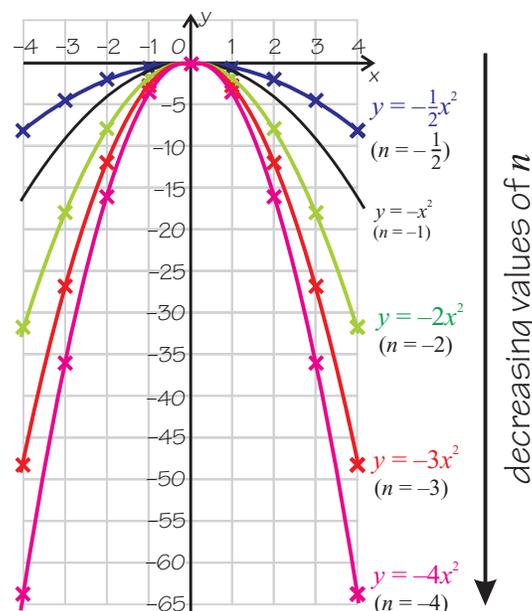
c) $y = -4x^2$

d) $y = -\frac{1}{2}x^2$

Solution

As always, make a table and plot the points.

x	$-2x^2$	$-3x^2$	$-4x^2$	$-\frac{1}{2}x^2$
0	0	0	0	0
1	-2	-3	-4	-0.5
2	-8	-12	-16	-2
3	-18	-27	-36	-4.5
4	-32	-48	-64	-8



Check it out:

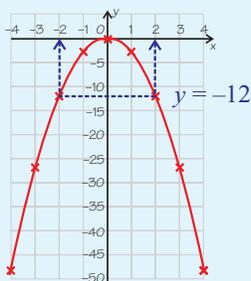
This time, only positive values of x have been included in the table. The values for negative x will be identical.

This time, since n is **negative**, all the graphs are “upside down u-shaped” parabolas. But all the graphs still have their **vertex** (the vertex is the **highest** point this time) at the same place, the **origin**.

Also, the **more negative** the value of n , the **steeper** and **narrower** the parabola will be.

Check it out:

To solve the equations in Exercises 9–14, first find the correct graph — you need the one with the matching equation. So for Exercise 9 ($-12 = -3x^2$) you need the $y = -3x^2$ graph. Then draw a line across the graph at the y -value you are given (in Exercise 9 it's -12) and read off the two x -values.



Guided Practice

On which of the graphs in Example 2 do the points in Exercises 1–8

lie? Choose from $y = -x^2$, $y = -2x^2$, $y = -3x^2$, and $y = -\frac{1}{2}x^2$.

- (1, -3)
- (-3, -4.5)
- (4, -32)
- (-5, -75)
- (-3, -27)
- (2, -2)
- (5, -75)
- (0, 0)

Solve the equations in Exercises 9–14 using the graphs in Example 2. There are two possible answers in each case.

- $-3x^2 = -12$
- $-\frac{1}{2}x^2 = -4.5$
- $-2x^2 = -32$
- $-\frac{1}{2}x^2 = -2$
- $-3x^2 = -27$
- $-3x^2 = -40$

Plot the graphs in Exercises 15–16 for x between -4 and 4 .

- $y = -5x^2$
- $y = -\frac{1}{3}x^2$

Graphs of $y = nx^2$ for $n > 0$ and $n < 0$ are Reflections

The graphs you've seen in this Lesson (of $y = nx^2$ for **negative** n) and those you saw in the previous Lesson (of $y = nx^2$ for **positive** n) are very closely related.

Example 3

Check it out:

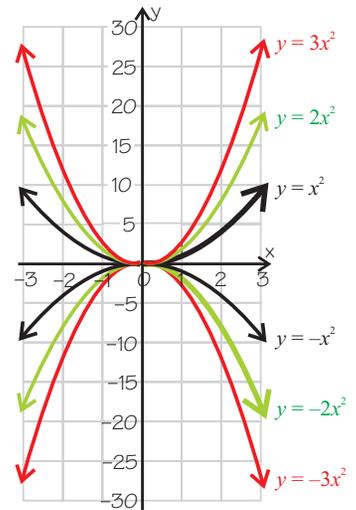
k is just "any number."
You could use n instead.

By plotting the graphs of the following equations on the same set of axes for x between -3 and 3 , describe the link between $y = kx^2$ and $y = -kx^2$.
 $y = x^2, y = -x^2, y = 2x^2, y = -2x^2, y = 3x^2, y = -3x^2$.

Solution

Plotting the graphs gives the diagram shown on the right.

For a given value of k , the graphs of $y = kx^2$ and $y = -kx^2$ are **reflections** of each other. One is a "**u-shaped**" graph **above the x -axis**, while the other is an "**upside down u-shaped**" graph **below the x -axis**.



Guided Practice

17. The point $(5, 100)$ lies on the graph of $y = 4x^2$. Without doing any calculations, state the y -coordinate of the point on the graph of $y = -4x^2$ with x -coordinate 5 .

18. Without plotting any points, describe what the graphs of the equations $y = 100x^2$ and $y = -100x^2$ would look like.

Independent Practice

1. Draw the graph of $y = -1.5x^2$ for values of x between -3 and 3 .

2. Without calculating any further y -values, draw the graph of $y = 1.5x^2$ for values of x between -3 and 3 .

3. What are the coordinates of the vertex of the graph of $y = -\frac{1}{4}x^2$?

4. If a circle has radius r , its area A is given by $A = \pi r^2$.

Describe what a graph of A against r would look like.

Check your answer by plotting points for $r = 1, 2, 3$, and 4 .

Now try these:

Lesson 5.4.2 additional
questions — p461

Round Up

Well, there were lots of pretty graphs to look at in this Lesson. The graphs of $y = nx^2$ are important in math, and you'll meet them again next year. But next Lesson, it's something similar... but different.

Lesson 5.4.3

Graphing $y = nx^3$

California Standards:

Algebra and Functions 3.1

Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

What it means for you:

You'll learn about how to plot graphs of equations with cubed variables in them, and how to use the graphs to solve equations.

Key words:

- parabola
- plot
- graph

Check it out:

The graph of $y = x^3$ has a different kind of symmetry to that of $y = x^2$ — rotational symmetry.

If you rotate the graph 180° about the origin, it will look exactly the same.

Check it out:

Graphs of $y = nx^2$ pass through **either** all positive values of y **or** all negative values of y , depending on the value of n .

Check it out:

Try to figure out the shape of the curve before you plot it. Think about the value of x^3 if x is negative. How is this different from the value of x^3 if x is positive?

For the last two Lessons, you've been drawing graphs of $y = nx^2$. Graphs of $y = nx^3$ are very *different*, but the *method* for actually drawing the graphs is exactly the same.

The Graph of $y = x^3$ is Not a Parabola

You can always draw a graph of an equation by **plotting points** in the normal way. First make a **table** of values, then **plot** the points.

Example 1

Draw the graph of $y = x^3$ for x between -4 and 4 .

Solution

First make a table of values:

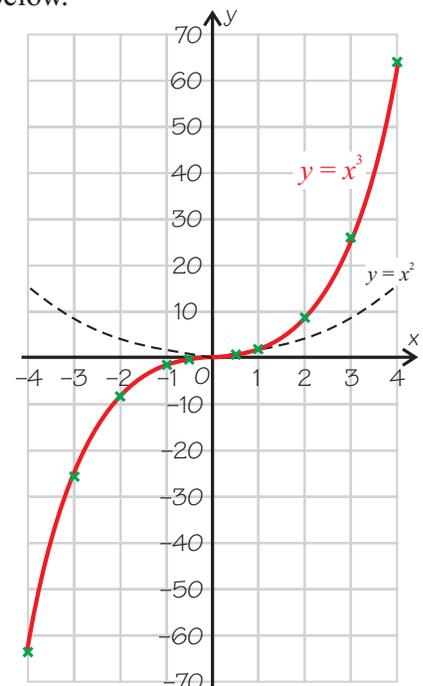
x	-4	-3	-2	-1	0	1	2	3	4
$y (= x^3)$	-64	-27	-8	-1	0	1	8	27	64

Then plot the points to get the graph below.

The graph of $y = x^3$ is completely different from the graph of $y = x^2$. It isn't "u-shaped" or "upside down u-shaped."

The graph still goes **steeply upward** as x gets more **positive**, but it goes **steeply downward** as x gets more **negative**.

The graph of $y = x^3$ passes through **all positive and negative** values of y .



The shape of the graph of $y = x^3$ is **not** a parabola — it is a curve that rises very quickly after $x = 1$, and falls very quickly below $x = -1$.

✓ Guided Practice

1. Draw the graph of $y = -x^3$ by plotting points with x -coordinates $-4, -3, -2, -1, -0.5, 0, 0.5, 1, 2, 3$, and 4 .

The Graph of $y = x^3$ Crosses the Graph of $y = x^2$

If you look really closely at the graphs of $y = x^3$ and $y = x^2$ you'll see that they **cross over** when $x = 1$.

Example 2

Draw the graph of $y = x^3$ for x values between 0 and 4.
Plot the points with x -values 0, 0.5, 1, 2, 3, and 4.

How does the curve of $y = x^3$ differ from that of $y = x^2$?

Solution

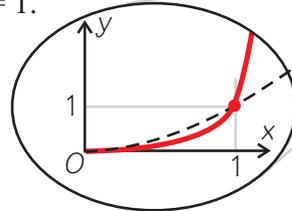
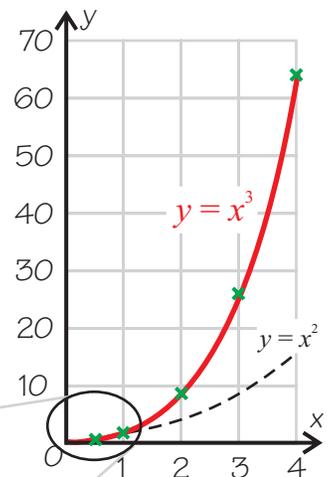
x	0	0.5	1	2	3	4
$y (= x^3)$	0	0.125	1	8	27	64

Plotting the points with the coordinates shown in the table gives you the graph on the right.

You can see that the graph of $y = x^3$ rises **much more steeply** as x increases than the graph of $y = x^2$ does.

But if you could **zoom in** really close near the origin, you'd see that the graph of $y = x^3$ is **below** the graph of $y = x^2$ between $x = 0$ and $x = 1$.

The two graphs **cross over** at the point $(1, 1)$, and cross again at $(0, 0)$.



Use Graphs of $y = x^3$ to Solve Equations

If you have an equation like $x^3 = 10$, you can **solve it** using a graph of $y = x^3$.

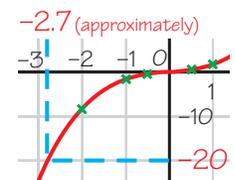
Example 3

Use the graph in Example 1 to solve the equation $x^3 = -20$.

Solution

First find -20 on the vertical axis.

Then find the corresponding value on the horizontal axis — this is the solution to the equation. So $x = -2.7$ (approximately).



Check it out:

There will only be one solution to equations like $x^3 = 10$ and $x^3 = -20$. This is because no two numbers can be cubed to give the same value.

✓ Guided Practice

Use the graph of $y = x^3$ to solve the equations in Exercises 2–7.

2. $x^3 = 64$

3. $x^3 = 1$

4. $x^3 = -1$

5. $x^3 = -27$

6. $x^3 = 30$

7. $x^3 = -50$

8. How many solutions are there to an equation of the form $x^3 = k$? Use the graph in Example 1 to justify your answer.

The Graph of $y = nx^3$ is Stretched or Squashed

The exact shape of the graph of $y = nx^3$ depends on the value of n .

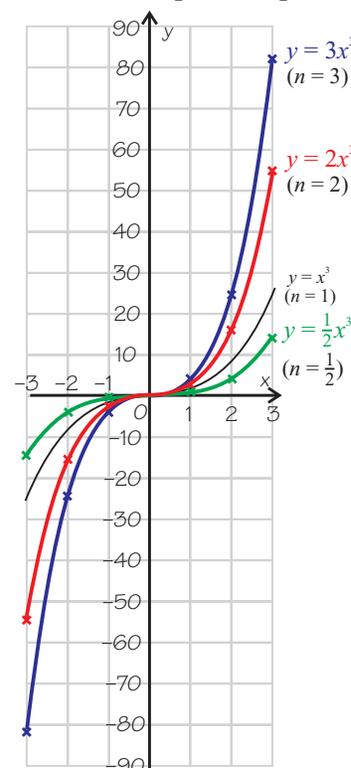
Example 4

Plot points to show how the graph of $y = nx^3$ changes as n takes the values 1, 2, 3, and $\frac{1}{2}$.

Solution

Using values of x between -3 and 3 should be enough for any patterns to emerge. So make a suitable table of values, and then plot the points.

x	$2x^3$	$3x^3$	$\frac{1}{2}x^3$
-3	-54	-81	-13.5
-2	-16	-24	-4
-1	-2	-3	-0.5
0	0	0	0
1	2	3	0.5
2	16	24	4
3	54	81	13.5



Don't forget:

The graph of $y = x^3$ ($n = 1$) is also plotted on these axes for comparison. The table of values is in Example 2 on the previous page.

As n increases, the curves get **steeper and steeper**.

However, the basic shape remains the same. All the curves have rotational symmetry about the origin.

✓ Guided Practice

Use the above graphs to solve the equations in Exercises 9–14.

9. $3x^3 = -60$

10. $2x^3 = 30$

11. $\frac{1}{2}x^3 = -10$

12. $\frac{1}{2}x^3 = 10$

13. $3x^3 = 40$

14. $2x^3 = -35$

15. How many solutions are there to an equation of the form $nx^3 = k$, where n and k are positive?

For $n < 0$, the Graph of $y = nx^3$ is Flipped Vertically

If n is **negative**, the graph of $y = nx^3$ is “upside down.”

Example 5

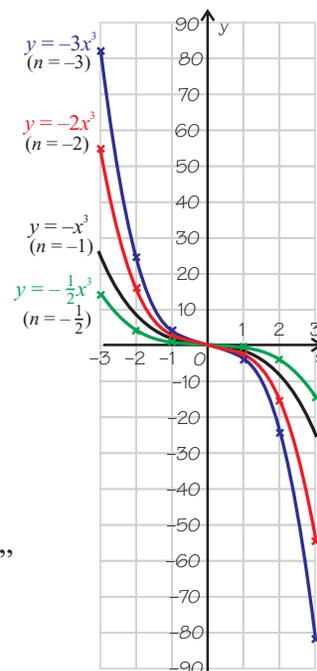
Plot points to show how the graph of $y = nx^3$ changes as n takes the values -1 , -2 , -3 , and $-\frac{1}{2}$.

Solution

The table of values looks very similar to the one in Example 4. The only difference is that all the numbers **switch sign** — so all the positive numbers become negative, and vice versa.

x	$-2x^3$	$-3x^3$	$-\frac{1}{2}x^3$
-3	54	81	13.5
-2	16	24	4
-1	2	3	0.5
0	0	0	0
1	-2	-3	-0.5
2	-16	-24	-4
3	-54	-81	-13.5

This change in sign of all the values means the curves all do a “vertical flip.”



Guided Practice

Use the above graphs to solve the equations in Exercises 16–18.

16. $-3x^3 = -50$

17. $-3x^3 = 50$

18. $-\frac{1}{2}x^3 = 10$

Independent Practice

Using a table of values, plot the graphs of the equations in Exercises 1–3 for values of x between -3 and 3 .

1. $y = 1.5x^3$

2. $y = -4x^3$

3. $y = -\frac{1}{3}x^3$

4. If the graph of $y = 8x^3$ goes through the point $(6, 1728)$, what are the coordinates of the point on the graph of $y = -8x^3$ with x -coordinate 6?

Now try these:

Lesson 5.4.3 additional questions — p461

Round Up

That’s the end of this Section, and with it, the end of this Chapter. It’s all useful information. You need to remember the general **shapes** of the graphs, and how they change when the n changes.

Chapter 5 Investigation

The Solar System

Some numbers are really, really large — like *distances in Space*. It'd take a long time to write such numbers out in full, and then they'd be hard to compare and work with. So *scientific notation* is used — it makes things much simpler.

The eight planets of the Solar System travel around the Sun in paths called **orbits**. The orbits are actually elliptical, but for this Investigation, you'll treat them as **circles**. The **average radius** of each planet's orbit is given below in miles.

Part 1:

The data on the right is presented in alphabetical order, as you might find in a reference book.

Make a more useful table by presenting the data so that:

- the planets are **in order of distance** from the Sun
- the distances are **given in scientific notation**.

Things to think about:

- Is it easier to convert the numbers into scientific notation **before** ordering them, or to order the numbers and **then** convert them into scientific notation?

Planet	Approximate mean distance from Sun (in miles)
Earth	92,960,000
Jupiter	483,800,000
Mars	141,700,000
Mercury	35,980,000
Neptune	2,793,000,000
Saturn	886,700,000
Uranus	1,785,000,000
Venus	67,240,000

Part 2:

Using the scientific notation figure, **compute the approximate area** inside the Earth's orbit and present it in scientific notation.

(Remember that the area of a circle is given by $A = \pi r^2$, where A is the area and r is the radius. Use $\pi = 3.14$.)

Extensions

- 1) The mean distance from Earth to the Sun is called an astronomical unit (AU) and is about 92.96 million miles. Add a column to your table to show all the distances converted into AUs.
- 2) Make a scale drawing of the Solar System using the scale of 1 cm = 1 AU. Place the Sun at one edge of the paper. Use dots to represent the planets.

Open-ended Extensions

- 1) Research the diameters of the planets. Write the diameters in miles and then rewrite them in scientific notation.
- 2) Make scale drawings showing the size of the planets. What scale did you use?
- 3) If you drew the diameter of Mercury as 1 mm, how large a sheet of paper would you need to accurately draw the entire Solar System with planet sizes and distances to the Sun all to the same scale?

Round Up

When you're working with very **big numbers**, it's usually easier if you put them in **scientific notation** first. This way, you can tell which is the biggest by comparing just the **exponents**, rather than counting all the digits each time. It's a similar situation with very small numbers.

Chapter 6

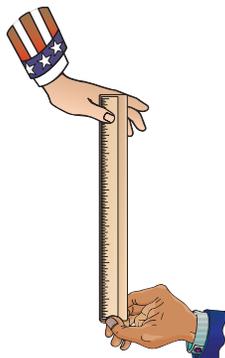
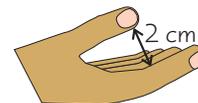
The Basics of Statistics

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Section 6.1 introduction — an exploration into: **Reaction Rates**

In this Exploration, you'll test your *reaction time* by catching a ruler dropped by another student. You'll collect data and calculate its *mean, mode, median, and range*. From this analysis you'll be able to *draw conclusions* about your typical reaction time.

The experiment requires one person to **drop** the ruler and another person to **catch it**. The catcher is seated with his or her arm resting on a table. The catcher's hand is off the table, with the distance between his or her thumb and pointer finger at **2 cm**.



The dropper holds a ruler so that **0 cm** is level with the catcher's finger and thumb. The dropper then **releases the ruler without warning**, and the catcher tries to catch it **as soon as possible**.

The dropper **records** the position of the catcher's pointer finger and thumb on the ruler.

Exercises

1. Repeat the experiment 12 times and record the results in a copy of the table below.

Trial	Distance (cm)	Trial	Distance (cm)
1		7	
2		8	
3		9	
4		10	
5		11	
6		12	

Switch jobs after the completion of the experiment. The catcher becomes the dropper and the dropper becomes the catcher.

2. What was the range for your data?
3. What is the median distance for your data?
4. What is the modal distance for your data?
5. What is the mean distance for your data?
6. Explain which value you think represents your typical reaction time best.
7. Were there any trials that did not seem to fit in with the rest of the results? If so, suggest possible reasons why.

Round Up

It's always a good idea to *repeat* experiments lots of times, then find the *average* of all the trials. This means your result is likely to be *more accurate*. You've found *three* types of average for your data from this experiment — one will often represent your data *better* than the others.

Lesson 6.1.1

Section 6.1 Median and Range

California Standards:

Statistics, Data Analysis,
and Probability 1.3

Understand the meaning of, and be able to compute, the **minimum**, the lower quartile, **the median**, the upper quartile, and **the maximum of a data set**.

What it means for you:

You'll learn the meaning of the terms minimum, median, maximum, and range, and how to find these values from data sets.

Key words:

- median
- minimum
- maximum
- range

Don't forget:

The average of a set of data is the sum of all the numbers divided by the number of data points. So when you work out the average of two numbers, you add them together, and then divide the result by two.

In grade 6, you learned about three different typical values of data sets — the mode, mean, and median. In this Lesson, you'll review the **median** in preparation for drawing **box-and-whisker plots** in the next Lesson.

The Median is the Value in the Middle of a Data Set

If you arrange a data set in order, the **middle value** is the **median**. It gives you an idea of a “typical” value for the data set.

Here's a reminder of the process you go through to find the median:

1. **Order** the data from smallest to largest.
2. **Count** the number of values in the data set.
3. If the number of values is **odd**, take the **middle value** as the median.
4. If the number of values is **even**, take the **average of the two middle values** as the median.

Example 1

Find the median of each data set below.

1. {4, 6, 8, 8, 12, 15, 19}
2. {12, 6, 4, 8, 15, 15, 8, 15}

Solution

1. The data is **already ordered** — and there are 7 values. This is an odd number, so the median is the **middle value**, which is **8**.

4, 6, 8, **8**, 12, 15, 19

2. The data **isn't ordered** — so you have to first order the data. There are 8 values in the data set — an even number. So the median is the **average** of the two middle values, which are **8 and 12**.

4, 6, 8, **8, 12**, 15, 15, 15

The average of these values is: $(8 + 12) \div 2 = 20 \div 2 = 10$.
So **the median is 10**.

Since the median is the middle of a data set, you know that **half** of the values in the data set are **below** the median, and **half** are **above** it.

✓ Guided Practice

1. A hospital measures the length of newborn babies on a daily basis. On one day the results in inches were:

19, 22, 20, 21, 22, 20, 24, 20, 17, 21.

What was the median length?

The Range Tells You About the Spread of the Data

The range of a data set tells you about the **spread** of the data. It tells you whether the data is **close together**, or **spaced out**. To calculate the range you first need to find the **minimum** and **maximum** values:

- The **smallest** value in a set is called the **minimum**
- The **largest** value in a set is called the **maximum**
- The **range** is the **difference between** the maximum and the minimum.

Don't forget:

You find the difference between two numbers by taking the smaller number from the larger number.

Check it out:

It's often safer to put the data into order first, rather than try to find the minimum and maximum values in a long jumbled set of data.

Example 2

Belinda had the following test scores on her first five tests:

92, 88, 96, 83, 91.

What is the range of her scores?

Solution

The minimum value is **83**, the maximum value is **96**.

The range is the **difference between** the maximum and the minimum.

The range is $96 - 83 = 13$.

Use Medians and Ranges to Compare Data Sets

Looking at the **medians** and the **ranges** can give you useful information about data sets.

Example 3

Jewelry Store A sells watches with a median price of **\$99** and a range of **\$60**.

Jewelry Store B sells watches with a median price of **\$99** and a range of **\$820**.

Describe what these statistics tell about the prices of the watches in each jewelry store.

Solution

Both stores have the **same median** price. But Store A has a **smaller range**, so the prices are all clustered more closely together.

The **minimum price** a watch could be in Store A is $\$99 - \$60 = \$39$, and the **maximum price** a watch could be in Store A is $\$99 + \$60 = \$159$.

Store B's price range is much **larger**, so the price of **at least one** of the watches it sells lies much further from the median than any of the watches in Store A. The **maximum price** a watch could be in Store B is $\$99 + \$820 = \$919$. But some of the watches may be very cheap — cheaper than the cheapest watch in Store A.

Check it out:

A possible data set that has a range of \$60, a median of \$99, and a minimum price of \$39 is: {\$39, \$99, \$99}.

A possible data set that has the same range and median, and a maximum price of \$159 is: {\$99, \$99, \$159}.

✓ Guided Practice

Don't forget:

If the values in the data set have units, you need to include units for the range and median.

A gardener is trying to grow large zucchinis. She has two sets of zucchinis that she treats with different fertilizers.

The lengths of the zucchinis in each set are shown below.

Set 1: {11 cm, 15 cm, 16 cm, 19 cm, 23 cm}

Set 2: {24 cm, 13 cm, 61 cm, 55 cm, 41 cm, 22 cm, 55 cm}

2. Find the range of the lengths in each set.
3. Find the median length for each set.
4. If you were only told the median length and the range of lengths for Set 1, what could you say about the minimum and maximum values of the set?

✓ Independent Practice

Find the median of the data sets in Exercises 1–4.

1. {11, 15, 16, 19, 23}

2. {8, 8, 9, 13, 15, 15}

3. {28, 11, 43, 21, 41, 53, 55}

4. {11, 13, 9, 12, 12, 19, 18, 17, 16, 5}

5. Frank had the following quiz scores: 18, 16, 15, 20, and 16. What was his median score?

6. Alyssa had the following number of rebounds over her last 8 games: 4, 8, 9, 3, 11, 5, 12, 5. What was the median number of rebounds?

Find the minimum, maximum, and range of the data sets in Exercises 7–8.

7. {8, 8, 9, 13, 15, 15}

8. {11, 13, 9, 12, 12, 19, 18, 17, 16, 5}

9. Store A sells fine pens with a median price of \$29 and a range of \$20. Store B sells fine pens with a median price of \$40 and a range of \$30. What could be the minimum and maximum possible prices of each store's pens?

10. Furniture Store A sells chairs with a median price of \$110 and a range of \$40. What is the lowest possible price for a chair in Furniture Store A?

Find the median and range of the sets of data in Exercises 11–15.

11. {86, 78, 81, 80, 80, 85, 72, 90}

12. {34, 35, 31, 32, 30, 35}

13. {101, 104, 107, 102, 98, 100}

14. {98, 97, 97, 97, 96, 95, 98, 96, 95, 98, 98}

15. {61, 60, 63, 65, 61, 62}

Don't forget:

The median shows you a "typical value" for a data set. The range shows you how spread out the data is.

Now try these:

Lesson 6.1.1 additional questions — p462

Round Up

The median and range are useful for *comparing two sets of data*. They can give you an idea of which set tends to have *higher values* and which has the *most spread-out values*. In a few Lessons, you'll see how *box-and-whisker plots* show this too, but in a more visual way.

Lesson 6.1.2

Box-and-Whisker Plots

California Standards:

Statistics, Data Analysis, and Probability 1.1

Know various forms of display for data sets, including a stem-and-leaf plot or **box-and-whisker plot**; use the forms to display a single set of data or to compare two sets of data.

Statistics, Data Analysis, and Probability 1.3

Understand the meaning of, and be able to compute, the minimum, the **lower quartile**, the median, the **upper quartile**, and the maximum of a data set.

What it means for you:

You'll learn how to find lower and upper quartiles, and how to make box-and-whisker plots of data sets.

Key words:

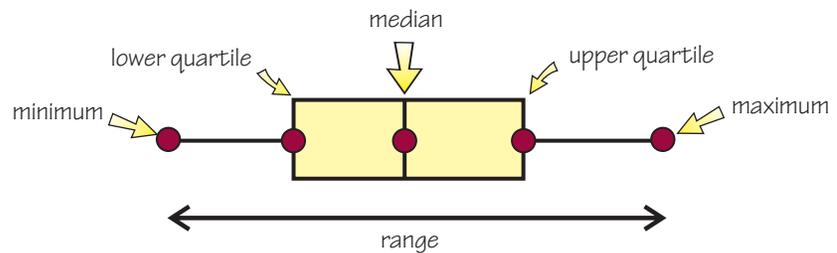
- box-and-whisker plot
- upper quartile
- lower quartile
- number line

Box-and-whisker plots are useful because you can use them to directly compare the medians and ranges of data sets. To plot them, you need five key values for the data set — the minimum, maximum, median, and two values you haven't met before — the lower and upper quartiles.

Box-and-Whisker Plots Show Five Values

Box-and-whisker plots are a way of displaying data sets. They have a central box, and two “whiskers” on either side.

There are **five important values** that are shown in a box-and-whisker plot:



Quartiles Split the Data into Four Equal Parts

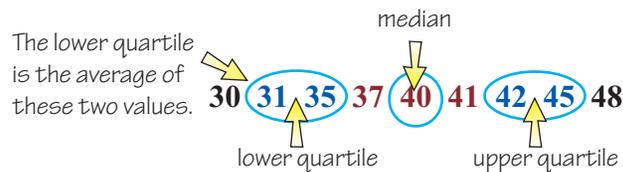
There are **three quartiles**. One of them is equal to the median, which splits the data into **two halves**. The other two quartiles are the **lower quartile** and the **upper quartile**:

- The **lower quartile** is the median of the **first half** of the data.
- The **upper quartile** is the median of the **second half** of the data.

To find the lower and upper quartiles:

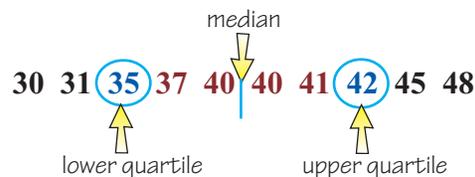
1. First **order the data** and find the **position of the median** of the full set.
2. Next find the **median value of each half of the data**:

If the total number of data points is **odd** —



When the number of points is odd, you **don't include the median value** when you work out the lower and upper quartiles...

If the total number of data points is **even** —



...but when you have an even number of points, you **do include the two middle values**.

Example 1

Find the lower and upper quartiles of the following data set:

20, 21, 21, 24, 25, 25, 27, 29, 30, 31, 33, 37

Solution

First, find the position of the median of the full data set.

20, 21, 21, 24, 25, 25, 27, 29, 30, 31, 33, 37

There's an even number of data points so the median is the average of the two middle values.

There are **six values on each side** of the median, so:

20, 21, 21, 24, 25, 25, 27, 29, 30, 31, 33, 37

Lower quartile = average of 3rd and 4th values = $(21 + 24) \div 2 = 22.5$

Upper quartile = average of 9th and 10th values = $(30 + 31) \div 2 = 30.5$

✓ Guided Practice

Find the lower and upper quartiles of the following data sets:

1. Test scores for class A: 56, 57, 57, 59, 62, 64, 64, 68, 69, 70, 72

2. Test scores for class B: 45, 52, 53, 53, 55, 57, 61, 61, 65, 68

Make a Box-and-Whisker Plot on a Number Line

You draw a box-and-whisker plot on a number line — this gives you a scale to line the numbers up on.

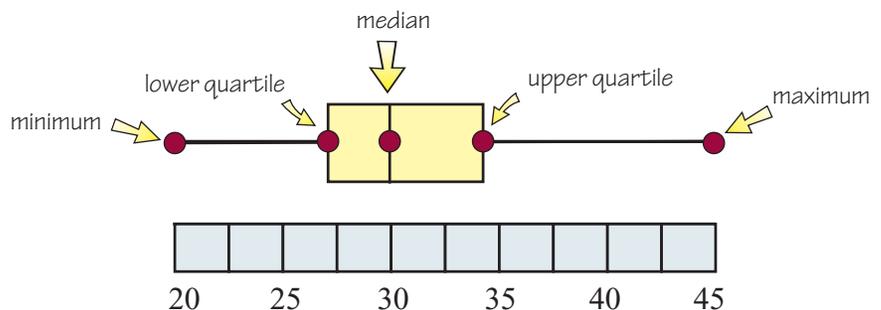
Follow these steps for making a box-and-whisker plot:

1. Find the **five important values for the data set** — the minimum, lower quartile, median, upper quartile, and maximum.
2. Draw out a **number line** that goes from the minimum to the maximum of the data.
3. **Plot** the five values on the number line, and draw a **box** from the lower quartile to the upper quartile. Mark the **median** across the box.
4. Draw **whiskers** to the minimum and maximum values.

Check it out:

The box represents half of the data. A quarter of the data is above the median line, and a quarter of the data is below the median line.

The **larger** the space from the quartile to the median, the **more spaced out** the data in that quarter is. If the quartile is **very close** to the median, it means that the data is very **concentrated**.



Example 2

Draw a box-and-whisker plot to illustrate the following data:

{45, 46, 47, 47, 49, 51, 51, 53, 55, 57, 57}

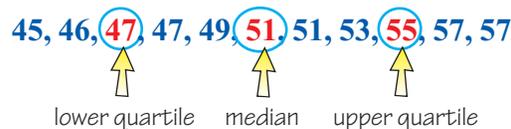
Solution

First find the five key values for the set:

The minimum and maximum are **45** and **57**.

There's an odd number of data points, so the median value is **51**.

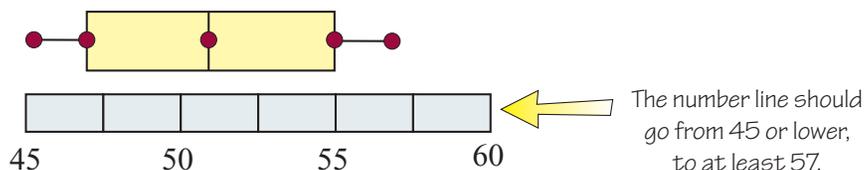
Now find the lower and upper quartiles:



Lower quartile = **47**

Upper quartile = **55**

Plot the data points and make the box-and-whisker plot:



Check it out:

This data set has an odd number of values. So the median is the middle value. You don't include this value when you're finding the lower and upper quartiles.

Guided Practice

3. Draw a box-and-whisker plot to illustrate the following data:

98, 76, 79, 85, 85, 81, 78, 94, 89

Independent Practice

1. Mrs. Walker wants to compare the test results of her period 1 and period 4 science classes. Find the maximum, minimum, median, and lower and upper quartiles of the sets of data, and display the data sets on box-and-whisker plots.

Period 1: 56, 78, 10, 43, 32, 20, 67, 65, 58, 72, 74, 67, 68, 55, 59, 49

Period 4: 75, 64, 65, 68, 62, 52, 42, 38, 53, 64, 64, 72, 73, 59, 59, 63

Now try these:

Lesson 6.1.2 additional questions — p462

Round Up

There's a lot of information in this Lesson. You need to remember the *five key values* for drawing box-and-whisker plots — the *minimum, lower quartile, median, upper quartile, and maximum*. Remember that the box goes from the *lower quartile to the upper quartile*, and there are *two whiskers* — one from the minimum to the lower quartile, and another from the upper quartile to the maximum. Next Lesson you'll see how box-and-whisker plots can be used to *analyze and compare sets of data*.

Lesson 6.1.3

More on Box-and-Whisker Plots

California Standards:

Statistics, Data Analysis, and Probability 1.1

Know various forms of display for data sets, including a stem-and-leaf plot or **box-and-whisker plot**; use the forms to display a single set of data or to **compare two sets of data**.

What it means for you:

You'll learn how you can compare data sets using box-and-whisker plots.

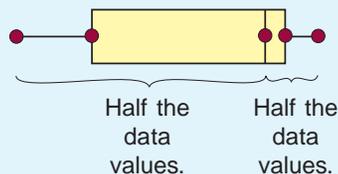
Key words:

- box
- whisker
- spread
- concentrated

Check it out:

Half the data values are between the minimum value and the median.

So:



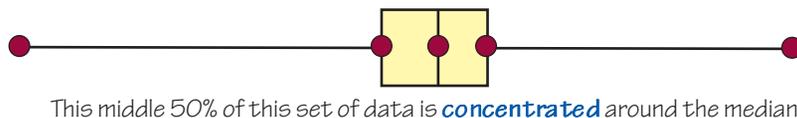
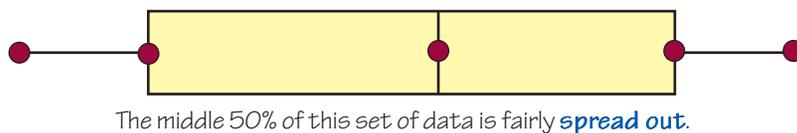
Last Lesson you learned how to make a **box-and-whisker plot** to display a set of data. In this Lesson you'll use the features of box-and-whisker plots to understand real-life data sets. You'll also see how box-and-whisker plots can be used to **compare two data sets**.

The Box Shows the Middle 50% of the Data Values

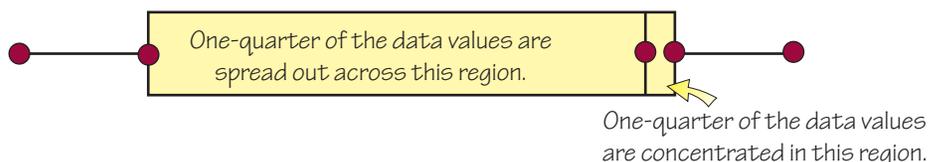
It's useful to be able to **compare** two or more data sets. Drawing two box-and-whisker plots on the **same number line** is a good way of doing this.

Remember these important points:

- The box represents the **middle 50%** of the data.
- The box length shows how **spread out** the middle 50% of the data is.

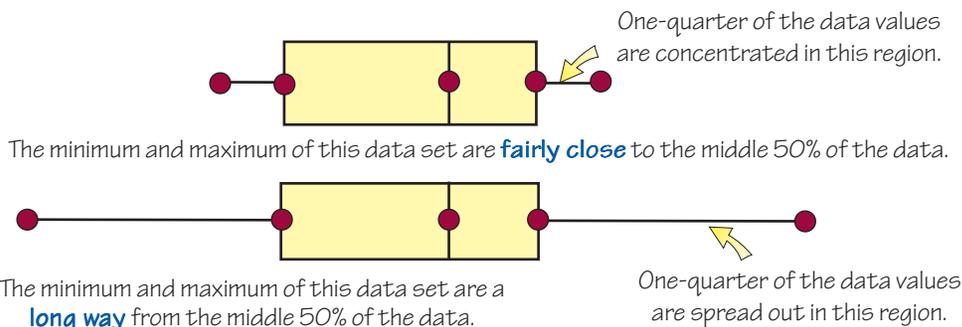


- If the median is at the **upper end of the box**, you know that one-quarter of the data values are **concentrated just above the median** value, and that the data is **spread out more below the median**.



The Whiskers Show the Full Range of the Data Values

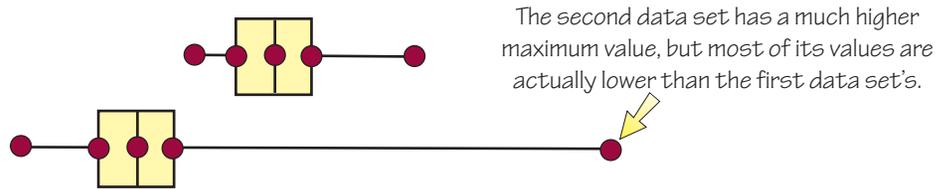
The lengths of the whiskers tell you how **far** the very lowest and very highest points are from the middle 50% of the data.



There are Different Ways to Compare Data Sets

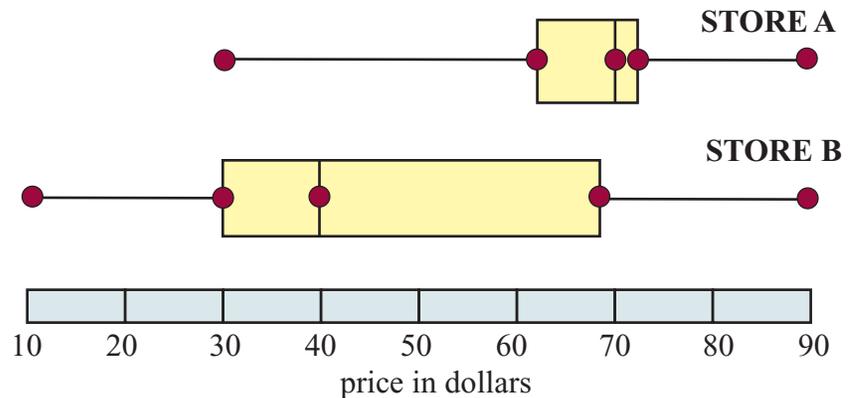
Comparing two sets of data can be quite complicated. There are many differences that you may need to think about.

For instance, if you **only** looked at the **minimum and maximum values** of data sets, you wouldn't get a complete picture. One set of data might have one unusually high value, with the rest of the data really low.



Example 1

These box-and-whisker plots show the prices of stock in two stores. What do they tell you about the price differences in the two stores?



Solution

Looking at the minimum and maximum values:

Store B has the **lowest priced item** (\$10). Store A's lowest price is much higher (\$30). Both stores sell their **most expensive item** for the same price (\$90). So Store B has a **greater range** of prices.

Looking at the medians:

Store A's stock has a median price of \$70, whereas Store B's stock has a median price of \$40. So Store B's items are **typically less expensive** than Store A's.

Another way of looking at this is that **half** of Store A's stock is **under \$70**, but **half** of Store B's stock is **under \$40**.

Looking at the quartiles:

The **middle 50%** of the prices in Store B are much more **spread out** than they are in Store A. They go from \$30 to nearly \$70.

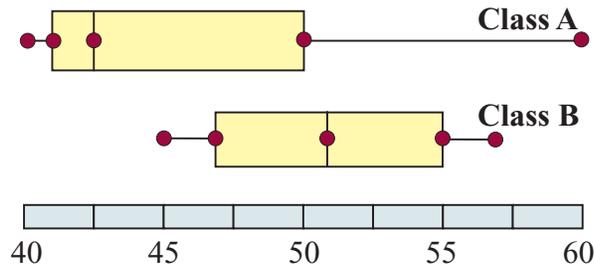
The middle 50% of the prices in Store A are **concentrated more tightly** around the median value of \$70.

Check it out:

The difference between the lower and upper quartile is called the "interquartile range."

Example 2

The box-and-whisker plots below show the test scores in two classes. Compare the two sets of scores.



Solution

- Class A's test scores had a much **larger range** than Class B's. Both the highest and lowest scores overall were found in Class A.
- Class B's scores were **generally higher** than Class A's. The **median** score for Class B was more than 50, but for Class A it was about 43.
- **More than half** the students in Class B scored **more than 50**, whereas in Class A only **one-quarter** scored more than 50.

Check it out:

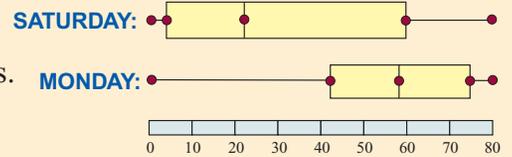
If one section of the box is longer than the other, it doesn't mean that they contain different amounts of data. It just means that the data in the larger section is more spread out.

Don't forget:

Always label your box-and-whisker plots so that you know which set of data belongs to which plot.

Guided Practice

1. These box-and-whisker plots show the ages of people using a public pool on two different days. Compare the differences in ages of the pool-users, and suggest why these differences are seen.



Independent Practice

1. The data below shows the ages of the people who subscribe to two different magazines. Draw a box-and-whisker plot of each set, and use them to compare the ages of the readers of each magazine.

Magazine 1: 20, 21, 32, 19, 47, 65, 34, 21, 33, 52, 24, 20, 19, 31, 23, 22

Magazine 2: 45, 67, 20, 72, 54, 37, 51, 54, 50, 52, 44, 39, 85, 29, 57, 60

2. A group of students took a test in March, then followed a special program for two months before retaking the test in June. Their scores are shown below.

Compare the sets of results by drawing box-and-whisker plots.

March test results: 20, 23, 28, 31, 24, 24, 25, 27, 25, 25, 23, 22, 25

June test results: 33, 26, 35, 28, 21, 30, 31, 35, 23, 26, 33, 29, 26

Now try these:

Lesson 6.1.3 additional questions — p462

Round Up

There has been a lot covered in the last two Lessons. Make sure you understand everything you've covered — like *how to find quartiles*, and *how to compare data shown on box-and-whisker plots*.

Lesson 6.1.4

Stem-and-Leaf Plots

California Standards:
Statistics, Data Analysis,
and Probability 1.1

Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

What it means for you:

You'll learn how to make a stem-and-leaf plot to display and compare data sets.

Key words:

- stem
- leaf
- tens digit
- ones digit

Check it out:

You don't usually draw the outlines of the stem and leaves — but these have been included here to show you why they're called stem-and-leaf plots.

Check it out:

The stem doesn't always contain tens digits — it depends on the data set. For example, the plot below represents this data set:

1.9, 2.4, 2.7, 3.6

1		9
2		4 7
3		6

Key: 1 | 9 represents 1.9

Other examples of what the stem may contain are hundreds or thousands.

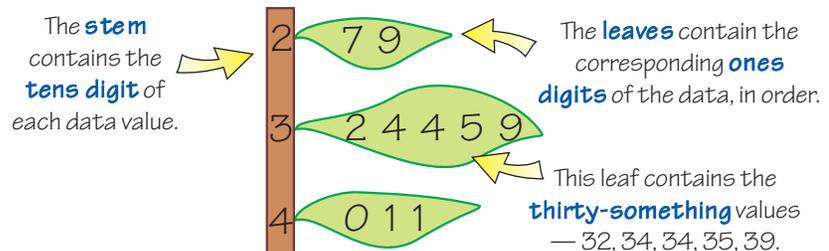
Stem-and-leaf plots are a way of displaying data sets so that you can see their main features more easily. Like box-and-whisker plots, stem-and-leaf plots are also useful for comparing two data sets.

A Stem-and-Leaf Plot Has a Stem and a Leaf

A **stem-and-leaf plot** is a way of displaying data so that you can see clearly **how widely spread** it is and which values are **more common**.

The diagram below displays the data set:

{27, 29, 32, 34, 34, 35, 39, 40, 41, 41}.



Key:

represents 32

You always need to include a **key** — this explains how the stem-and-leaf plot should be read.

Example 1

Make a stem-and-leaf plot to display the following data:

34, 36, 36, 37, 41, 45, 46, 49, 50, 50

Solution

The data contains values in the 30s, 40s, and 50s.

So give the stem **3 rows** — 3 tens, 4 tens, and 5 tens.

Now fill in the leaves. For example, the row with 3 on the stem has a leaf that contains 4, 6, 6, and 7 to represent 34, 36, 36, and 37.

3		4 6 6 7
4		1 5 6 9
5		0 0

Key: 4 | 5 represents 45

✓ Guided Practice

1. Draw a stem-and-leaf plot of the following set of data.

98, 76, 79, 85, 85, 81, 78, 94, 89

Find the Median and Range from Stem-and-Leaf Plots

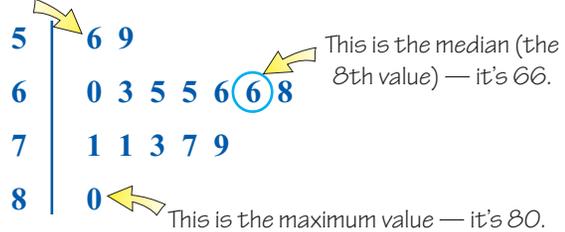
You can find the **median, minimum, maximum, and range** of the data from a stem-and-leaf plot. The example below shows you how.

Example 2

Use the stem-and-leaf plot below to find the:

- median of the data,
- minimum, maximum, and range of the data.

This is the minimum value — it's 56.



Key: 5 | 6 represents 56

Solution

- a)** You find the median in exactly the same way as usual, except the data points are now spread out over a number of rows.

First count the number of data points, and decide which is the **middle data point**. There are 15 points on this stem-and-leaf plot — this is **odd**, so the median is the middle value.

The middle value is the 8th value, which is **66**.

- b)** The minimum value is the first number in the top row.
So the minimum value = **56**.

The maximum value is the last number in the bottom row.
So the maximum value = **80**.

The range is the difference between these numbers.
So the range = $80 - 56 = 24$.

Check it out:

From the shape of the stem-and-leaf plot you can see that most values are in the 60s and 70s, with just a few in the 50s and 80s.

Guided Practice

- 2.** The stem-and-leaf plot below shows the number of children who attended an after-school program each week. Find the median number of children and the range.



- 3.** Find the median and the range of the data shown on the stem-and-leaf plot you made in Guided Practice Exercise 1.

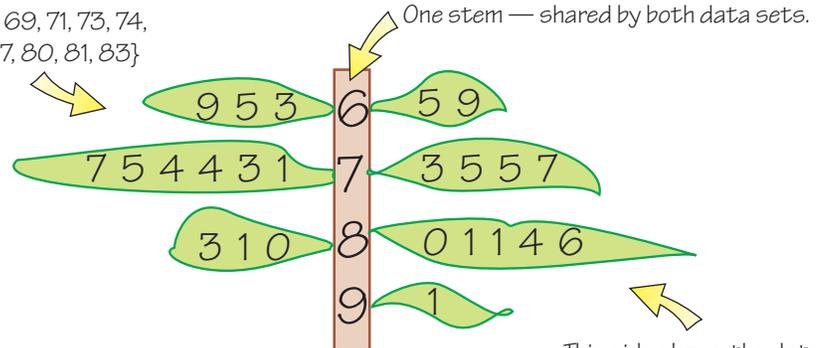
Use Stem-and-Leaf Plots to Display Two Data Sets

A stem-and-leaf plot with leaves on **both sides of the stem** can be used to compare data sets. This is called a **back-to-back** stem-and-leaf plot.

The leaves share the **same stem**, with one data set displayed on one side, and the other on the other side.

This side shows the data set:

{63, 65, 69, 71, 73, 74,
74, 75, 77, 80, 81, 83}



This side shows the data set:
{65, 69, 73, 75, 75, 77,
80, 81, 81, 84, 86, 91}

Check it out:

The larger values are further away from the stem on each side of a back-to-back stem-and-leaf plot.

Don't forget:

A good first step when drawing stem-and-leaf plots is to order your data from smallest to largest. This way, you are less likely to make mistakes like leaving numbers out.

Key:



represents 63 from the first data set, and 65 from the second.

✓ Guided Practice

4. Draw a back-to-back stem-and-leaf plot of the following sets of data.

Test scores for class A: {56, 57, 57, 59, 62, 64, 64, 68, 69, 70, 72}

Test scores for class B: {45, 52, 53, 53, 55, 57, 61, 61, 65, 68, 68}

Compare Data Sets Shown on Stem-and-Leaf Plots

To compare the data sets, look at the **shape** of the back-to-back stem-and-leaf plot.

Sometimes there will be a few leaves that are much **longer** than the others — this means the data is more **concentrated** around a certain value.

Other sets of data will have **lots of shorter leaves** — which means the data is more **spread out**.

You can also see on the diagram which data set contains the **highest number**, and which data set contains the **lowest number**.

For example:

The data on this side is clustered tightly around the high-30s and 40s.

9 8 8
7 7 5 5 5 3 2

1	9 9
2	0 2 8
3	1 5 7
4	0 8

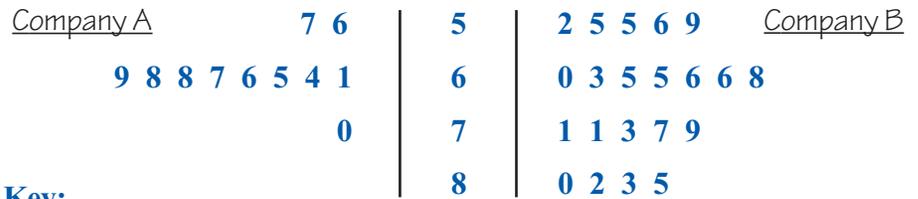
The data on this side is fairly evenly spread between 19 and 48.

Key:

8 | 3 | 1 represents 38 from the first data set, and 31 from the second.

Example 3

Two holiday companies each organized a trip to visit the pyramids in Egypt for people aged over 50. The ages of the passengers on each trip are shown on the stem-and-leaf plot below. Compare the ages of the people who traveled with each company.



Key:

1 | 6 | 0 represents 61 from Company A, and 60 from Company B.

Solution

The ages of the people who traveled with **Company B** were fairly **evenly spread between 52 and 85**. (The leaves are all of a similar length.)

The people who traveled with **Company A** were typically **younger**, and were **closer together** in age — most of them were in their 60s. (This is shown by a very long leaf representing the people in their 60s.)

Check it out:

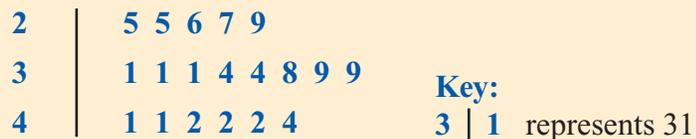
The median age for each trip was 66. The age range on Company A's trip was 14. This was much smaller than Company B's, which was 33. This supports what the shape of the stem-and-leaf plot shows.

Guided Practice

5. Look at the back-to-back stem-and-leaf plot of the test scores that you drew in Guided Practice Exercise 4. Compare the two sets of data.

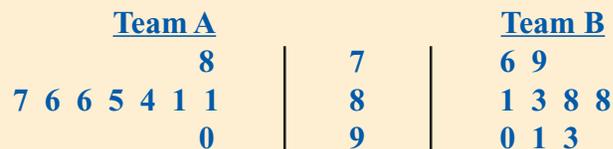
Independent Practice

1. List all the individual data values that are contained in the stem-and-leaf plot below.



2. In the stem-and-leaf plot in Exercise 1, how many of the values lie between 30 and 35?

3. Below is a back-to-back stem-and-leaf plot of golf scores of individuals on two teams. (In golf, the lower the score the better.) Which team has players of a more similar standard? Explain your answer.



Key: 8 | 7 | 6 represents 78 from Team A, and 76 from Team B.

Now try these:

Lesson 6.1.4 additional questions — p463

Round Up

Stem-and-leaf plots are great for showing the trends in data sets. They show data in a much more visual way than lists of numbers do. And displaying two data sets back to back makes the main differences nice and clear.

Lesson 6.1.5

Preparing Data to Be Analyzed

California Standards:

Statistics, Data Analysis,
and Probability 1.1

Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

What it means for you:

You'll practice preparing a real-life set of data ready to be analyzed.

Key words:

- analyze

Check it out:

From looking at the raw data in the table, you can see that some people's cholesterol did decrease, but other people's stayed the same, while some people's cholesterol actually increased. It's hard to tell what the general trend is though.

For data to be *meaningful*, you usually need to collect quite a lot of it. For instance, if you want to find out the most popular song, then just asking your three closest friends *won't* give you a very reliable result.

In this Lesson you'll *display* some larger sets of real-life data, using the methods that you've learned in this Section so far. Then next Lesson you'll think about what the data actually shows.

Real-Life Data is Used to Answer Questions

Real-life data can be collected to try to **answer a question**.

For instance, a company has produced a new medicine designed to **lower cholesterol**. They want to find out if it works, so their question is:

“Does the medicine lower cholesterol when taken daily for two months?”

Before entering into a large study, the company gives the medicine to **25 volunteers**. They record each person's **cholesterol level** before they take the drug, and again after they have been taking it for two months.

The data is shown in the tables below:

Person	Before (units)	After (units)
1	190	185
2	210	210
3	185	185
4	190	170
5	215	210
6	205	205
7	210	165
8	220	205
9	190	185
10	200	205
11	195	200
12	200	180
13	215	210
14	215	185
15	205	165

Person	Before (units)	After (units)
16	215	175
17	220	200
18	190	190
19	195	185
20	210	195
21	190	180
22	215	210
23	210	185
24	200	180
25	185	195

They hope to use this data to answer their question.

Example 1

The company wants to make a box-and-whisker plot of the cholesterol levels **before** taking the medicine, and a box-and-whisker plot of the cholesterol levels **after** taking the medicine.

Find the minimum and maximum values, the median, and the lower and upper quartiles for each data set.

Solution

The first step is to put the data in order:

“Before” data: 185, 185, 190, 190, 190, 190, 190, 195, 195, 200, 200, 200, 205, 205, 210, 210, 210, 210, 215, 215, 215, 215, 215, 220, 220

“After” data: 165, 165, 170, 175, 180, 180, 180, 185, 185, 185, 185, 185, 185, 190, 195, 195, 200, 200, 205, 205, 205, 210, 210, 210, 210.

The minimum and maximum values:

- Minimum of **“before”** data is **185**, maximum is **220**
- Minimum of **“after”** data is **165**, maximum is **210**

The median:

There are 25 pieces of data in each set, which is odd, so the median is the middle value. This is the 13th data point of each set.

- Median of **“before”** data is **205**
- Median of **“after”** data is **185**

The upper and lower quartiles:

Each half has 12 data points, which is even, so the upper and lower quartiles are the average of the 6th and 7th data points in each half.

- Lower quartile of **“before”** data = $(190 + 190) \div 2 = \mathbf{190}$
- Lower quartile of **“after”** data = $(180 + 180) \div 2 = \mathbf{180}$
- Upper quartile of **“before”** data = $(215 + 215) \div 2 = \mathbf{215}$
- Upper quartile of **“after”** data = $(205 + 205) \div 2 = \mathbf{205}$

Check it out:

It's easy to leave out a value when putting them in order. Check you still have 25 values in each ordered set.

Don't forget:

Finding the minimum, maximum, median, lower quartile, and upper quartile was covered in the last few Lessons. Look back if you can't remember how to find them.

Check it out:

There's an odd number in the full data set — so you don't include the median value when working out the quartiles.

✓ Guided Practice

1. Marissa is growing sunflowers in her yard. She treats half of the sunflowers with water and a new plant food, and the other half with just water. Marissa wants to answer this question:

“Does the new plant food make sunflowers grow taller?”

The data below shows the heights of the sunflowers at the end of her experiment. Find the values needed to draw a box-and-whisker plot for each set of data.

With food (in cm): 230, 210, 180, 196, 204, 202, 185, 180, 120, 156, 178, 195, 205, 250, 236, 226, 210, 207, 197, 180

Without food (in cm): 205, 230, 210, 196, 186, 198, 204, 176, 134, 156, 202, 185, 182, 178, 208, 165, 174, 182, 110, 162

Display Your Data Sets to Show the Trends Clearly

Once you've prepared your data, you can **display it**.

Example 2

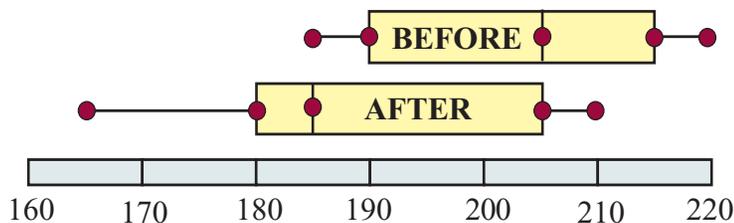
Display the “before” and “after” data in box-and-whisker plots on the same number line.

Solution

The key values for the “before” and “after” data sets were worked out in Example 1:

“Before” data: minimum = 185, lower quartile = 190, median = 205, upper quartile = 215, maximum = 220.

“After” data: minimum = 165, lower quartile = 180, median = 185, upper quartile = 205, maximum = 210.



Check it out:

You should choose the type of plot or chart that shows the important features of your data most clearly. Don't forget about the types of charts you've met in previous grades — such as bar graphs.

Check it out:

The number line needs to include the lower minimum (165) and also the higher maximum (220).

Guided Practice

2. Draw box-and-whisker plots to display Marissa's sets of sunflower height data from Exercise 1.

Independent Practice

1. A local man is conducting a survey to compare happiness in two nearby towns — Town A and Town B. Inhabitants were asked to rate their happiness on a scale of 1–10. The results are below. Prepare them for analyzing, and display the data in a box-and-whisker plot.

Town A: 2, 5, 7, 3, 6, 9, 2, 4, 5, 5, 6, 4, 4, 3, 7, 8, 6, 5, 5, 3, 1, 8, 9, 5

Town B: 5, 7, 7, 8, 8, 6, 6, 5, 4, 4, 6, 9, 9, 10, 3, 7, 8, 8, 9, 6, 7, 4, 8, 9

2. Use a back-to-back stem-and-leaf plot to display the “before” and “after” sets of cholesterol data from p329.

Now try these:

Lesson 6.1.5 additional questions — p463

Round Up

Once you've *displayed* your data, then it's ready for *analyzing*. This means working out what it means, and seeing whether it answers your question. In the next Lesson, you'll analyze the data sets that you've displayed in this Lesson.

Lesson 6.1.6

Analyzing Data

California Standards:

Statistics, Data Analysis,
and Probability 1.1

Know various forms of
display for data sets,
including a stem-and-leaf
plot or box-and-whisker
plot; use the forms to
display a single set of data
or to compare two sets of
data.

Mathematical Reasoning
2.6

Express the solution clearly
and logically by using the
appropriate
mathematical notation and
terms and **clear language**;
support solutions
with evidence in both verbal
and symbolic work.

What it means for you:

You'll learn how to analyze
data displayed in box-and-
whisker plots and stem-and-
leaf plots. You'll also
consider whether or not the
data answers the original
question.

Key words:

- compare
- contrast
- conclusion
- limitation

*It's easy to think you're done when you have your data displayed as nice neat plots, but you need to think about what the display is telling you. You also have to **think back** to why you collected the data in the first place, and see if it answers your question.*

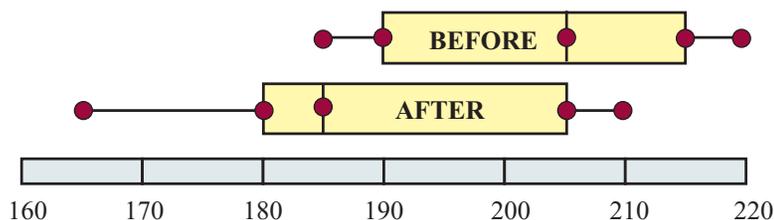
Compare the Similarities; Contrast the Differences

To analyze the results of a study with **two sets of data**, you need to **compare** the displays of each set of data. You have to look at what is **similar** between the data sets and what is **different**.

Example 1

In the previous Lesson, you drew box-and-whisker plots to show the cholesterol levels of a group of people **before and after** they took a certain medicine for two months. These box-and-whisker plots are shown below.

What do they show you about cholesterol levels before and after taking the drug?



Solution

There is a clear **difference** between the two data sets.

The “before” box is much further toward the **higher** end of the scale. This means that cholesterol levels were generally higher **before** the medicine was taken.

The **median** is much lower in the “after” box, which indicates the typical cholesterol level was **reduced** by the medicine.

Guided Practice

1. Look back at the box-and-whisker plots that you drew last Lesson to illustrate the height of Marissa’s sunflowers. Compare the data shown on the plots.
2. Plot Marissa’s data in a back-to-back stem-and-leaf plot. Which diagram do you think best shows the differences and similarities between the two sets of data? Explain your answer.

Draw Conclusions After Analyzing Data

Conclusions bring all your analysis together, and relate what you’ve found out to the **original question**.

Check it out:

You need to be able to back up your conclusions with evidence from the plots of data.

Example 2

The cholesterol data on p329 was collected to try to answer this question:

“Does the medicine lower cholesterol when taken daily for two months?”

Does the data support an answer to this question?

Solution

The data showed that there was a **general reduction** in people’s cholesterol levels after taking the medicine.

You can conclude that **yes, the medicine does tend to reduce cholesterol levels when taken daily for two months**.

There were people in the data set for whom the medicine **didn’t work**. You can’t tell this from the box-and-whisker plots though, because they don’t show individual data points. You can just see the **overall trend**.

Think About Any Limitations of Your Investigation

Your investigation is **unlikely** to give you perfect results. It’s important to understand some reasons for this:

- You might not have used a **big enough sample**. The bigger your sample, the more **accurate and reliable** your results are likely to be.
- Your sample **might not represent the population** well. For instance, if all the people in the cholesterol study were **women**, or **all aged 40**, you couldn’t say if the results were likely to be true for everyone.

Don’t forget:

The sample is all the people or things you collect data about. It’s usually a small part of the whole population. The population is the entire group that you want to know about. You usually use a sample because the whole population is too large for you to be able to collect data on every member.

✓ Guided Practice

3. In Exercise 1, you analyzed the results from Marissa’s sunflower experiment. Draw some conclusions from your analysis — do you think the plant food has made the sunflowers grow taller?

✓ Independent Practice

The data below shows the number of vehicles that passed along Road A during each day of August and December.

August: 73, 67, 79, 86, 78, 54, 65, 63, 73, 75, 79, 69, 62, 63, 75, 59, 78, 79, 72, 75, 64, 68, 69, 62, 56, 75, 78, 84, 82, 78, 65

December: 65, 68, 53, 52, 49, 67, 73, 62, 65, 59, 54, 71, 60, 60, 56, 57, 43, 51, 63, 54, 58, 69, 56, 58, 58, 62, 61, 53, 41, 47, 53

Joe, the local town planner, wants to know whether there is a higher demand for the road during the summer months.

1. Find the minimum, lower quartile, median, upper quartile, and maximum of each data set.
2. Plot the data on a box-and-whisker plot.
3. Plot the data on a back-to-back stem-and-leaf plot.
4. Compare the sets of data on each of the plots you have made. Is there a higher demand for the road during the summer months?

Moesha wants to compare the heights of the seventh grade boys in her school with the heights of the seventh grade girls.

5. Write down a clear question that Moesha might want to know the answer to.
6. What sets of data could Moesha collect?

7. Two running clubs both believe that their members are faster at running 100 m than the other club’s members. The data below shows the personal best times (in seconds) for the members of each club.

Club A: 12.5, 12.3, 11.3, 11.2, 12.9, 12.7, 12.4, 11.9, 12.0, 11.6, 11.5, 10.7, 10.9, 11.0, 11.2, 12.4, 13.1

Club B: 10.1, 11.9, 13.1, 12.0, 12.2, 12.3, 12.6, 11.9, 12.9, 13.0, 13.5, 13.4, 13.9, 12.6, 12.5, 13.4, 12.2

Display the results clearly and explain whether you think they show which club’s members are faster.

Now try these:

Lesson 6.1.6 additional questions — p463

Round Up

You can *collect*, *display*, and *analyze* data to try to answer a question. You’ve got to be aware that the conclusions you draw have *limitations* — they can only be definitely true for the sample you used.

Section 6.2 introduction — an exploration into: Age and Height

The purpose of this Exploration is to determine whether there is a *relationship* between the *age* of a person and their *height*. You can find out if a relationship exists by plotting points on a graph called a scatter plot. This provides a *visual check* of whether one variable affects the other variable.

A collection of points on a graph sometimes fall in a **diagonal band**. This means the pair of variables represented by the points are **related**.

Example

Plot Variable 1 against Variable 2 on a scatter plot. Say whether these variables are related.

Solution

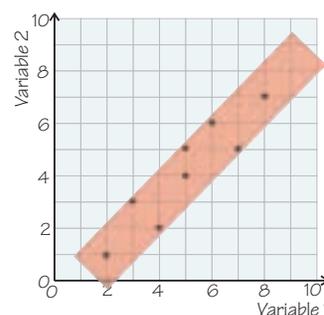
The following coordinates have been taken from the data set.

(2, 1) (3, 3) (4, 2) (5, 4) (5, 5) (6, 6) (7, 5) (8, 7)

These points can be plotted on a graph.

The points form a diagonal band on the graph — so **the variables are related**. As one variable increases, the other does too.

Variable 1	2	3	4	5	5	6	7	8
Variable 2	1	3	2	4	5	6	5	7



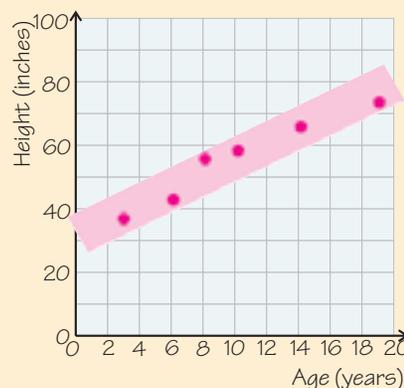
Exercises

- This table shows the ages and heights of six children in a family.

Family member	Maura	Tobias	George	Spencer	Chloe	Wendy
Age (years)	14	3	8	19	10	6
Height (inches)	63	36	54	72	57	42

Copy these axes onto graph paper.
Plot the age and corresponding height of each family member.

- Does there appear to be a relationship between the age and height of a person? Explain your answer.
- Plot your age and corresponding height on the graph. Label the point with your name.
- Does your age and height fit with the trend shown in the scatter plot?
- Based on the graph, predict what the height of a 12 year old would be.



Round Up

Some variables are *related* to each other — the examples on this page are *positively related* (or *correlated*). When one increases, so does the other. Other variables are *negatively correlated* — when one increases, the other *decreases*. There's more on this later in the Section.

Lesson 6.2.1

Section 6.2 Making Scatterplots

California Standards:

Statistics, Data Analysis,
and Probability 1.2

Represent two numerical variables on a scatterplot

and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

What it means for you:

You'll use scatterplots to display sets of data.

Key words:

- conjecture
- scatterplot
- scale
- axes

Check it out:

A conjecture is supposed to be an educated guess, so make sure you have a reason for thinking it might be true.

You'd expect some variables to be *related* to each other.

For example, it might not be a surprise to learn that as grade level increases, so does the average amount of homework that's set.

Scatterplots are a way of *displaying* sets of data to see if and how the variables are *related* to each other.

Some Things May Be Related to Each Other

Some variables are **related** to other variables. You can make **conjectures**, or educated guesses, about how things might be related.

For example,

- The hotter the day is, the more ice cream will be sold.
- The faster you drive a car, the fewer miles you'll get to the gallon.
- The older a child, the later his or her bedtime.

You Can Collect Data to Test Your Conjecture

To see if your conjecture is correct, you first need to **collect data**.

For example, to see if it's true that ice cream sales increase on hotter days, you need to find the **average temperature** for a number of days, and the **number of ice creams sold** on each of these days. You might end up with a **table** of data that looks like this:

Average temperature of day (°F)	41	63	55	73	70	90	48	66	87	79
Number of ice creams sold that day	16	67	80	101	100	170	36	73	123	114

Example 1

What data could you collect to test the conjecture “**the taller a person, the bigger their feet?**”

Solution

You would need to collect data on the **heights** of a set of people, and on the **size of their feet**.

✓ Guided Practice

1. What data would you need to collect to test the conjecture, “the older a child, the later his or her bedtime”?
2. Design a table in which to record this data.

Mark Data Pairs on a Scatterplot

You can display two sets of data values on a **scatterplot**. The values need to be in **pairs**. A scatterplot is a really good way of seeing if the data sets are related — there’s a lot more on this in the next two Lessons.

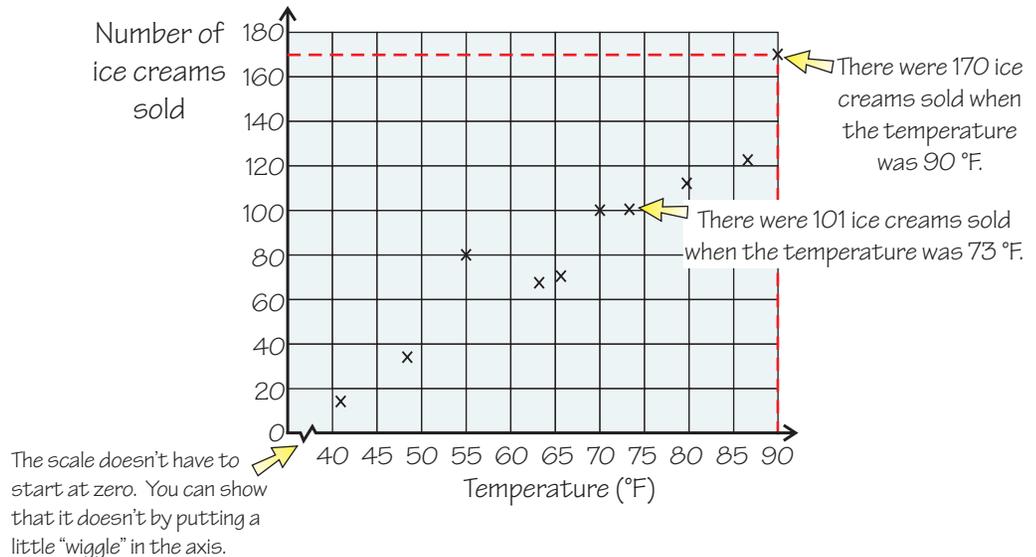
Below is the scatterplot showing the **number of ice creams sold against the temperature**.

Each cross represents a **pair** of data values — the **number of ice creams sold** on a day of a **certain temperature**.

Check it out:

Each pair of data values is like a coordinate — you plot them in exactly the same way. So 16 ice creams sold at 41 °F can be thought of as (41, 16).

You’ve got to make sure you get the data in the right order though — temperature is on the horizontal axis (*x*-axis).



Scatterplots Have Two Axes with Different Scales

You have to think carefully about the **scale** of each axis.

Each axis represents a different thing and is likely to have a different scale. Here’s how to choose a sensible scale for an axis.

1. Look at the **minimum** and **maximum** values of the data set.

You have to choose a **starting point** and an **ending point** for the scale that fits all of the data.

Your scale **doesn't** have to start at zero. If it doesn't, you include a little "wiggle," as above.

The temperatures above were all between 41 °F and 90 °F — so 40 °F and 90 °F were suitable start and end points. It made sense not to start the axis from zero.

2. Choose a sensible **step size**. It must be small enough so that you can show your data clearly, but big enough to fit on your piece of paper.

The temperatures above only had a range of about 50 °F, so 5 °F steps were used. The number of ice creams sold had a much bigger range, so steps of 20 were used.

Don't forget:

The range is the maximum value minus the minimum value.

So the temperature range is: $90 - 41 = 49$ °F

Don't forget to **label** each axis clearly. Once you've done all this you can start **plotting your data**.

Example 2

Make a scatterplot of the data below relating people's ages and heights.

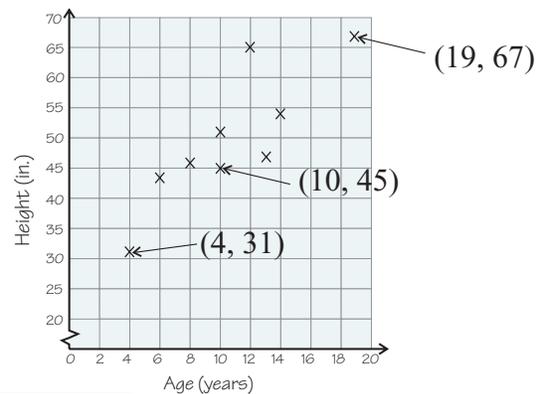
Age (years)	10	19	4	6	10	8	14	12	13
Height (in.)	45	67	31	43	51	46	54	65	47

Solution

First you need to decide on a **scale** for each axis.

The ages go from 4 to 19, so a scale might run from 0 to 20, in steps of 2. The heights go from 31 in. to 67 in. This range is larger, so the scale might go from 20 to 70 in steps of 5 inches.

Then you can **plot the values**. Think of each pair of values as coordinates with the form $(age, height)$, instead of (x, y) . The first three values in the table would be plotted at $(10, 45)$, $(19, 67)$, and $(4, 31)$.



Don't forget:

Take extra care when you're plotting not to get confused between different scales on your axes. It's really easy to mark something in the wrong place.

Guided Practice

3. Use the data below to make a scatterplot relating foot length to height.

Foot length (in.)	7	8	8.3	9	8.5	7.4	6.5	7.5	8.1
Height (in.)	64	68	70.1	72	69	57	59	63	68
Foot length (in.)	8.2	9.1	7.5	7	6.8	7.8	8.4	9.5	8.2
Height (in.)	62	73	61	65	67	71	69	75	70

Independent Practice

Now try these:

Lesson 6.2.1 additional questions — p464

1. Miguel makes the conjecture, "the more people there are in a household, the heavier their recycling bins." What data would you collect to test Miguel's conjecture?

2. The data below was collected to test this conjecture:

"The older a child, the less time he or she will sleep per day."

Draw a scatterplot of this data.

Age (years)	4.5	0.5	8	15	11	14	10.5	2	6
Hours of sleep	11	16	10	8	9	7	9	12	10

Round Up

Now you know what a scatterplot is and how to draw one. The next Lesson shows you how to interpret scatterplots, and how to decide whether, or how closely, the variables are related.

Lesson 6.2.2

Shapes of Scatterplots

California Standards:

Statistics, Data Analysis,
and Probability 1.2

Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

What it means for you:

You'll learn about different types of correlation and what they look like on scatterplots.

Key words:

- slope
- positive correlation
- negative correlation
- strong correlation

Check it out:

If two things are correlated, it doesn't necessarily mean one causes the other. For instance, shark attacks and ice cream sales may show positive correlation, but one doesn't cause the other. They're both increased by hot weather, which makes people more likely to swim in the sea, and to buy ice cream.

In the last Lesson, you learned how to make *scatterplots* from sets of data. By looking at the *pattern of the points* in a scatterplot, you can decide *how the variables are related* — for example, whether ice cream sales really do increase on hot days.

Positive Slope Means Positive Correlation

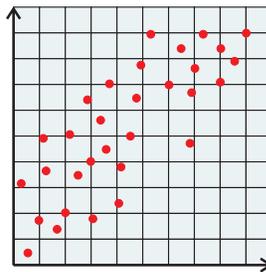
If two things are **correlated**, they are related to each other — if one changes, the other will too.

Two variables are **positively correlated** if one variable **increases** when the other does. For example, children's heights are positively correlated with their ages — because **older** children are typically **taller** than younger ones.

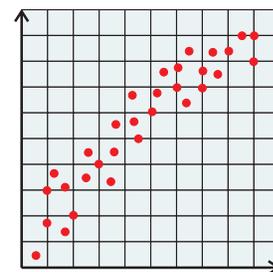
Variables are **positively correlated** if one variable **increases** as the other does.

If two **positively correlated** variables are plotted on a scatterplot, the points will lie in a band from **bottom left** to **top right**. If you were to draw a line through the points it would have a **positive slope**.

The **thinner the band** of points on the scatterplot, the more **strongly correlated** the data is.



This graph shows **positive correlation**.



This graph shows **strong positive correlation**.

Negative Slope Means Negative Correlation

Negative correlation is when one quantity increases as another decreases. For example, values of cars usually **decrease** as their age **increases**.

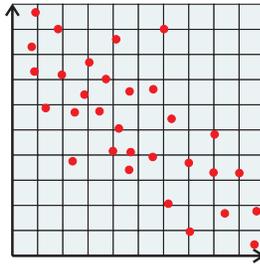
Variables are **negatively correlated** if one variable **increases** as the other **decreases**.

If a scatterplot shows **negative correlation**, the points will lie in a band from **top left** to **bottom right**. They'll follow a line with a **negative slope**.

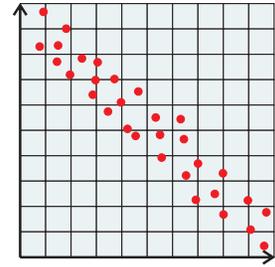
Check it out:

The scatterplot you drew in Independent Practice Exercise 2 in the last Lesson showed negative correlation. The ages of children and the amount of time they sleep are negatively correlated.

The **thinner the band** of points, the more **strongly correlated** the data is.



This graph shows **negative correlation**.

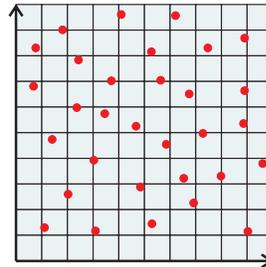


This graph shows **strong negative correlation**.

No Obvious Correlation Means Random Distribution

When points seem to be **spread randomly** all over the scatterplot, then it is said that there is **no obvious correlation**.

For example, people's heights and their test scores are not correlated — the height of a person has no effect on their expected test score.



This graph shows **no obvious correlation**.

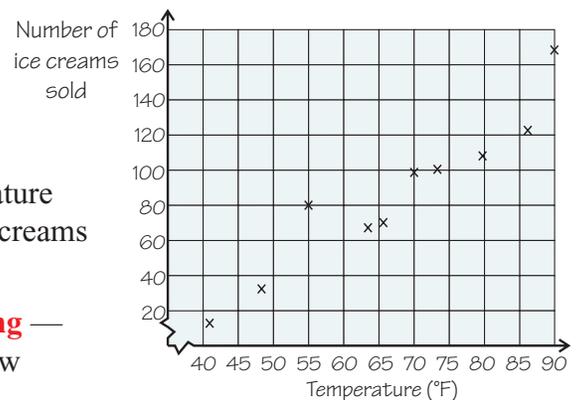
Example 1

Describe the correlation shown in the scatterplot opposite.

Solution

The plot shows **positive correlation**. (As the temperature **increases**, the number of ice creams sold tends to **increase**.)

The correlation is fairly **strong** — the points lie in a fairly narrow band.

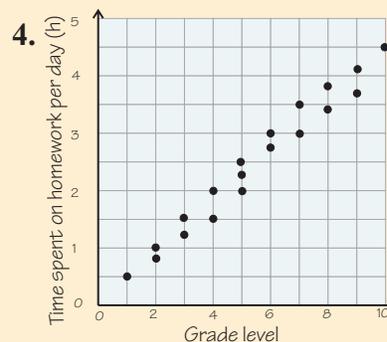
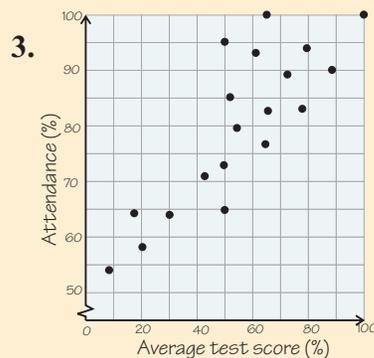
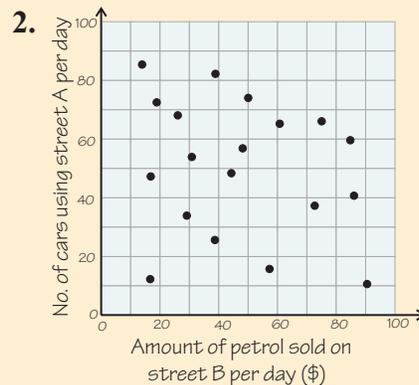
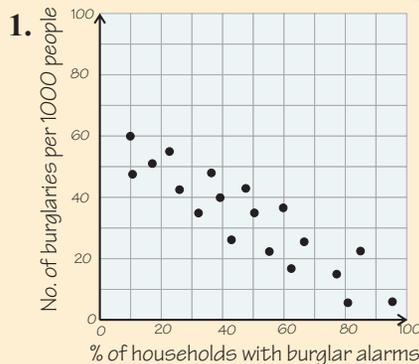


Check it out:

The correlation isn't perfect here. If it was perfect the points would lie in a straight line.

✓ Guided Practice

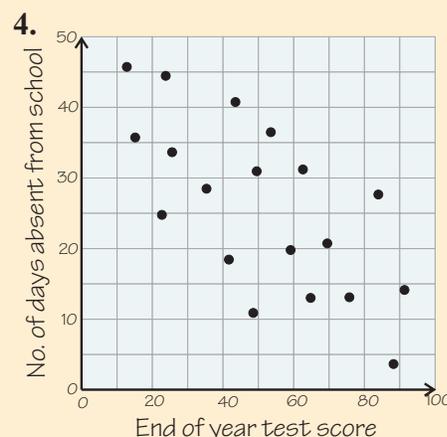
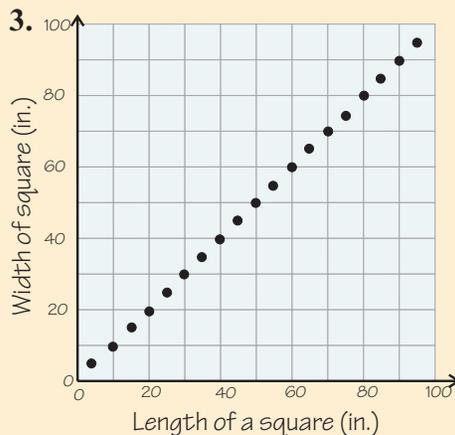
In Exercises 1–4, describe the type of correlation.



✓ Independent Practice

- Brandon investigates the relationship between the number of spectators at a football game and the amount of money taken at the concession stand. What kind of correlation would you expect?
- If every job you do takes one job off your to-do list, what kind of correlation would you expect between the number of jobs you do and the number of jobs on your to-do list?

In Exercises 3–4, describe the correlation shown.



Now try these:

Lesson 6.2.2 additional questions — p464

Round Up

If the points lie roughly in a diagonal line across a *scatterplot*, it means the variables are correlated. An “uphill” band means positive correlation, whereas a “downhill” band means negative correlation.

Lesson 6.2.3

Using Scatterplots

California Standards:

Statistics, Data Analysis,
and Probability 1.2

Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

Mathematical Reasoning
2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

What it means for you:

You'll predict data values using scatterplots. You'll also practice finding the highest and lowest values in a data set.

Key words:

- prediction
- scatterplot
- line of best fit

If you have many pairs of values plotted on a scatterplot, and they all fall in a neat band, you know the two variables are *correlated*. If you plotted some more pairs of values, you'd expect them to lie within the band of points. You can use this idea to predict values. For instance, from the scatterplot of ice cream sales against average temperature, you could predict how many ice creams would be sold when the temperature was 50 °F.

Finding the Highest and Lowest Values

Box-and-whisker plots **don't** show all the raw values — just the maximum and minimum values and the general trends. **Scatterplots** are different — they show the **raw data values**, as well as trends.

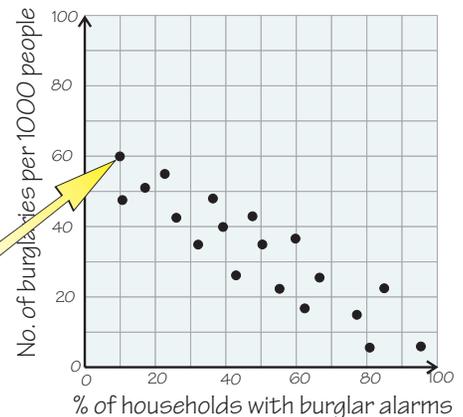
Example 1

The scatterplot below shows the number of burglaries per 1000 people against the percentage of households that have burglar alarms installed. What was the greatest number of burglaries per 1000 people recorded?

Solution

The greatest number of burglaries recorded is the point that lies furthest up the vertical axis on the graph.

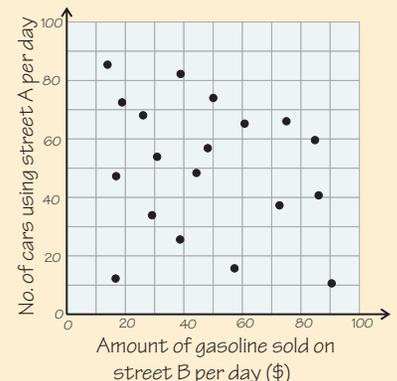
The highest number of burglaries recorded per 1000 people is **60**.



Guided Practice

For Exercises 1–3, refer to the scatterplot on the right.

1. What was the highest number of cars recorded using street A on a single day?
2. What was the greatest amount of money spent on gasoline in street B on any day?
3. How many cars used street A on the day when the least amount of gasoline was sold on street B?

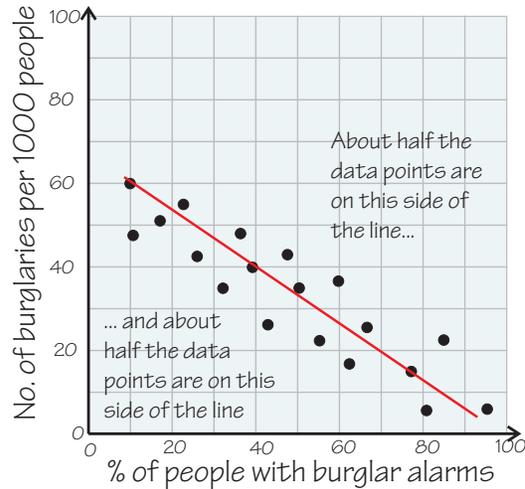


A Line of Best Fit Shows the Trend in the Data

Not many sets of data are perfectly correlated, so a **line of best fit** is used to show the **trend**. If the data was perfectly correlated you'd expect all the points to lie on this line.

Example 2

Draw a line of best fit on the scatterplot below.



Check it out:

A clear plastic ruler is useful for drawing lines of best fit. You can adjust it until you think the edge is in the right place.

The scatterplot shows **negative correlation**, so the line of best fit will have a **negative slope**.

The line of best fit splits the data approximately in **half**. You should have roughly the same number of points on each side of the line.

You can't add a line of best fit to data that has no correlation.

✓ Guided Practice

4. The hand spans of 11 students are measured, together with the lengths of their arms. The measurements are recorded in the table below.

Hand span (cm)	19	18	20	15	21	22	16	17	20	24	26
Arm length (cm)	50	46	56	40	60	63	48	44	48	57	60

Plot a scatterplot of this data. Add a line of best fit to your scatterplot.

5. The ages and values of a particular type of car are recorded below.

Age of car (years)	0	2	7	12	11	6	8	1	6	3	3	8	9
Value of car (dollars)	10,000	9000	4000	1000	4000	6000	7000	8000	7000	6000	8000	5000	4000

Plot a scatterplot of this data. Add a line of best fit to your scatterplot.

Use Lines of Best Fit to Make Predictions

You can use a line of best fit to predict what other data points might be.

Example 3

Predict the number of burglaries per 1000 people if 50% of households have burglar alarms.

Solution

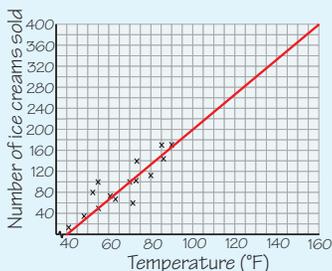


Start at **50%** on the **horizontal axis**.
Read up to the line of best fit.
Read across from the line of best fit to the **vertical axis**.

When 50% of households have burglar alarms, the number of burglaries per 1000 people is expected to be around 33.

Check it out:

You can use scatterplots to predict data points outside of the range you were given. You have to take care though as you don't know whether the data will continue in the same pattern. For instance, you could extend the axes and line of best fit in the graph below and predict that 400 ice creams would be sold at a temperature of 160 °F. This is obviously wrong as people wouldn't survive if it was that hot.



Now try these:

Lesson 6.2.3 additional questions — p464

✓ Guided Practice

In Guided Practice Exercise 4, you drew a scatterplot of arm length against hand span. Use your line of best fit to predict:

- the arm length of a student with a 23 cm hand span.
- the hand span of a student with a 52 cm arm length.

In Guided Practice Exercise 5, you drew a scatterplot of values against ages for a certain type of car. Use your line of best fit to predict:

- the expected value of a 5-year-old car of this type.
- the age of a car that is valued at \$5500.

✓ Independent Practice

The table below shows the height (in feet) of mountains with their cumulative snowfall on April 1st (in inches).

- Create a scatterplot of the data.
- Draw in a line of best fit for the data.
- A mountain has a height of 7200 feet. What would you expect its cumulative snowfall to be on April 1st?

Height (ft)	6700	7900	7600	6800	6200	5800	8200	6700
Snowfall (in.)	153	174	249	172	128	32	238	162

Round Up

Lines of best fit follow the **trend** for the data. You can use them to **predict** values — but remember, chances are your predictions won't be totally accurate. They can give you a good idea though.

Chapter 6 Investigation

Cricket Chirps and Temperature

Displaying data in a *visual way* makes it easier to see whether trends and patterns exist.

For a long time, people have believed that you can estimate the **temperature** from the **number of times a cricket chirps** in a set period of time.

At the same time of evening on 15 consecutive days, a student took the temperature outside and counted the number of times a cricket chirped in **15 seconds**. The results are shown below.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of cricket chirps	15	17	19	13	14	15	17	20	21	21	19	18	16	16	16
Temperature (degrees Fahrenheit)	72	76	83	70	75	80	82	87	93	88	90	85	75	81	72

The data isn't very useful in this form — you can't see any patterns clearly.

You are going to make a **visual display** that makes the data easier to interpret.

Make a **scatterplot** to compare the number of cricket chirps and the temperature.

Explain what, if any, **correlation** exists among the data.

Do you think that you can **estimate** the temperature from the number of cricket chirps?

Extensions

- 1) Draw a line of best fit on your scatterplot.
Use the line to predict the temperature for each number of cricket chirps from 10 to 25.
Which of these predictions do you have the most confidence in?
- 2) There are several different formulas for working out temperature from cricket chirps.
One is:
“Count the number of chirps a cricket makes during a 15-second period.
Then, add 45 to the number of chirps. This gives you an estimate of the temperature in degrees Fahrenheit.”
Does this formula agree with the data above?

Open-ended Extensions

- 1) Can you create a formula that fits the line you drew in the first Extension above?
- 2) There are many formulas for estimating the temperature from cricket chirps.
See how many different ones you can find using reference books, almanacs, or the internet.
Do any of them match the data above?

Round Up

When you've displayed your data in an appropriate way, you'll often immediately be able to see patterns that you couldn't see before. Scatterplots only work when you've got two data sets that are paired. If you haven't, you have to use a different form of display.

Chapter 7

Three-Dimensional Geometry

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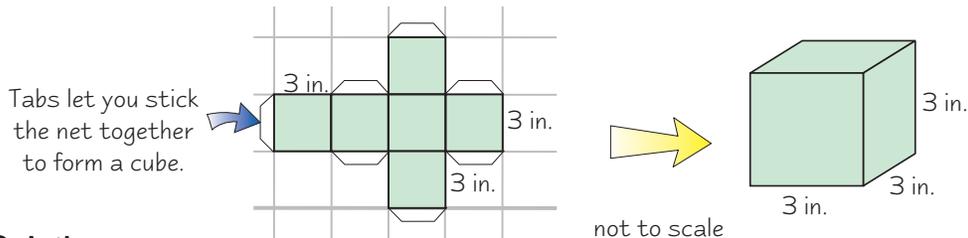
Section 7.1 introduction — an exploration into: Nets

In this Exploration, you will be using *two-dimensional figures* called *nets* to make *three-dimensional rectangular prisms*. You'll construct nets for rectangular prisms with given dimensions.

The example below shows a **net** of a cube. You make the cube by cutting the net out, folding it along the lines and taping it together.

Example

What dimensions will the cube made from the net below have?

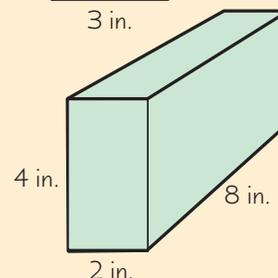
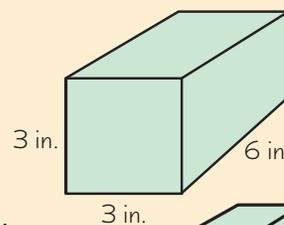
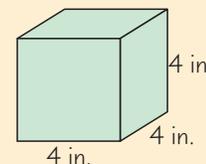


Solution

The cube made by this net has a length, width, and height of 3 inches.

Exercises

- Create a net for a cube that has a length, width, and height of 4 inches. Then build your cube.
- Find a rectangular prism that isn't a cube and draw around each face. It may be useful to mark each face after you have drawn around it.
 - How many faces are there?
 - What shapes are the faces?
- Measure the length and width of each face you drew in Exercise 2. Do any of the faces match each other?
- This rectangular prism has twice the length of the cube in the example at the top of the page. Its height and width are the same. What dimensions do the faces of this rectangular prism have?
- Create a net for the prism shown on the right. Then build the prism.
- The prism shown on the right has the following dimensions: Length = 8 in., width = 2 in., height = 4 in. What dimensions do the faces of this prism have?
- Create a net for a rectangular prism with the dimensions given in Exercise 6. Then build the prism.



Round Up

Cubes have six identical square faces — so their nets are made of six identical squares. You have to make sure you *join* them to each other *correctly* so that the net folds into a cube though. *Rectangular prisms* are trickier to draw nets for. They usually have *three different pairs* of faces.

Lesson 7.1.1

Section 7.1

Three-Dimensional Figures

Real life is full of three-dimensional figures. This Lesson's about some of the special figures that have mathematical names.

California Standards:

Measurement and Geometry 2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of **basic three-dimensional figures**, including rectangles, parallelograms, trapezoids, squares, triangles, circles, **prisms**, and **cylinders**.

Measurement and Geometry 3.6

Identify elements of **three-dimensional geometric objects** (e.g., **diagonals of rectangular solids**) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

What it means for you:

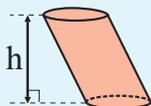
You'll practice recognizing some important 3-D shapes, such as prisms, cylinders, cones and pyramids.

Key words:

- prism
- cylinder
- cone
- pyramid
- diagonal
- right
- oblique

Check it out:

The height of a three-dimensional solid is always the perpendicular distance from the top to the bottom. So the height of cylinder C is the distance straight down from the circle at the top to the circle at the bottom. It doesn't matter that there isn't a side that goes straight down.



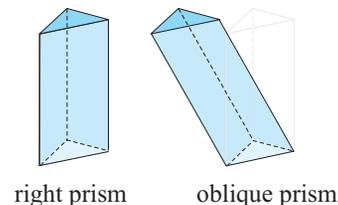
Prisms and Cylinders Have Two Identical Ends

A **prism** is a 3-D shape formed by joining two **congruent polygon faces** that are **parallel** to each other. The polygon faces are called the **bases** of the prism.

• If the edges joining the bases are at right angles to the bases, it is called a **right prism**.

• If the edges joining the bases are not at right angles to the bases, it's an **oblique prism**.

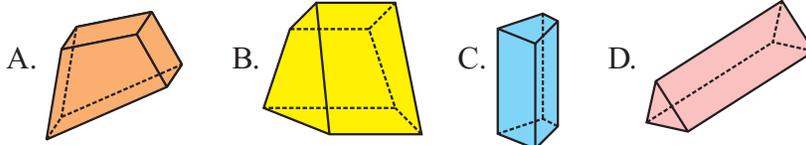
Oblique prisms appear to lean to one side.



Prisms are often named according to their bases — if the bases are rectangles, it's a **rectangular prism**. If the bases are triangles, it's a **triangular prism**.

Example 1

Which of these figures is **not** a prism? Explain why not.



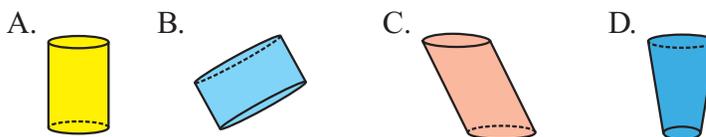
Solution

B is not a prism. The shape at one end is not the same as the shape at the other. **A, C, and D are prisms.** D is a triangular prism.

A **cylinder** is just like a prism, except that the bases have curved, rather than straight, edges. All the cylinders in this book will be **circular cylinders** with circle bases, though it is possible to have cylinders with ellipse bases. As with prisms, cylinders can be **right** or **oblique**.

Example 2

Which of these figures is not a cylinder? Explain why not.

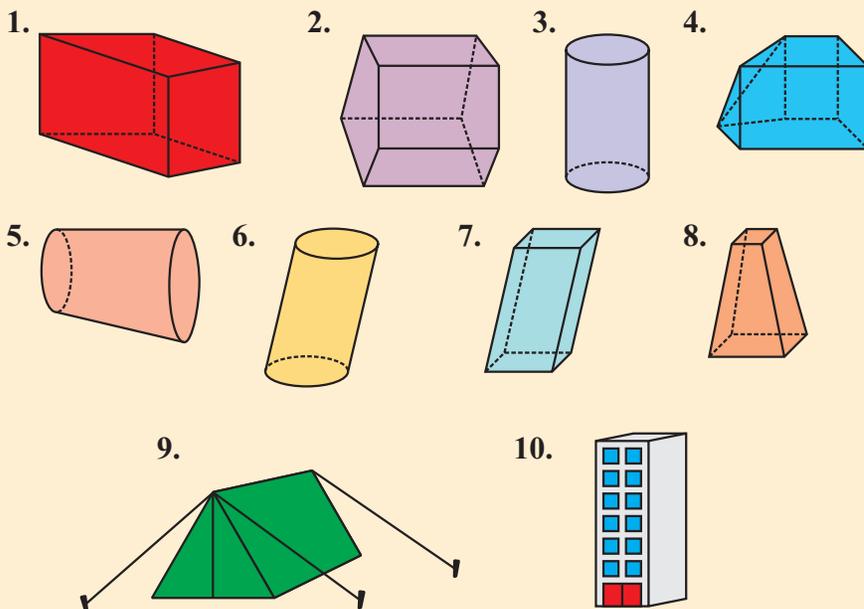


Solution

D is not a cylinder because the circle at the bottom is not the same size as the circle at the top. **A, B, and C are cylinders** because they have congruent circular faces that are parallel to each other. A and B are right cylinders, and C is an oblique cylinder.

✓ Guided Practice

In Exercises 1–10, identify each shape as either a prism, a cylinder, or neither.



Don't forget:

A polygon is any shape that is made from straight lines that have been joined, end-to-end into a closed shape. See Section 3.1 for a reminder on polygons.

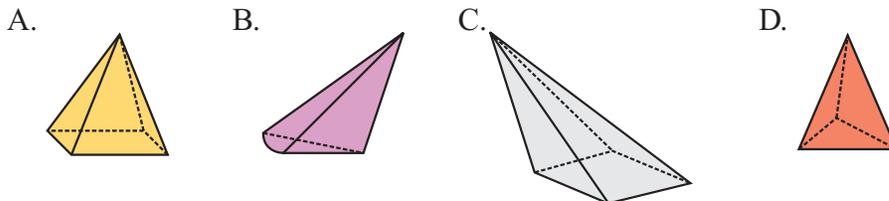
Pyramids and Cones Have Points

A **pyramid** is a three-dimensional shape that has a **polygon** for its base, and all the other faces come to a **point**. The point **doesn't** have to be over the base.

A pyramid with a **rectangular base** is known as a **rectangular pyramid**. Similarly, a pyramid with a **triangular base** is a **triangular pyramid**.

Example 3

Which of these figures is not a pyramid? Explain why not.



Solution

B is not a pyramid. Its base has a curved edge, so it isn't a polygon. The base of a pyramid is always a polygon.

A, C and D are all pyramids. It doesn't matter that C leans, as the sides all still come to a point.

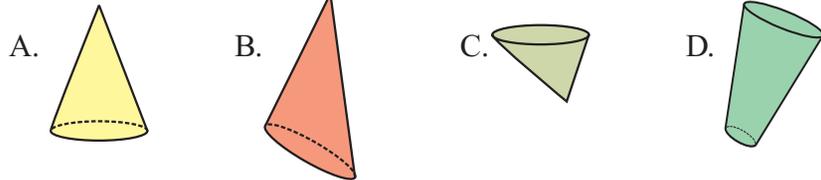
A **cone** is like a pyramid, but instead of having a polygon for a base, the base has a curved edge. All of the cones in this book will be **circular cones** with bases that are **circles**.

Check it out:

Pyramids and cones which have their point directly above the center of their base are called “right pyramids” or “right cones.”

Example 4

Which of these figures is not a cone? Explain why not.

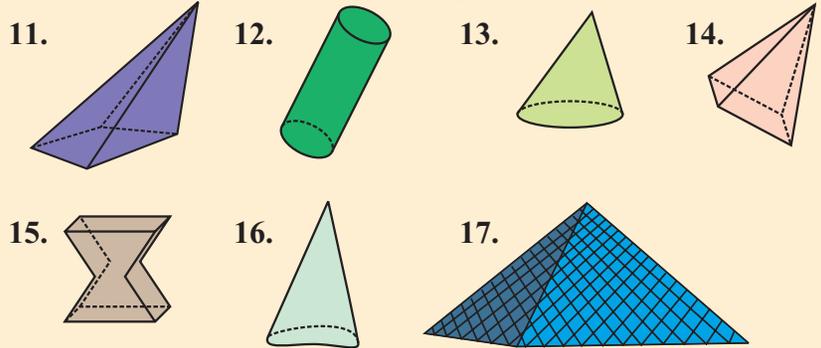


Solution

D is not a cone because it doesn't make a point at the end. **A, B, and C are all cones.** It doesn't make any difference that B and C are leaning to the side.

Guided Practice

Identify the shapes below as either pyramids, circular cones, or neither.



Check it out:

Pyramids and prisms are polyhedrons. The sides of polyhedrons are all polygons, and are known as **faces**. The line where two faces meet is called an **edge**. A point where edges meet is called a **vertex**.

Diagonals Go Through the Insides of Solids

Diagonals are a type of line segment. In a 3-D shape, they connect two **vertices** that **aren't on the same face**.

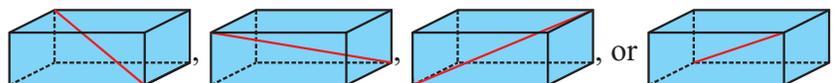
Example 5

The diagram below shows a rectangular prism or cuboid. Mark a diagonal on it.



Solution

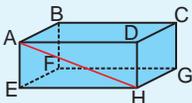
There are four diagonals that you could mark:



There are no other possible diagonals.

Check it out:

Diagonals must pass through the inside of the shape. So the line from A to H **isn't** a diagonal.



✓ Guided Practice

Check it out:

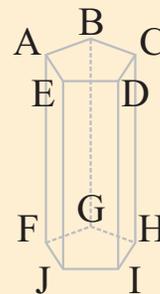
For Exercises 19–20 — start with vertex A and see how many diagonals you can draw from it. The same number of diagonals will start from vertices B, C, D, and E.

18. Crystal says that any line that goes through a prism is a diagonal of the prism. Is she correct?

Exercises 19–20 are about the prism shown on the right.

19. How many diagonals does this shape have?

20. Name all the diagonals by giving their starting vertex and ending vertex.



✓ Independent Practice

1. What is similar about a cylinder and a prism?

2. What is different about a cylinder and a prism?

In Exercises 3–13, identify each shape as a prism, cylinder, pyramid, cone, or as none of those.

3.



4.



5.



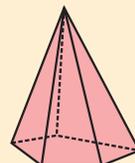
6.



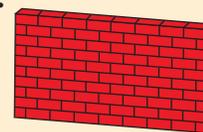
7.



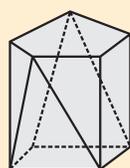
8.



9.



10.



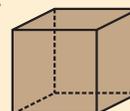
11.



12.



13.



In Exercises 14–17, say whether each statement is true or false. If any are false, explain why.

14. A cylinder is a type of prism.

15. All cubes are prisms.

16. Pyramids have no diagonals.

17. A cylinder is any shape with two circles for bases.

Now try these:

Lesson 7.1.1 additional questions — p465

Round Up

You need to be able to *identify* the different kinds of three-dimensional figures. The hard part is remembering precisely when you can use each name and when you can't. Once you've mastered that, you can try *nets* — which are like flat "patterns" that can be folded to make 3-D figures.

Lesson 7.1.2

California Standards:

Measurement and
Geometry 3.5

Construct two-dimensional
patterns for three-
dimensional models, such
as cylinders, prisms, and
cones.

What it means for you:

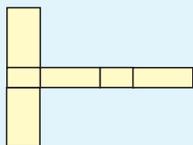
You'll draw two-dimensional
patterns that can be folded up
to make three-dimensional
shapes, such as prisms.
You'll use the formula you
learned for finding the
circumference of a circle to
draw the pattern for a
cylinder.

Key words:

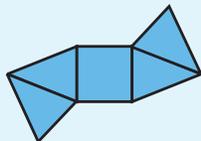
- net
- cone
- cylinder
- pyramid
- two-dimensional
- three-dimensional
- rectangular pyramid

Check it out:

There's often more than one
net for a shape.
Here's a different net for the
cuboid in Example 1:



And here's one for the
pyramid in Example 2:



Nets

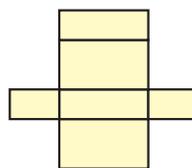
Sometimes you might want to make a model 3-D shape out of card. To do this, you need to figure out which two-dimensional shapes you need for the faces, how big they have to be, and how they should be joined together.

2-D Nets Can Be Folded Into 3-D Figures

A two-dimensional shape pattern that can be folded into a three-dimensional figure is called a **net**. The lines on the net mark the **fold lines** — these are the **edges** of the faces.

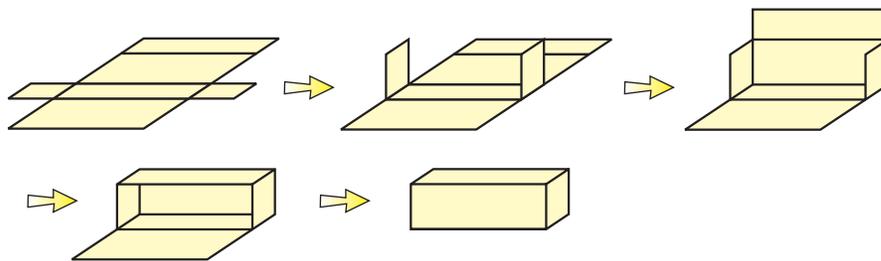
Example 1

What three-dimensional
shape is this the net of?



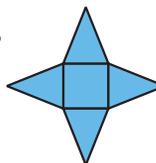
Solution

If you fold along all of the marked lines then you get a prism with a rectangular base. So **this is the net of a rectangular prism or cuboid**.



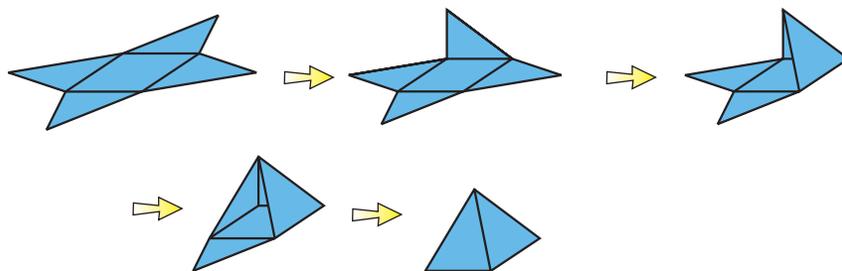
Example 2

What shape is this the net of?



Solution

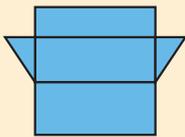
This is the net of a square-based pyramid (a special type of rectangular pyramid), which is a pyramid with a square base.



✓ Guided Practice

In Exercises 1–4, say what shape is made by each net.

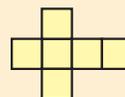
1.



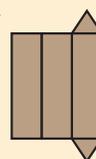
2.



3.



4.



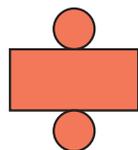
The Net of a Cylinder Has Two Circles

The net of a circular cylinder has **two circles** — one for the top of the cylinder and one for the bottom.

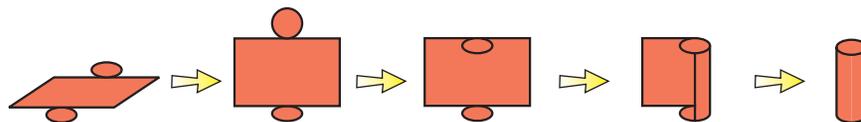
Example 3

Sketch the net of a right circular cylinder.

Solution



The net of a circular cylinder looks like a rectangle with a circle on top and a circle on the bottom. If you were to fold this shape up it would make a cylinder.



Don't forget:

A **right cylinder** is one in which the two bases are directly above each other. A right cylinder doesn't lean to one side.

The rectangle needs to be wide enough to be **wrapped around the outside** of the circles. So its length needs to be equal to the circumference of the circles.

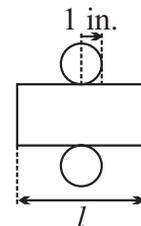
Example 4

Work out the missing length, l , to the nearest hundredth.

Solution

The rectangle needs to wrap around the circle. So it has to have the **same length as the circumference of the circle**.

$$\begin{aligned} C &= 2\pi r \\ l &= 2 \times \pi \times r \\ &= 2 \times \pi \times 1 \\ &= \mathbf{6.28 \text{ in. (to the nearest hundredth)}} \end{aligned}$$

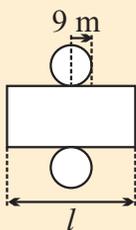
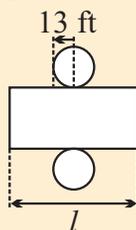
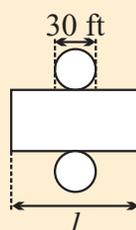
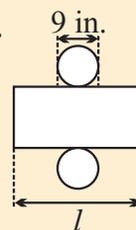


Don't forget:

The circumference of a circle is given by the formula $C = 2\pi r$, (where r = radius). That's the same as $C = \pi d$, (where d = diameter).

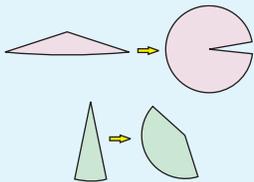
✓ Guided Practice

Work out the missing measurements in Exercises 5–8. Use $\pi = 3.14$.

5.  6.  7.  8. 

Check it out:

In Example 5, cutting along the cone makes a circle with exactly $\frac{1}{4}$ missing. But the amount of the circle that is missing could be anything. For very flat cones only a small amount will be missing. For very tall cones a very large amount will be missing.



Cutting a Cone Makes Part of a Circle

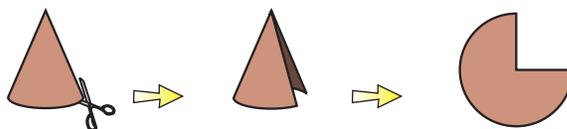
The net of a **circular cone** includes part of a **circle**.

Example 5

Sketch the net of a circular cone.

Solution

Imagine cutting up the side of a cone with no base and laying it flat.

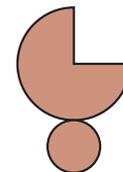


You get **a circle with part missing**.

To make a full cone, you need a base as well — **the base is a circle**.

The base circle can't be as large as the one with a sector cut out, or there'd be no way to roll it up and still have the base fit.

So the net of a cone is a circle with part missing and a smaller circle underneath.



Check it out:

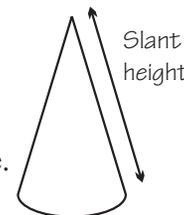
The circumference of the base circle must be the same as the length of the curved edge of the part-circle.

The sketched net above has a part-circle that is $\frac{3}{4}$ of a full circle.

If you're making a cone with a certain **base-radius** and a certain **slant height**, you can work out what **fraction** of a circle you need:

$$\text{Fraction of circle} = \text{radius of base} \div \text{slant height}$$

You use the **slant height** itself as the radius of the part-circle.

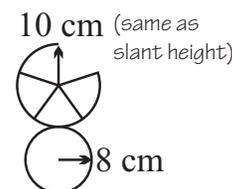


Example 6

Draw the net of a circular cone with slant height 10 cm and base radius 8 cm.

Solution

If slant height is 10 cm and base radius is 8 cm then the top circle should be $\frac{8}{10}$ of a complete circle. That's the same as $\frac{4}{5}$. **So draw the top circle with $\frac{1}{5}$ missing.**



✓ Guided Practice

In Exercises 9–11, say whether each statement is true or false.

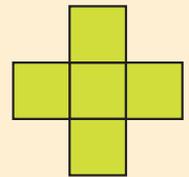
9. The net of a cone is two circles.
10. You can draw the net of a cone without knowing the vertical height of the cone.
11. The part-circle in the net of a cone is smaller than the full circle.

Work out what fraction of a circle is needed for the part-circle in the net of each circular cone described in Exercises 12–14.

12. A cone with slant height 20 cm and base radius 16 cm.
13. A cone with slant height 15 inches and base radius 3 inches.
14. A cone with slant height 2 feet and base radius 1 foot.

✓ Independent Practice

1. Explain why the net shown on the right doesn't make a cube.



In Exercises 2–4, say which net could make each three-dimensional figure.

2.
 - A.
 - B.
 - C.
3.
 - A.
 - B.
 - C.
4.
 - A.
 - B.
 - C.

Fill in the missing measurements in Exercises 5–7. Use $\pi = 3.14$.

- 5.
- 6.
- 7.

Now try these:

Lesson 7.1.2 additional questions — p465

Work out what fraction of a circle is needed for the part-circle in the net of each circular cone described in Exercises 8–9.

8. A cone with slant height 3 inches and base radius 1 inch.
9. A cone with slant height 11 inches and base radius 3 inches.

Round Up

Nets sound complicated, and some of them even look complicated. The key is to imagine folding along each of the lines, and think about what shape you would get.

Lesson 7.1.3

Surface Areas of Cylinders and Prisms

California Standards:

Measurement and Geometry 2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Measurement and Geometry 3.5

Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

Mathematical Reasoning 1.3

Determine when and how to break a problem into simpler parts.

What it means for you:

You'll see how to work out the surface area of 3-D shapes like cylinders and prisms.

Key words

- net
- surface area
- cylinder
- prism

Don't forget:

The area of a rectangle is given by $A = lw$.

The area of a triangle is given

$$\text{by } A = \frac{1}{2}bh.$$

Nets are very useful for finding the **surface area** of 3-D shapes. They change a 3-D problem into a 2-D problem.

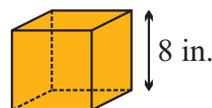
Draw a Net to Work Out the Surface Area

The **surface area** of a **three-dimensional solid** is the total area of all its faces — it's the area you'd paint if you were painting the shape.

The **net** of a **three-dimensional solid** can be folded to make a hollow shape that looks exactly like the solid. So one way to work out the surface area of the solid is to work out **the surface area of the net**.

Example 1

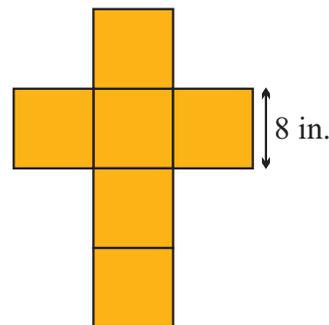
What is the surface area of this cube?



Solution

The net of the cube is six squares. So the surface area of the cube is equal to the area of six squares.

The area of each square is $8 \times 8 = 64 \text{ in}^2$. So the surface area of the entire cube is $6 \times 64 = 384 \text{ in}^2$.



Example 2

What is the surface area of this prism?

Solution

The net of this prism has three identical rectangles. The area of each rectangle is $10 \times 20 = 200 \text{ cm}^2$.

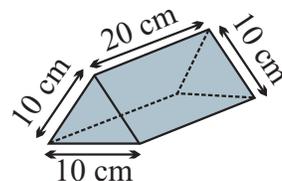
So the total surface area of the three rectangles is $3 \times 200 = 600 \text{ cm}^2$.

There are also two identical triangles. Each has a base of 10 cm and a height of 8.7 cm. The area of each triangle is

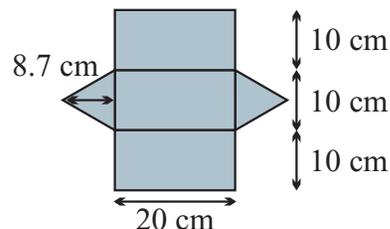
$$\frac{1}{2} \times 10 \times 8.7 = 43.5 \text{ cm}^2.$$

So the surface area of both the triangles together is $2 \times 43.5 = 87 \text{ cm}^2$.

So the total surface area of the prism is $600 + 87 = 687 \text{ cm}^2$.

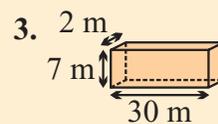
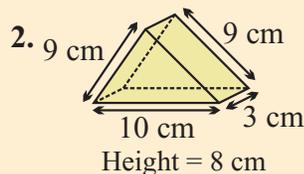
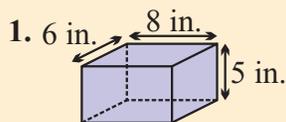


Vertical height = 8.7 cm



✓ Guided Practice

Work out the surface area of the shapes shown in Exercises 1–3.

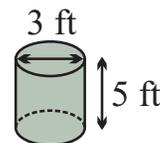


Finding the Surface Area of Cylinders

The net of a circular cylinder has a **rectangle** and **two circles**. So you need to use the **formula for the area of a circle** to find its surface area.

Example 3

What is the surface area of this cylinder? Use $\pi = 3.14$.

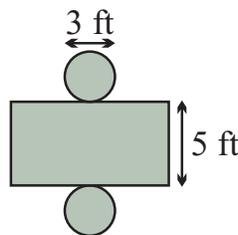


Don't forget:

Use the formula for the circumference of a circle, $C = \pi d$, (where d = diameter) to work out the length of the rectangle in the net of a cylinder. The rectangle has to be long enough to reach all the way around the circle.

Solution

The net of the cylinder has one rectangle and two identical circles.



To work out the area of the rectangle, you need to know its length. It's the same as the circumference of the circles, so it is $3 \times \pi = 9.42$ ft.

So the area of the rectangle is $9.42 \times 5 = 47.1$ ft².

The circles have a diameter of 3 feet. So they have a radius of 1.5 feet. The area of each circle is $\pi \times 1.5^2 = \pi \times 2.25 = 7.065$ ft².

Together the two circles have a surface area of $2 \times 7.065 = 14.13$ ft².

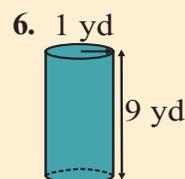
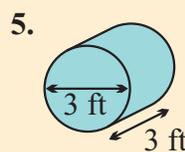
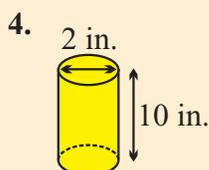
So the total surface area of the cylinder is $47.1 + 14.13 = 61.23$ ft².

Don't forget:

The area of a circle is given by $A = \pi r^2$ (r = radius).

✓ Guided Practice

Find the surface areas of the cylinders in Exercises 4–6. Use $\pi = 3.14$.

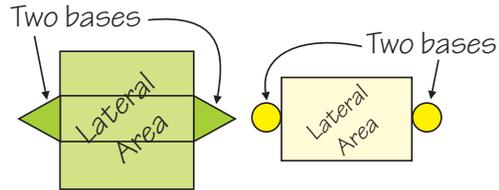


Use Formulas For Prism and Cylinder Surface Areas

The way you work out the surface area of a cylinder, and the way you work out the surface area of a prism are similar.

The surface area of either is **twice** the **area of the base** plus the area of the **part between the bases** of the net.

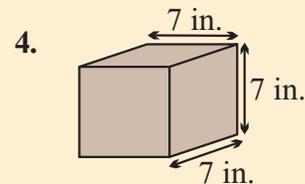
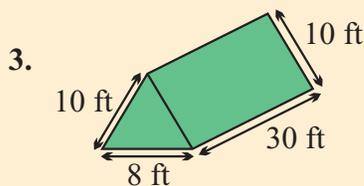
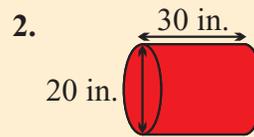
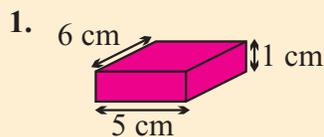
The part between the bases is sometimes called the **lateral area**.



$$\text{Area} = (2 \times \text{base}) + \text{lateral area}$$

Independent Practice

Work out the surface areas of the shapes shown in Exercises 1–4. Use $\pi = 3.14$.

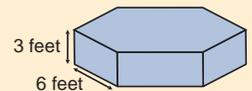


Vertical height = 6.8 feet

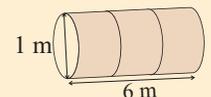
Now try these:

Lesson 7.1.3 additional questions — p466

5. A statue is to be placed on a marble stand, in the shape of a regular-hexagonal prism. Find the area of the stand's base, given that the stand has a surface area of 201.5 square feet and dimensions as shown.



The inside of a large tunnel in a children's play area is to be painted. The tunnel is 6 meters long and 1 meter tall. It is open at each end.



6. What is the area to be painted?

7. Cans of paint each cover 5 m^2 . How many cans do they need to buy?

Round Up

Working out the surface area of a 3-D shape means **adding together** the area of every part of the outside. One way to do that is to add together the areas of different parts of the **net**. Just make sure you can remember the **triangle**, **rectangle**, and **circle area formulas**.

Lesson 7.1.4

Surface Areas & Edge Lengths of Complex Shapes

California Standards:

Measurement and Geometry 2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Measurement and Geometry 2.2

Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

Measurement and Geometry 2.3

Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids.

Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

Mathematical Reasoning 1.3

Determine when and how to break a problem into simpler parts.

What it means for you:

You'll work out the surface area and edge lengths of complex figures.

Key words:

- net
- surface area
- edge
- prism

Prisms and cylinders can be stuck together to make *complex shapes*. For example, a house might be made up of a rectangular prism with a triangular prism on top for the roof. You can use lots of the skills you've already learned to find the total edge length and surface area of a complex shape — but there are some important things to watch for.

Finding the Total Edge Length

An **edge** on a solid shape is a line where **two faces meet**.

The tricky thing about finding the **total edge length** of a solid, is making sure that you include **each edge length only once**.

Example 1

Find the total edge length of the rectangular prism shown.

Solution

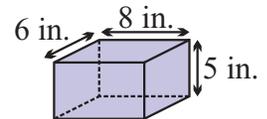
There are four edges around the top face:

$$6 + 6 + 8 + 8 = \mathbf{28 \text{ in.}}$$

The bottom is identical to the top, so this also has an edge length of **28 in.**

There are four vertical edges joining the top and bottom: $4 \times 5 = \mathbf{20 \text{ in.}}$

$$\text{So total edge length} = 28 + 28 + 20 = \mathbf{76 \text{ in.}}$$



When complex shapes are formed from simple shapes, some of the edges of the simple shapes might “disappear.” These need to be subtracted.

Example 2

A wedding cake has two tiers. The back and front views are shown below. The cake is to have ribbon laid around its edges. What is the total length of ribbon needed?

Solution

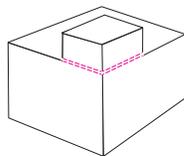
Total edge length of the top tier:

$$(10 \times 8) + (5 \times 4) = \mathbf{100 \text{ cm}}$$

Total edge length of the bottom tier:

$$(20 \times 8) + (15 \times 4) = \mathbf{220 \text{ cm}}$$

But, two of the 10 cm edges on the top tier aren't edges on the finished cake. You have to subtract these “shared” edge lengths from both the top and the bottom.



$$\begin{aligned} \text{Total edge length} &= \text{top tier edge lengths} \\ &+ \text{bottom tier edge lengths} - (2 \times \text{shared edge lengths}) \\ &= 100 + 220 - (2 \times 10 \times 2) = \mathbf{280 \text{ cm}} \end{aligned}$$

So 280 cm of ribbon is needed.

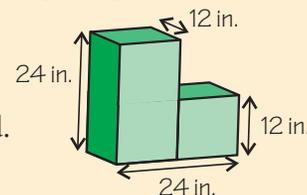
✓ Guided Practice

Don't forget:

All the edges on a cube are the same length.

A display stand is formed from a cube and a rectangular prism.

1. Find the total edge lengths of the cube and rectangular prism before they were joined.
2. Find the total edge length of the display stand.



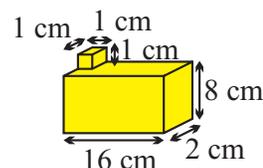
Break Complex Figures Up to Work Out Surface Area

You work out the **surface areas** of complex shapes by breaking them into **simple shapes** and finding the surface area of each part.

The place where two shapes are stuck together **doesn't** form part of the complex shape's surface — so you need to **subtract** the area of it from the areas of **both** simple shapes.

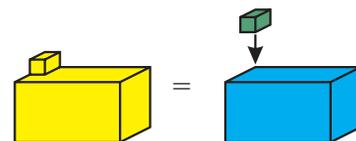
Example 3

What is the surface area of this shape?



Solution

The complex shape is **like two rectangular prisms stuck together**. You can **work out the surface area of each individually, and then add them together**.



But **the bottom of the small prism is covered up, as well as some of the top of the large prism**. So you lose some surface area.

The amount covered up on the big prism must be the same as the amount covered up on the small prism.

So you have to **subtract the area of the bottom face of the small prism twice** — once to take away the face on the small prism, and once to take away the same shape on the big prism.

The surface area of the big prism is **352 cm²**.

The surface area of the small prism is **6 cm²**.

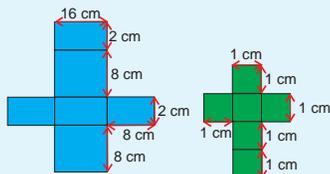
The surface area of the bottom face of the small prism is **1 cm²**.

Total surface area = surface area of big prism + surface area of small prism – (2 × bottom face of small prism).

So the surface area of the shape is **352 + 6 – (2 × 1) = 356 cm²**.

Don't forget:

To find the surface areas of the prisms, first draw the nets:



Big prism:

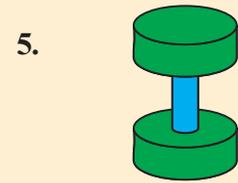
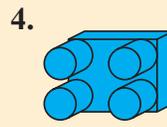
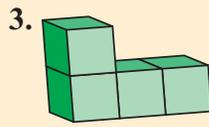
$$\text{Surface area} = 2(16 \times 2) + 2(16 \times 8) + 2(8 \times 2) = 352 \text{ cm}^2$$

Small prism (actually a cube):

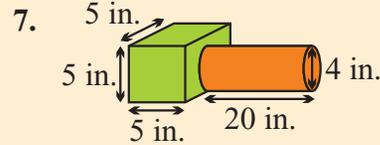
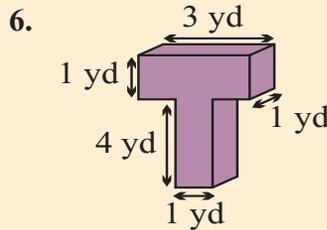
$$1 \times 1 \times 6 = 6 \text{ cm}^2$$

✓ Guided Practice

In Exercises 3–5, suggest how the complex figure could be split up into simple figures.



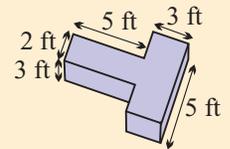
In Exercises 6–7, work out the surface area of each shape. Use $\pi = 3.14$.



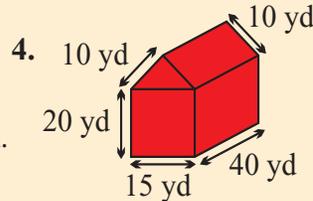
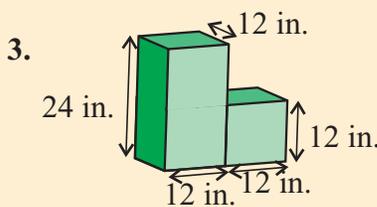
✓ Independent Practice

A kitchen work center is made from two rectangular prisms. It is to have a trim around the edge.

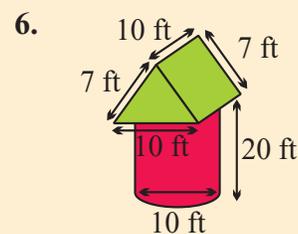
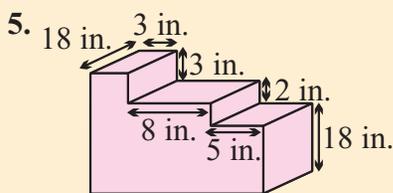
- Find the total edge length of each of the prisms separately.
- Find the length of trim needed for the work center.



Work out the surface areas of the shapes shown in Exercises 3–6. Use $\pi = 3.14$.



Total height of shape = 26.6 yd



Total height of shape = 24.9 ft

Don't forget:

The edges that “disappear” when the complex shape is formed are lost from both prisms.

Now try these:

Lesson 7.1.4 additional questions — p466

Round Up

So to find the *total edge length* of a complex shape, first break the shape up into *simple shapes*. Then you can find the edge length of each piece separately. But then you have to think about which edges “*disappear*” when the complex shape is made. The surface area of complex shapes is found the same way. Remember — the edge lengths and surface areas that “*disappear*” need to be subtracted *twice* — once from each shape.

Lesson 7.1.5

Lines and Planes in Space

California Standards:

Measurement and Geometry 3.6

Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

What it means for you:

This Lesson is all about the different ways planes and lines can be arranged in space.

Key words:

- coplanar
- skew lines
- perpendicular
- intersects

Imagine two endless, flat sheets of paper in space. Unless they are *parallel* to each other, they'll end up *meeting* sooner or later. *Planes* are similar to these endless sheets of paper — except they can pass through one another. This Lesson's about all the different ways that *lines* and *planes* can meet.

Planes and Lines Can Meet in Different Ways

Planes are **flat 2-D surfaces** in the **3-D world**. They go on forever.

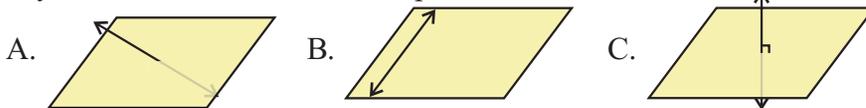
Lines are **1-D shapes in a 2-D plane**. They extend forever in both directions.

There are three ways for a line and a plane to **meet**:

1. The line might **rest on the plane**, so every point of the line is touching the plane.
2. The line might **pass through the plane** — so it **intersects** the plane.
3. The line might **intersect the plane at a right angle to it**. In this case, the line is **perpendicular** to the plane.

Example 1

Say how each line relates to the plane.

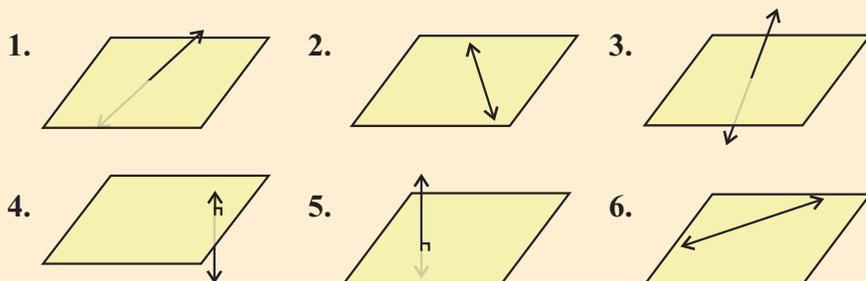


Solution

- A. **The line intersects the plane.**
Part of it is above the plane and part of it is below the plane.
- B. **The line rests on the plane.** Every point on the line touches the plane.
- C. **The line intersects the plane at right angles to it.**
So the line is **perpendicular** to the plane.

Guided Practice

In Exercises 1–6, say how each line relates to the plane.



Lines Can Be Coplanar or Skew

Two lines are **coplanar** if there is a plane that they both lie on — imagine trying to hold a giant piece of paper so that both lines lie on it. Lines that **intersect** or are **parallel** are always coplanar — there's always one plane on which they both lie.

If two lines neither intersect nor are parallel, they're **skew**. This means there's no plane that they both lie on.

Check it out:

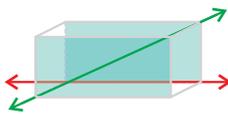
Parallel lines go on forever and never meet — they always have the same slope.



Example 2

Say whether each pair of lines is coplanar or skew. Explain your answers.

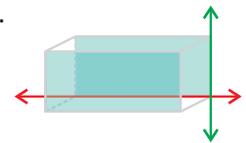
A.



B.



C.



Solution

A. **The lines never intersect and aren't parallel.** You can't find a single plane that both rest on. That means they aren't coplanar.

So they are skew lines.

B. **The lines are parallel. That means they are coplanar.**

Both lines rest on the plane at the back of the diagram.

C. **The lines intersect. That means they are coplanar.**

They both rest on the plane at the back of the diagram.

✓ Guided Practice

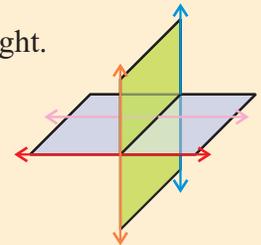
In Exercises 7–10, say whether each statement is true or false.

If it is false, explain why.

- All lines are either coplanar or skew.
- Skew lines can be drawn on the coordinate plane.
- Lines that intersect are always coplanar.
- Lines that don't intersect are never coplanar.

Exercises 11–12 are about the lines shown on the right.

- How many lines are skew with the blue line?
- How many lines are coplanar with the red line?



Planes Can Meet in Different Ways Too

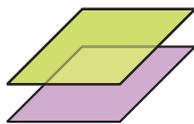
Two planes are **parallel** if they **never meet**.

If they do meet, then the two planes are **intersecting**. Intersecting planes look like one plane going through another. Planes are **perpendicular** if they make a **right angle** where they meet.

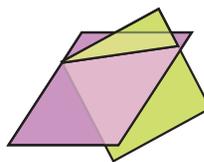
Example 3

Say whether the planes in each pair are parallel or intersecting.

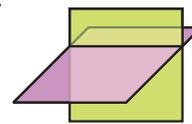
A.



B.



C.

**Solution**

A. **The planes are parallel.** They never touch.

B. **The planes intersect** — one goes through the other.

C. **The planes intersect** — one goes through the other. They make a right angle where they meet, so **they are perpendicular planes.**

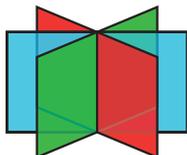
When two planes intersect, they always meet along a **line**.

When three planes all intersect at the same place, they can either meet along a **line** or at a **single point**.

Example 4

Say if each of these sets of planes intersect, and if so whether they meet along a line or at a point.

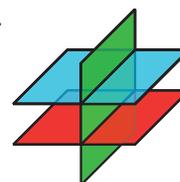
A.



B.



C.

**Solution**

A. **The planes all intersect. They meet along a line.**

The line is shown vertically down the middle of the picture.

B. **The planes all intersect. They meet at a point.**

The point is at the centre of the picture.

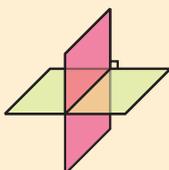
C. **The planes do not all intersect.**

The blue plane never intersects with the red plane (they're parallel).

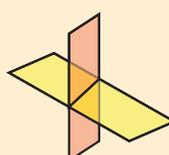
✓ Guided Practice

In Exercises 13–18, say whether the planes meet, and if so describe how.

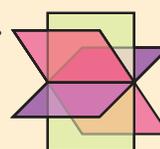
13.

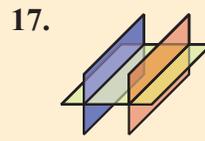
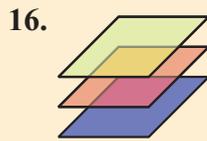


14.



15.





Fill in the missing words in Exercises 19–21.

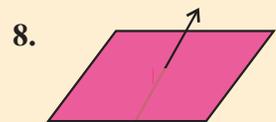
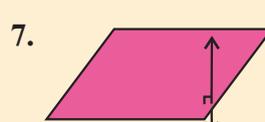
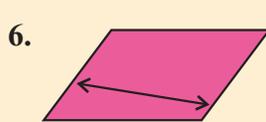
19. Planes that never touch are _____.
20. Two intersecting planes always meet along a _____.
21. Three intersecting planes meet either along a _____ or at a _____.

✓ Independent Practice

In Exercises 1–5, say whether each statement is true or false. If any are false, explain why.

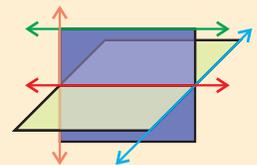
- When a line goes through a plane, it is said to be perpendicular to the plane.
- Skew lines never lie on the same plane.
- Two lines are coplanar if they both intersect the same plane.
- If two planes aren't parallel, then they meet along a line.
- Three planes always all meet each other at a line or a point unless they're all parallel to each other.

In Exercises 6–8, say how each line relates to the plane.



Exercises 9–11 are about the lines shown on the right.

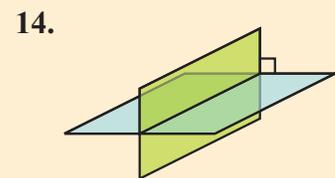
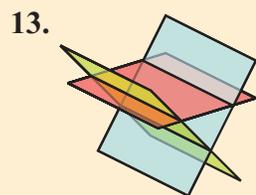
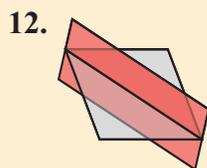
- How many lines are coplanar with the red line?
- How many lines are coplanar with the green line?
- How many lines are skew to the blue line?



Now try these:

Lesson 7.1.5 additional questions — p467

In Exercises 12–14, identify whether the planes meet and if so say how.



Round Up

It's hard to imagine the 2-D drawings in 3-D with lines and planes that go on forever. You've got to try to visualize the planes and lines intersecting in space — then you'll be most of the way there.

Section 7.2 introduction — an exploration into: Build the Best Package

When companies design packaging, they often aim to *maximize volume* and *minimize the amount of material used*, so that they spend as little money on it as possible. In this Exploration you'll be asked to design the *best package* when given specific volume requirements.

To find the **amount of material** needed to make a box, you calculate its **surface area**.

Example

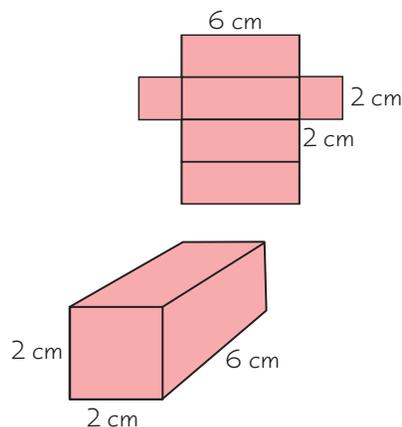
The net on the right can be folded to make a rectangular prism. Calculate the **surface area** and **volume** of the prism.

Solution

The **surface area** is the total area of all the faces. There are four 6 cm by 2 cm faces, and two 2 cm by 2 cm faces. So, **surface area** = $4 \times (6 \times 2) + 2 \times (2 \times 2) = 48 + 8 = 56 \text{ cm}^2$.

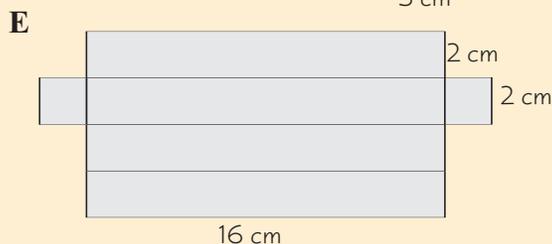
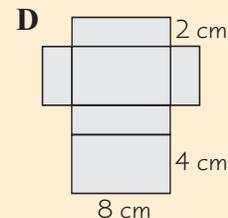
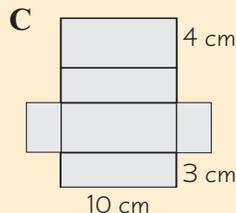
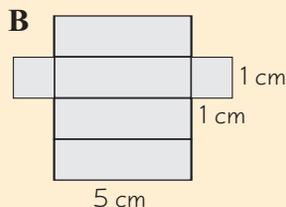
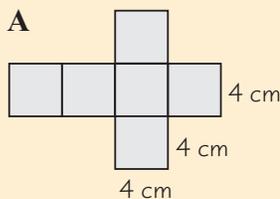
The net folds up to make the rectangular prism on the right.

Volume = length \times width \times height
= $6 \times 2 \times 2 = 24 \text{ cm}^3$



Exercises

- Calculate the volumes and surface areas of the rectangular prisms that can be made from these nets. Record your calculations in a copy of the table below.



Rectangular Prism	Surface Area	Volume
A		
B		
C		
D		
E		

- Is it possible to build packages with the same volume using different amounts of material? Explain your answer.
- Design and construct a box that has a volume of 24 cm^3 and uses the least material possible. The length, width, and height of the package must be whole numbers.

Round Up

Volumes and surface areas of rectangular prisms depend on the dimensions — they don't always increase together. You can change volume without changing surface area, and vice-versa.

Lesson 7.2.1

Section 7.2 Volumes

California Standards:

Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Mathematical Reasoning 2.2
Apply strategies and results from simpler problems to more complex problems.

Mathematical Reasoning 3.2
Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

What it means for you:

You'll be reminded what volume is, and then see how you can work out the volume of prisms and cylinders.

Key words:

- volume
- cubic units
- prism
- cylinder

Don't forget:

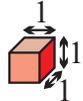
Cubic units can be written as unit^3 — so cubic yards can be written as yd^3 . You multiply three length measurements, such as length, width, and height, to get a volume, so that's why volume is measured in unit^3 .

The *volume* of a 3-D object, like a box, a swimming pool, or a can, is a measure of the *amount of space* that's contained inside it. Volume is measured in units like cubic feet (ft^3) or cubic centimeters (cm^3). This Lesson, you'll learn how to find the volume of *prisms and cylinders*.

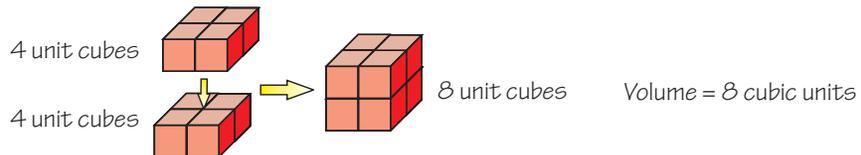
Volume Measures Space Inside a Figure

The amount of **space inside** a 3-D figure is called the **volume**.

Volume is measured in **cubic units**. One cubic unit is the volume of a **unit cube** — a cube with a side length of 1 unit.

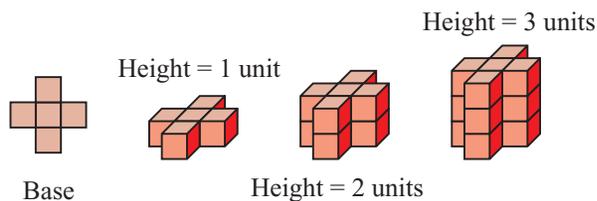


The **number of unit cubes** that could fit inside a solid shape and fill it completely is the volume in cubic units.



The Volume of a Prism is a Multiple of its Base Area

You can work out the volume of a **prism** from the **area of its base**.



The base is made of **5 unit squares**. So it has an **area** of 5 square units.

When the prism's height is 1 unit, it has a **volume** of 5 cubic units because it would take 5 unit cubes to make it.

When the prism's height is 2 units, it has a **volume** of 10 cubic units because it would take 10 unit cubes to make it.

When the prism's height is 3 units, it has a volume of 15 cubic units.

Every time you **increase the height** by 1 unit you **add** an extra 5 unit cubes.

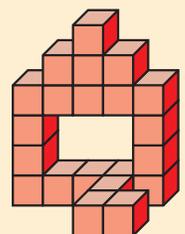
✓ Guided Practice

1. The figure on the right is constructed from unit cubes. What is its volume?

A prism is one yard high. It has volume 4 yd^3 .

2. What is the area of the prism's base?

3. A prism with an identical base has a volume of 16 yd^3 . How tall is this prism?



Area Formulas Help Work Out the Volume of a Prism

When you count the number of unit cubes that make a shape, you find the **number of unit cubes that make up the base layer**, and **multiply it by the height in units**.

You can't always count the number of unit cubes that are inside a shape, because **not all shapes can fit an exact number of unit cubes inside them**. Instead you can work out the volume of any prism by **multiplying the area of the base** by the **height**.

$$\text{Volume of prism} = \text{Base area} \times \text{Height}$$

Example 1

What is the volume of this prism?

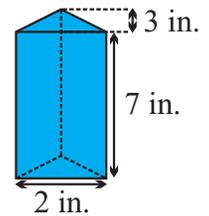
Solution

The base of this prism is a triangle. So use the area of a triangle formula to work out its area.

$$\text{Area of base} = \frac{1}{2}bh = \frac{1}{2} \times 2 \times 3 = 3 \text{ in}^2.$$

Then just multiply that area by the height of the prism.

$$\text{Volume of prism} = \text{base area} \times \text{height} = 3 \times 7 = 21 \text{ in}^3.$$



Don't forget:

The height you use to work out the area of the base is the height of the base triangle. The height you're using when you work out the volume of the prism is the height of the prism itself.

It doesn't matter if the prism looks like it is **lying down** — the same method of finding volume can still be used.

Example 2

What is the volume of this prism?

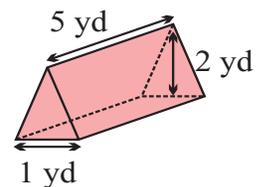
Solution

Treat the triangle as the base of the prism, and the length of 5 yards as the height.

The base is always the shape that is the same through the entire prism.

$$\text{Area of base} = \frac{1}{2}bh = \frac{1}{2} \times 1 \times 2 = 1 \text{ yd}^2.$$

$$\text{So, volume of prism} = \text{base area} \times \text{height} = 1 \times 5 = 5 \text{ yd}^3.$$

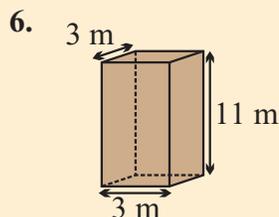
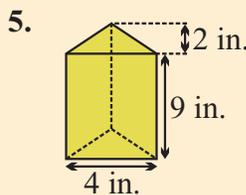
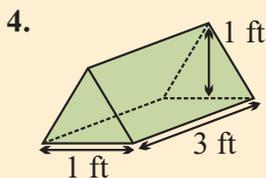


Check it out:

You've done dimensional analysis before — it's where you check that the units of your answer match the units you should get. When you're figuring out volumes, the answer should always be in units cubed — like cm^3 , m^3 , ft^3 , or yd^3 .

✓ Guided Practice

Work out the volumes of the figures in Exercises 4–6.



Find the Volume of a Cylinder in the Same Way

Circular cylinders are similar to prisms — the only difference is that the base is a **circle** instead of a **polygon**. So you can work out the volumes of cylinders in the same way as the volumes of prisms — by **multiplying the base area by the height**.

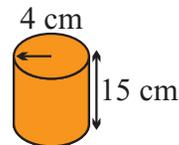
You use the **area of a circle formula** to get the base area of a cylinder.

Don't forget:

Make sure you use the radius of the circle in the area formula and not the diameter. If you're given the diameter, you need to halve it to find the radius first of all.

Example 3

What is the volume of this cylinder? Use $\pi = 3.14$.



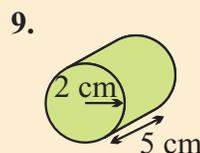
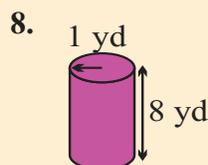
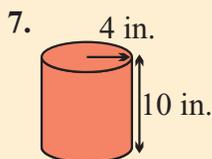
Solution

$$\text{Area of base} = \pi r^2 = \pi \times 4^2 = \pi \times 16 = 50.24 \text{ cm}^2.$$

$$\text{Height} = 15 \text{ cm, so volume of cylinder} = 50.24 \times 15 = 753.6 \text{ cm}^3.$$

✓ Guided Practice

Work out the volumes of the figures in Exercises 7–9. Use $\pi = 3.14$.



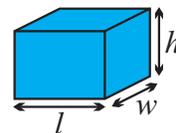
Rectangular Prisms and Cubes are Special Cases

The area of the base of a rectangular prism is **length (l)** \times **width (w)**.

If you multiply that by height to get the volume then you get

$$\text{Volume} = \text{length } (l) \times \text{width } (w) \times \text{height } (h)$$

$$V (\text{rectangular prism}) = lwh$$

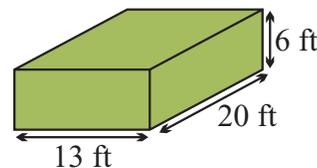


Example 4

What is the volume of this rectangular prism?

Solution

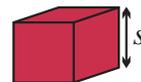
$$\text{Volume} = lwh = 13 \times 20 \times 6 = 1560 \text{ ft}^3.$$



Check it out:

It doesn't matter which dimensions you use as width, height, or length — you can multiply them in any order.

All sides of a cube are **the same length**. For a cube with side length s , the base area is $s \times s = s^2$, and the height is also s , so the volume is $s^2 \times s = s^3$.



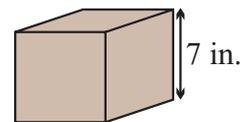
$$V(\text{cube}) = s^3 \quad \text{where } s \text{ is the side length.}$$

Example 5

What is the volume of this cube?

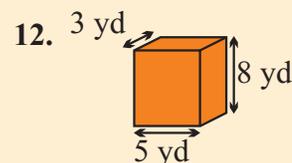
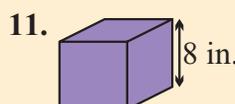
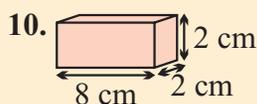
Solution

$$\text{Volume} = s^3 = 7^3 = 343 \text{ in}^3.$$



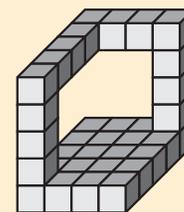
Guided Practice

Work out the volumes of the figures in Exercises 10–12. Figures with only one side length shown are cubes.



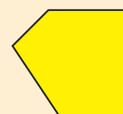
Independent Practice

1. The figure on the right is constructed from cubes with a volume of 1 in^3 . What is its volume?

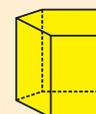


2. How many unit cubes can you fit inside a figure with dimensions 3 units \times 3 units \times 5 units?

3. What is the volume of the prism shown on the right?



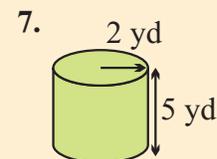
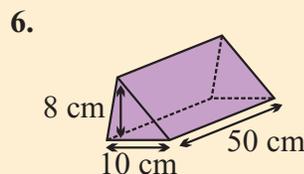
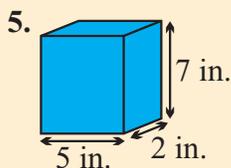
Base Area = 500 ft^2



Height = 70 ft

4. A cylinder of volume 32 in^3 is cut in half. What is the volume of each half?

Work out the volumes of the figures shown in Exercises 5–7. Use $\pi = 3.14$.



8. What is the volume of a cube with side length 3 yd?

Now try these:

Lesson 7.2.1 additional questions — p467

Round Up

Volume is the amount of space inside a 3-D figure, and it's measured in cubed units. For cylinders and prisms, you can multiply the base area by the height of the shape to find the volume.

Lesson 7.2.2

Graphing Volumes

California Standards:

Algebra and Functions 3.2

Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

What it means for you:

You'll plot graphs to show how volume changes as the dimensions of different shapes change. You'll also use graphs to estimate the dimensions of shapes with given volumes.

Key words:

- approximately
- volume

Don't forget:

\approx means "is approximately equal to." You can use it when you know that the answer is near to a number but you don't know exactly what the answer is.

In Section 5.4, you saw the graphs of $y = nx^2$ and $y = nx^3$. When you graph volume against the side length of a cube, or against the radius of a cylinder, you get these types of graphs too.

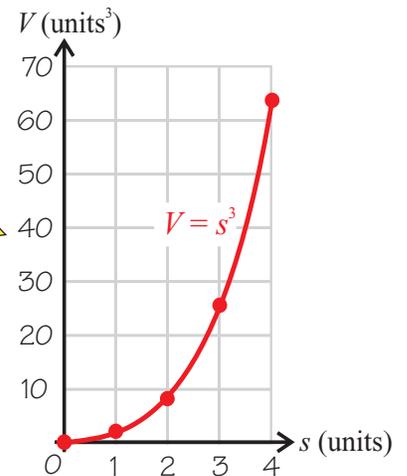
The Volume of a Cube Makes an x^3 Graph

If you **increase the side length** of a cube, the cube's **volume increases**. You can **plot a graph** to see exactly how volume changes with side length.

The volume of a cube with sides of length s is given by $V = s^3$. The first step in plotting a graph is **making a table of values**.

s (units)	V (units ³)
1	1
2	8
3	27
4	64

Plot these points on a graph — put the s -values on the x -axis. Join the points with a smooth curve.



The graph shows that V goes **steeply upward** as s gets bigger — it's the $y = x^3$ graph that you've seen before.

The graph can be used to **find the volume** of other size cubes without doing any multiplication.

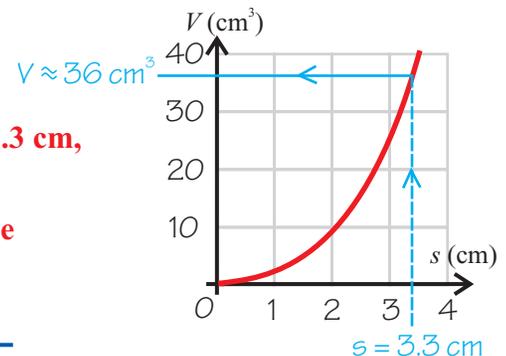
Example 1

Using the graph, approximately what is the volume of a cube with side length 3.3 cm?

Solution

Reading off the graph, **when $s = 3.3$ cm, $V \approx 36$ cm³.**

So **the volume of a cube with side length 3.3 cm is approximately 36 cm³.**



You can also use the graph to find the side length of a cube if you know the volume.

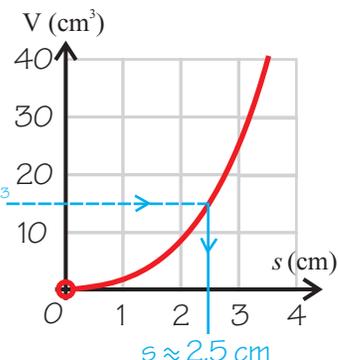
Example 2

What is the side length of a cube with volume 15 cm^3 ?

Solution

Reading from the graph,
when $V = 15 \text{ cm}^3$, $s \approx 2.5 \text{ cm}$.

So, a cube with volume 15 cm^3 $V = 15 \text{ cm}^3$
has a side length of
approximately 2.5 cm .



Guided Practice

1. If volume was plotted against side length, with side length along the x -axis, explain how you would find the volume of a cube of side 4 m.
2. A cube has a volume of 20 ft^3 . Use the graph of volume against side length to find the approximate length of each edge of the cube.

You Can Graph the Volume of Prisms and Cylinders

You can graph the volumes of **prisms** and **cylinders** as **one of their dimensions**, such as height, **changes**. The rest of the dimensions have to be **kept the same**.

Example 3

Graph the volume against the radius of cylinders of height $\frac{1}{2} \text{ cm}$.

Solution

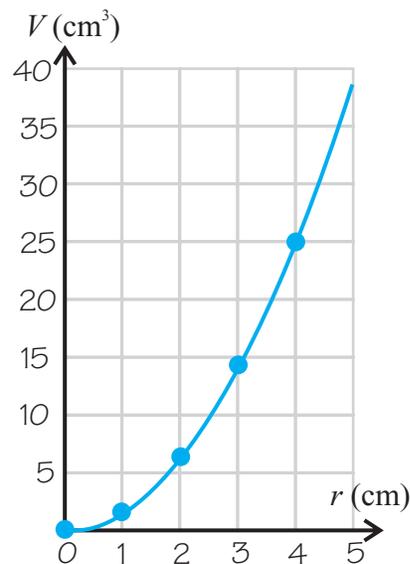
The volume of a cylinder is $V = \pi r^2 h$.

So **cylinders which are $\frac{1}{2} \text{ cm}$ high**
have the volume $\frac{1}{2} \pi r^2$.

Make a table of values to plot:

radius (cm)	volume (cm^3)
1	1.57
2	6.28
3	14.13
4	25.12

Since the radius is squared in the formula for volume, you get a $y = nx^2$ graph — which is a parabola.



Don't forget:

See Lesson 5.4.1 to remind yourself about graphing $y = nx^2$ equations.

Check it out:

Volume graphs are only drawn for positive values — you can't have negative lengths or volumes.

You can **use the graph** to find the **volume of a cylinder** of a given height if you know the radius, and you can **find the radius** if you know the volume.

Example 4

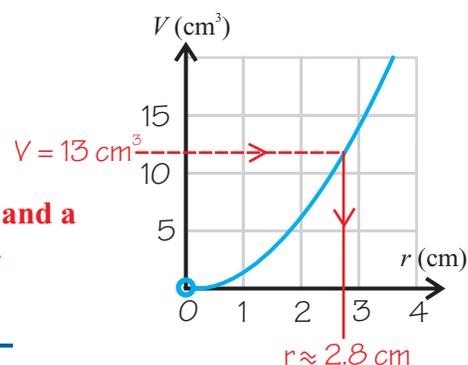
What radius of cylinder with height $\frac{1}{2}$ cm has a volume of 13 cm^3 ?

Solution

Use the graph from Example 3.

Reading from the graph, **when**
 $V = 13 \text{ cm}^3$, $r \approx 2.8 \text{ cm}$.

So, **a cylinder with height $\frac{1}{2}$ cm and a volume of 13 cm^3 has a radius of approximately 2.8 cm.**



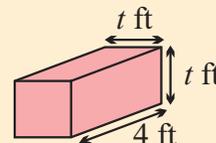
✓ Guided Practice

3. Graph volume against radius for cylinders with height $\frac{2}{\pi}$ units.

4. Use your graph from Exercise 3 to estimate the radius of a cylinder with height $\frac{2}{\pi}$ centimeters that has a volume of 30 cubic centimeters.

5. Graph the volume against t for the figure shown.

6. Use your graph from Exercise 5 to estimate the value of t that makes a volume of 48 cubic feet.



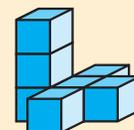
✓ Independent Practice

1. Graph volume against height for prisms with base area 6 units².

2. Graph volume against radius for cylinders with height 3 units.

3. Use your graph from Exercise 1 to find the approximate height of a prism that has a volume of 10.5 cm^3 and a base area of 6 cm^2 .

4. The building on the right is constructed from 7 cubes. Each cube has a side length of s inches. Graph the volume of the building against s . Use the graph to find the side length of cubes needed for a building of volume 36 cubic inches.



Now try these:

Lesson 7.2.2 additional questions — p468

Round Up

If you *increase the length* of one of the dimensions of a 3-D figure, you *increase its volume*. You can use a graph to show the relationship between the *length of a dimension* and the *volume*.

Next Lesson you'll learn about *similar solids* — these are solids of different sizes, which have each of their dimensions in proportion with the *corresponding* dimensions on the other solids.

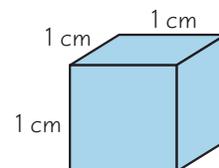
Section 7.3 introduction — an exploration into: Growing Cubes

Cubes have a length, width, and height that are equal. When you change these dimensions, the volume and surface area also change. The goal of this Exploration is to find out if changing the dimensions by a given amount produces a predictable change in the volume and surface area.

To investigate this, you have to find the **volumes** and **surface areas** of some cubes of **different sizes**.

Example

A cube with a length, width, and height of **1 centimeter** is shown on the right. Calculate its **surface area** and **volume**.



Solution

The **surface area** is the total area of all the faces.

There are six 1 cm by 1 cm faces. So, **surface area** = $6 \times (1 \times 1) = 6 \text{ cm}^2$.

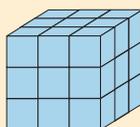
$$\begin{aligned} \text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 1 \times 1 \times 1 = 1 \text{ cm}^3 \end{aligned}$$

Exercises

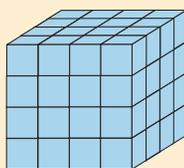
- Build the following with centimeter cubes. Calculate the surface area and volume of each and record them in a copy of the table shown.



$2 \times 2 \times 2$
cube



$3 \times 3 \times 3$
cube



$4 \times 4 \times 4$
cube

Cube dimensions (cm)	Scale Factor	Volume (cm ³)	Surface area (cm ²)
original — $1 \times 1 \times 1$	1	1	6
$2 \times 2 \times 2$	2		
$3 \times 3 \times 3$	3		
$4 \times 4 \times 4$	4		

- What is the volume of the original $1 \times 1 \times 1$ cm cube multiplied by if the cube is enlarged by:
 - a scale factor of 2?
 - a scale factor of 3?
 - a scale factor of 4?
- How are the scale factor and the number the original volume is multiplied by connected?
- What is the surface area of the original cube multiplied by if the cube is enlarged by:
 - a scale factor of 2?
 - a scale factor of 3?
 - a scale factor of 4?
- What is the connection between the scale factor and the number the original surface area is multiplied by?
- Predict the new volume and surface area if the original cube is enlarged by a scale factor of 5.
- Check your predictions by calculating the volume and surface area of a $5 \times 5 \times 5$ cm cube.

Round Up

*When the dimensions of a cube are increased, the **surface area** and **volume** always get bigger. The pattern's **more complicated** than a linear change though. It involves **square and cube numbers**.*

Lesson 7.3.1

Section 7.3 Similar Solids

California Standards:

Measurement and
Geometry 1.2

Construct and read
drawings and models made
to scale.

What it means for you:

You'll learn about using scale factors to change the size of solid figures.

Key words:

- similar
- congruent
- corresponding
- solid
- scale factor
- image

Don't forget:

For more about scale factors, and similar and congruent figures, look back at Lessons 3.4.3 and 3.4.6.

Applying a scale factor makes an *image* of a shape that is a *different size* from the original — you saw this with 2-D shapes in Chapter 3. You can also use scale factors with 3-D shapes to produce *similar solids* of different sizes.

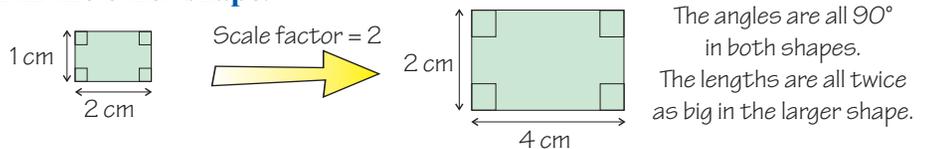
Scale Factors Produce Similar Figures

You've looked at the effect of scale factors on 2-D shapes. You're going to review this before seeing how scale factors affect 3-D shapes.

Two shapes are **similar** if one can be multiplied by a **scale factor** to make a shape that is **congruent** to the other one.

Two shapes are *congruent* if they're exactly the same — if all the corresponding sides and angles are equal.

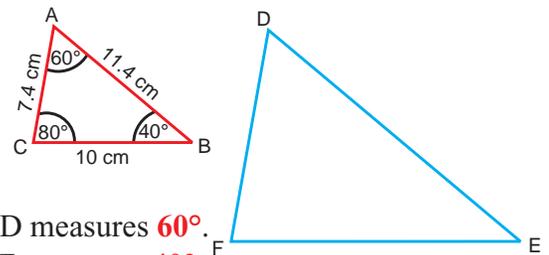
Similar shapes have **corresponding angles that are equal**. Also, **any length in one shape is equal to the scale factor times the corresponding length in the other shape**.



Example 1

Triangle ABC is multiplied by a scale factor of 3 to make triangle DEF.

Find the measures of all the sides and angles of triangle DEF. The triangles are not drawn to scale.



Solution

D corresponds to A, so angle D measures 60° .

E corresponds to B, so angle E measures 40° .

F corresponds to C, so angle F measures 80° .

To find the side lengths of DEF, multiply the side lengths of ABC by the **scale factor**.

$$\text{length of DE} = 3 \times (\text{length of AB}) = 3 \times 11.4 \text{ cm} = \mathbf{34.2 \text{ cm}}$$

$$\text{length of EF} = 3 \times (\text{length of BC}) = 3 \times 10 \text{ cm} = \mathbf{30 \text{ cm}}$$

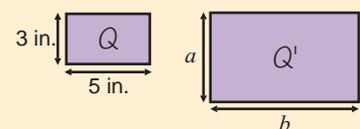
$$\text{length of DF} = 3 \times (\text{length of AC}) = 3 \times 7.4 \text{ cm} = \mathbf{22.2 \text{ cm}}$$

Guided Practice

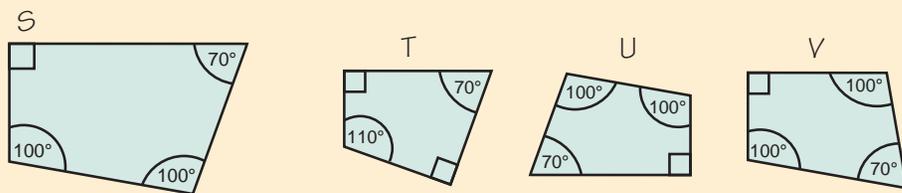
Rectangle Q is multiplied by a scale factor of 4 to give the image Q'.

1. How long is the side marked *a*?

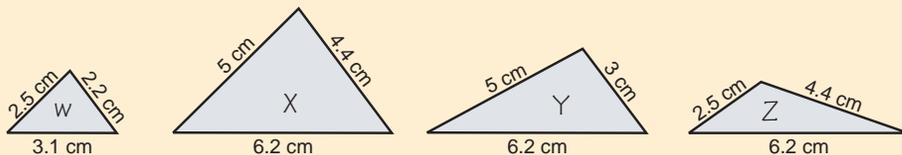
2. How long is the side marked *b*?



3. Which of the quadrilaterals below is similar to S: T, U, or V?



4. Which triangle below is similar to W: X, Y, or Z?



3-D Figures Can Be Multiplied by Scale Factors

If you multiply a **3-D figure** by a scale factor you get a **similar figure**.

Just like with 2-D shapes, **corresponding angles** in similar 3-D shapes have **equal measures**. And like 2-D shapes, you multiply any **length** in the original by the **scale factor** to get the corresponding length in the **image**.

$$\text{length in image} = \text{scale factor} \times \text{length in original}$$

Don't forget:

When you transform any figure, including changing the size by a scale factor, the new figure is called the image. The image of A is often called A' (read as "A prime").

Check it out:

You could also have used the corresponding lengths 3 cm and 9 cm to find the scale factor.

Example 2

A and A' are similar. Find x .

Solution

First you need to find the **scale factor**. The lengths in the **original** are multiplied by the **scale factor** to get the lengths in the **image**.

The length of 15 cm in A' corresponds to the length of 5 cm in A.

Rearranging the formula above gives:

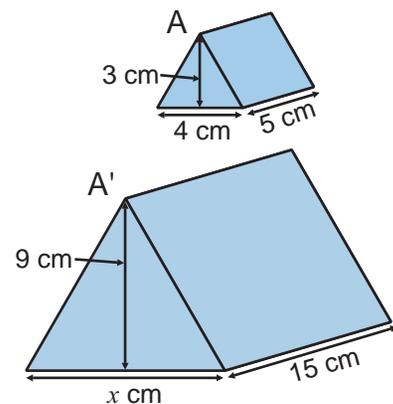
$$\begin{aligned} \text{Scale factor} &= \text{length in A}' \div \text{length in A} \\ &= 15 \text{ cm} \div 5 \text{ cm} \\ &= 3 \end{aligned}$$

The length of x cm in A' corresponds to the length of 4 cm in A.

Length in A' = scale factor \times length in A

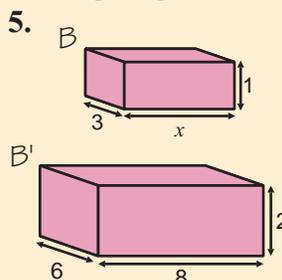
$$x \text{ cm} = 3 \times 4 \text{ cm}$$

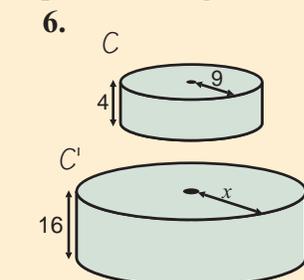
$$x = 12$$

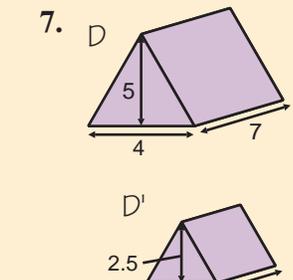


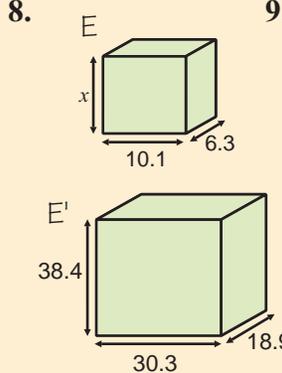
✓ Guided Practice

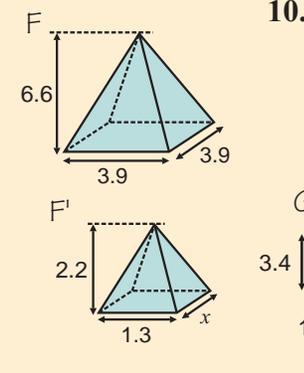
Use the equation to find the scale factor that has been used to produce the image in each of the following pairs of similar solids. Find the missing length, x , in each pair. All lengths are measured in cm.

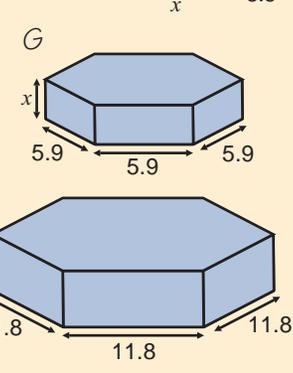
5. 

6. 

7. 

8. 

9. 

10. 

You Can Check Whether Two Solids are Similar

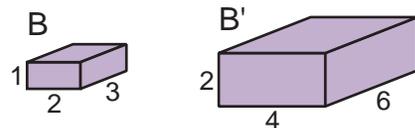
You need to check that all the lengths have been multiplied by the **same scale factor**, and that the **corresponding angles are the same**.

Example 3

Are these rectangular prisms similar?

Solution

If the figures are similar, then all the lengths will have been multiplied by the same scale factor.



$$\text{scale factor} = \text{length in image} \div \text{length in original}$$

Start with the shortest edge length: $2 \div 1 = 2$

Then the next shortest edge: $4 \div 2 = 2$

And last, the longest edge: $6 \div 3 = 2$

The scale factors are all the same, and all the angles are 90° in each, so **the figures are similar**.

B has been enlarged by **scale factor 2** to make the image B'.

Check it out:

Be careful when you use this formula to check if figures are similar. You must make sure that you are comparing corresponding lengths.

✓ Guided Practice

In each set of solids below, say which of the numbered figures is similar to the first figure, and what scale factor created the image. The figures are not drawn to scale. All lengths are measured in cm.

11.

12.

13.

✓ Independent Practice

Copy the following sentences and fill in the missing words:

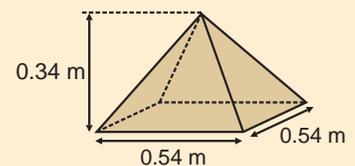
1. A scale _____ of 1 produces an image that is the same _____.
2. The _____ will be _____ than the original if the scale factor is between 0 and 1.
3. If the scale factor is _____ 1, the image is bigger than the _____.

Mr. Freeman's history class made a model pyramid, shown below. The class makes a second pyramid, which is twice the size of the first.

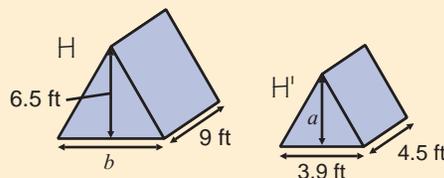
- 4.** What is the height of the second model?

The class builds a third model, which is 3 times the size of the second one.

5. What is the height of the third model?
6. How long is the base of the third model?
7. What can you say about the angles in all three models?



8. What scale factor has been used to produce the image H' from prism H?
9. What is the length marked a ?
10. What is the length marked b ?



Now try these:

Lesson 7.3.1 additional questions — p468

Round Up

Applying *scale factors to solid shapes* changes their size — and therefore their *surface area* and *volume*. You'll see exactly how *surface area* and *volume* are affected by scale factors next Lesson.

Lesson 7.3.2

Surface Areas & Volumes of Similar Figures

California Standards:

Measurement and Geometry 1.2

Construct and read drawings and models made to scale.

Measurement and Geometry 2.1

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Measurement and Geometry 2.3

Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

What it means for you:

You'll find out the effect that applying scale factors has on the surface area and volume of a 3-D figure.

Key words:

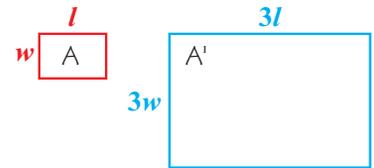
- scale factor
- area
- volume
- square
- cube

You've seen already how applying a scale factor affects the perimeter and area of 2-D figures. Well, when you multiply a solid figure by a scale factor, the surface area and the volume are changed. It's not surprising really — bigger shapes have bigger surface areas and volumes. This Lesson looks at exactly how they change.

Area is Multiplied by the Square of the Scale Factor

When a 2-D figure is multiplied by a scale factor, the area gets multiplied by the square of the scale factor.

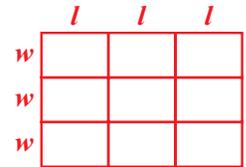
Here, rectangle A is enlarged by a scale factor of 3 to give the image A'.



The area of A is lw .

The area of A' is $3l \times 3w = 9lw$.

The side lengths in the image are 3 times greater than the original, but the area of the image is $3 \times 3 = 9$ times greater.

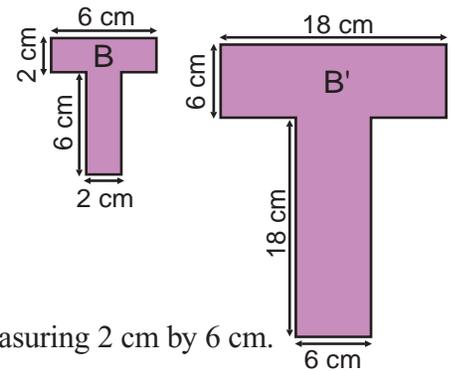


Example 1

Figure B is multiplied by a scale factor of 3 to produce the image B'.

Find the area of figure B.

Use the area of B and the scale factor to find the area of B'.



Solution

B is made up of 2 rectangles, each measuring 2 cm by 6 cm.

So the area of B is: $2 \times (2 \times 6) = 2 \times 12 = 24 \text{ cm}^2$

The area of B' is: $\text{area of B} \times (\text{scale factor})^2$
 $= 24 \text{ cm}^2 \times 3^2$
 $= 24 \text{ cm}^2 \times 9 = 216 \text{ cm}^2$

Check: Area of B' = $2 \times (6 \times 18) = 2 \times 108 = 216 \text{ cm}^2$ — Correct

Guided Practice

Ignacio draws a figure with an area of 5 in^2 . Find the area of the image if Ignacio multiplies his figure by the following scale factors:

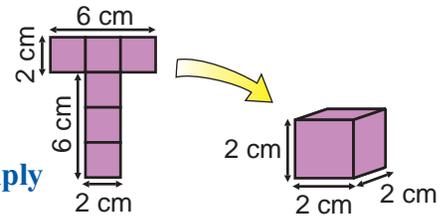
1. 2
2. 10
3. 6
4. $\frac{1}{2}$

Surface Area is Multiplied by the Scale Factor Squared

Figure B from Example 1 is the **net** of a **2 cm cube**. The **surface area of the cube** is the same as the **area of the net**.

The image B' is the net of a **6 cm cube**.

A 6 cm cube is what you get if you **multiply** a 2 cm cube by a **scale factor of 3**.



So, if you multiply a cube by a scale factor, the **surface area** is multiplied by the **square** of the **scale factor**. This is true for the surface area of **any solid**.

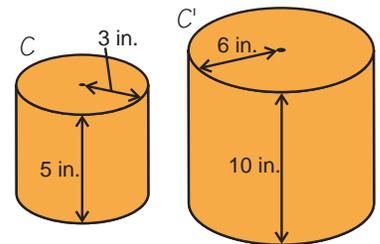
Example 2

Cylinder C' is an enlargement of cylinder C by a scale factor of 2.

Find the surface area of cylinder C.

Use the surface area of C and the scale factor to find the surface area of C'.

Use $\pi = 3.14$.



Solution

First find the **surface area of C**:

$$\begin{aligned} \text{Area} &= (2 \times \text{base area}) + \text{lateral area} \\ &= (2 \times \pi r^2) + (2\pi r \times h) \\ &= (2 \times 3.14 \times 3^2) + (2 \times 3.14 \times 3 \times 5) \\ &= 56.52 + 94.2 \\ &= \mathbf{150.72 \text{ in}^2} \end{aligned}$$

To find the surface area of C', just multiply the surface area of C by the **square of the scale factor**:

$$A = 150.72 \times 2^2 = 150.72 \times 4 = \mathbf{602.88 \text{ in}^2}$$

You can calculate the surface area of cylinder C' to **check** the answer:

$$\begin{aligned} A &= (2 \times \pi r^2) + (2\pi r \times h) \\ &= (2 \times 3.14 \times 6^2) + (2 \times 3.14 \times 6 \times 10) \\ &= 226.08 + 376.8 = \mathbf{602.88 \text{ in}^2} \text{ — Correct} \end{aligned}$$

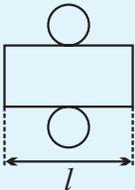
Don't forget:

The formula for surface area of a cylinder in Example 2 uses the formulas for area and circumference of a circle.

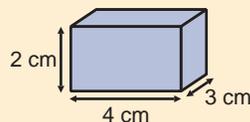
$$\begin{aligned} \text{They are: } A &= \pi r^2 \\ C &= 2\pi r \end{aligned}$$

where r is the circle's radius.

Remember — the length of the rectangle (l) that is used to find the "lateral area" is equal to the circumference of the base circles. See Lesson 7.1.3.



Guided Practice



5. What is the surface area of the prism on the left?

Find the surface area of the image if this figure is multiplied by:

6. Scale factor 3

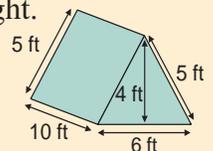
7. Scale factor 4

8. Find the surface area of the triangular prism on the right.

Calculate the surface area of the image if this figure is multiplied by:

9. Scale factor 2

10. Scale factor $\frac{1}{2}$



Volume is Multiplied by the Cube of the Scale Factor



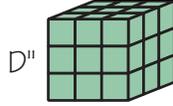
Cube D is a **unit cube**. Its volume is **1 cubic unit**.



D is multiplied by **scale factor 2** to get D'.

Cube D' has a volume of **8 cubic units**.

Multiplying by scale factor 2 increases the volume of D by **$2^3 = 8$ times**.



D is multiplied by **scale factor 3** to get D''.

The volume of D'' is **27 cubic units**.

Multiplying by scale factor 3 increases the volume of D by **$3^3 = 27$ times**.

When you multiply any solid figure by a scale factor, the **volume** is multiplied by the **cube of the scale factor**.

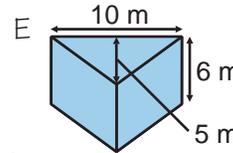
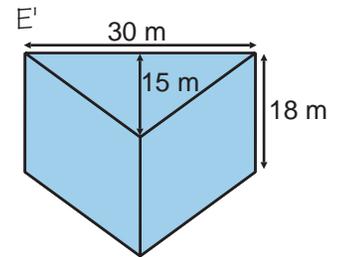
This works for **any** solid figure.

Example 3

Prism E' is an enlargement of prism E by a scale factor of 3.

Find the volume of E.

Use the volume of E and the scale factor to find the volume of E'.



Solution

First find the **volume of E**:

$$\begin{aligned}
 V &= \text{area of base} \times \text{height} \\
 &= \left(\frac{1}{2} \times \text{base of triangle} \times \text{height of triangle}\right) \times \text{height of prism} \\
 &= \left(\frac{1}{2} \times 10 \times 5\right) \times 6 \\
 &= 25 \times 6 = \mathbf{150 \text{ m}^3}
 \end{aligned}$$

To find the volume of E', multiply the volume of E by the **cube of the scale factor**:

$$V = 150 \times 3^3 = 150 \times 27 = \mathbf{4050 \text{ m}^3}$$

You can calculate the volume of E' to **check** the answer:

$$\begin{aligned}
 V &= \left(\frac{1}{2} \times \text{base of triangle} \times \text{height of triangle}\right) \times \text{height of prism} \\
 &= \left(\frac{1}{2} \times 30 \times 15\right) \times 18 \\
 &= 225 \times 18 = \mathbf{4050 \text{ m}^3} \text{ — Correct}
 \end{aligned}$$

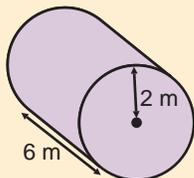
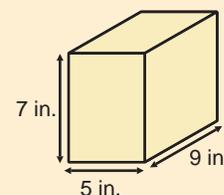
✓ Guided Practice

11. What is the volume of this prism?

Find the volume of the image if this figure is multiplied by:

12. Scale factor 2

13. Scale factor 3



14. Calculate the volume of this cylinder.

Use 3.14 for π .

Find the volume of the image if this figure is multiplied by:

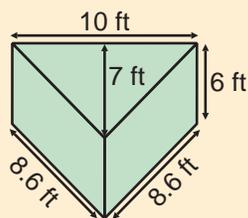
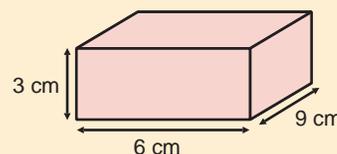
15. Scale factor 2

16. Scale factor $\frac{1}{3}$

✓ Independent Practice

Find:

1. The surface area of this prism.
2. The surface area of the image, if the prism is multiplied by a scale factor of 2.
3. The volume of this prism.
4. The volume of the image, if the prism is multiplied by a scale factor of 2.



Calculate:

5. The surface area of this prism.
 6. The surface area of the image, if the prism is multiplied by a scale factor of 3.
 7. The volume of this prism.
 8. The volume of the image, if the prism is multiplied by a scale factor of 3.
9. The surface area of a cube is 480 cm^2 . The cube is enlarged by a scale factor of k . What is the surface area of the new cube?
10. A pyramid has a volume of 24 cm^3 . If you double all the dimensions of the pyramid, what will be the volume of the new pyramid?
11. A scale factor enlargement of a cylinder is produced. The surface area of the image is 9 times the surface area of the original. The base of the original cylinder has a radius of 5 in. What is the radius of the base of the image?
12. A scale model of a building has a surface area of 6 ft^2 . If the real building has a surface area of 600 ft^2 , what scale factor has been used to make the model?

Now try these:

Lesson 7.3.2 additional questions — p469

Round Up

One way to remember how *scale factor* affects *surface area* and *volume* is to think about the *units* you use to measure them. *Surface area* is given in *square units* like m^2 or in^2 , so the scale factor is *squared*. *Volume* is given in *cubic units* like cm^3 or ft^3 , so the scale factor is *cubed*.

Lesson 7.3.3

Changing Units

California Standards:

Measurement and
Geometry 2.4

Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1 \text{ ft}^2] = [144 \text{ in}^2]$, 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in}^3] = [16.38 \text{ cm}^3]$).

What it means for you:

You'll learn how to convert between different units of area and volume.

Key words:

- area
- volume
- square
- cube

Don't forget:

For a reminder of how to use conversion factors, see Lesson 4.2.3.

Check it out:

Another way to do this conversion is to multiply the area measurement by a conversion fraction equal to 1. This method involves dimensional analysis and was explained in Lesson 4.3.4.

$$500 \text{ cm}^2 \times \frac{1 \text{ m}^2}{10,000 \text{ cm}^2} = \frac{500}{10,000} \text{ m}^2 = 0.05 \text{ m}^2$$

conversion fraction

You wouldn't measure the area of a country in cm^2 — km^2 would be a better unit to use. Similarly, you wouldn't measure the volume of a die in m^3 . That's why this Lesson's about *converting units of area and volume* — so you can keep your answers in a *sensible range*.

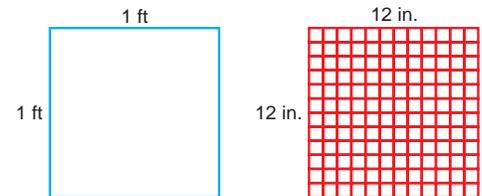
You Can Convert Between Units of Area

You've seen that: **1 foot = 12 inches**

So 1 foot is 1 inch multiplied by a **scale factor of 12**.

If you multiply a **1 inch square** by scale factor 12, you get a **1 foot square**.

$$\begin{aligned} \text{So the area} &= 1 \text{ ft}^2 \\ &= 1 \text{ in}^2 \times 12^2 \\ &= 1 \text{ in}^2 \times 144 \\ &= 144 \text{ in}^2 \end{aligned}$$



You can do the same with metric units: **1 meter = 100 centimeters**

So 1 m is 1 cm multiplied by a **scale factor of 100**.

So: **$1 \text{ m}^2 = 1 \text{ cm}^2 \times 100^2 = 1 \text{ cm}^2 \times 10,000 = 10,000 \text{ cm}^2$**

$1 \text{ m}^2 = 10,000 \text{ cm}^2$ is a **conversion factor**, which you can use to change m^2 to cm^2 , or cm^2 to m^2 .

Example 1

A cube has a surface area of 500 cm^2 . What is this surface area in m^2 ?

Solution

$1 \text{ m}^2 = 10,000 \text{ cm}^2$, so the ratio of cm^2 to m^2 is $10,000 : 1$ or $\frac{10,000}{1}$.

Write a proportion where there are $x \text{ m}^2$ to 500 cm^2 : $\frac{10,000}{1} = \frac{500}{x}$

Cross multiply and solve for x : $10,000 \times x = 500 \times 1$
 $10,000x = 500$
 $x = 500 \div 10,000 = 0.05$

The surface area of the cube is **0.05 m^2** .

Guided Practice

In Exercises 1–3, convert the areas to cm^2 .

1. 3 m^2 2. 0.25 m^2 3. 1.8 m^2

In Exercises 4–6, convert the areas to ft^2 .

4. 72 in^2 5. 864 in^2 6. 252 in^2

7. Luisa makes a scale drawing of a park. The drawing has an area of 240 in^2 . What is the area of the drawing in ft^2 ?

8. TJ buys 1.2 m^2 of fabric to make a Halloween costume. What is this area in cm^2 ?

You Can Convert Between Metric and Customary Units

You can convert areas between the metric and customary systems:

$$1 \text{ in} = 2.54 \text{ cm}$$

So $1 \text{ in}^2 = 2.54 \text{ cm} \times 2.54 \text{ cm} = 6.45 \text{ cm}^2$

$$1 \text{ ft} = 0.3 \text{ m}$$

So $1 \text{ ft}^2 = 0.3 \text{ m} \times 0.3 \text{ m} = 0.09 \text{ m}^2$

Example 2

A cube has a surface area of 500 cm^2 . What is this surface area in in^2 ?

Solution

$1 \text{ in}^2 = 6.45 \text{ cm}^2$, so the ratio of cm^2 to in^2 is $6.45 : 1$ or $\frac{6.45}{1}$.

Write a proportion where there are $x \text{ in}^2$ to 500 cm^2 : $\frac{6.45}{1} = \frac{500}{x}$

Cross multiply and **solve** for x : $6.45 \times x = 500 \times 1$
 $6.45x = 500$
 $x = 500 \div 6.45 = 77.52$

The surface area of the cube is 77.52 in^2 .

Check the reasonableness: 1 in^2 is around 6.5 cm^2 , and $77.52 \text{ in}^2 \approx 80 \text{ in}^2$. $80 \times 6.5 = 520$, so **the answer is reasonable**.

Don't forget:

You should always check the reasonableness of your answers — this just means checking it's about the right size.

Don't forget:

There are two ways you could do Exercise 15 — either convert ft^2 to m^2 first, then convert m^2 to cm^2 , or convert ft^2 to in^2 then in^2 to cm^2 . You can do Exercise 16 in two ways also. You'll get slightly different answers depending on which way around you do it — due to rounding errors.

Guided Practice

In Exercises 9–11, convert the following areas to in^2 :

9. 12.9 cm^2 10. 225 cm^2 11. 92 cm^2

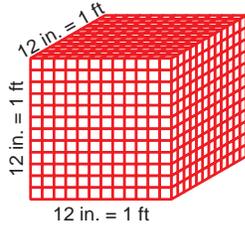
In Exercises 12–14, convert the following areas to m^2 :

12. 5 ft^2 13. 132 ft^2 14. 66.7 ft^2

15. What is 1 ft^2 in cm^2 ?

16. What is 1 m^2 in in^2 ?

You Can Also Convert Units of Volume



If you multiply a **1 inch cube** by **scale factor 12**, you get a **1 foot cube**.

$$\text{So, } 1 \text{ ft}^3 = 1 \text{ in}^3 \times 12^3 = 1 \text{ in}^3 \times 1728 = 1728 \text{ in}^3$$

In the same way, if you multiply a **1 cm cube** by **scale factor 100**, you get a **1 m cube**.

$$\text{So, } 1 \text{ m}^3 = 1 \text{ cm}^3 \times 100^3 = 1 \text{ cm}^3 \times 1,000,000 = 1,000,000 \text{ cm}^3$$

Example 3

A cylinder has a volume of 4 ft^3 . What is this volume in in^3 ?

Solution

$1 \text{ ft}^3 = 1728 \text{ in}^3$, so the ratio of in^3 to ft^3 is $1728 : 1$ or $\frac{1728}{1}$.

Write a proportion where there are $x \text{ in}^3$ to 4 ft^3 : $\frac{1728}{1} = \frac{x}{4}$

Cross multiply and **solve** for x : $1728 \times 4 = x \times 1$
 $6912 = x$

The volume of the cylinder is **6912 in^3** .

Check the reasonableness: There are about 1700 in^3 in 1 ft^3 .
 $1700 \times 4 = 6800$, so **the answer is reasonable**.

You Can Also Convert Volume Units Between Systems

You can convert volumes between the metric and customary systems:

$$1 \text{ in} = 2.54 \text{ cm}$$

$$\text{So } 1 \text{ in}^3 = 2.54 \text{ cm} \times 2.54 \text{ cm} \times 2.54 \text{ cm} = 16.39 \text{ cm}^3$$

$$1 \text{ ft} = 0.3 \text{ m}$$

$$\text{So } 1 \text{ ft}^3 = 0.3 \text{ m} \times 0.3 \text{ m} \times 0.3 \text{ m} = 0.027 \text{ m}^3$$

Example 4

A cylinder has a volume of 4 ft^3 . What is this volume in m^3 ?

Solution

$1 \text{ ft}^3 = 0.027 \text{ m}^3$, so the ratio of m^3 to ft^3 is $0.027 : 1$ or $\frac{0.027}{1}$.

Write a proportion where there are $x \text{ m}^3$ to 4 ft^3 : $\frac{0.027}{1} = \frac{x}{4}$

Cross multiply and **solve** for x : $0.027 \times 4 = x \times 1$
 $0.108 = x$

The volume of the cylinder is **0.108 m^3** .

Check the reasonableness: There are about 0.03 m^3 in 1 ft^3 .
 $0.03 \times 4 = 0.12$, so **the answer is reasonable**.

✓ Guided Practice

In Exercises 17–20, convert the areas to cm^3 .

17. 10 m^3 18. 21.5 m^3 19. 8 in^3 20. 14.3 in^3

In Exercises 21–24, convert the areas to ft^3 .

21. 2592 in^3 22. 1000 in^3 23. 135 m^3 24. 5.8 m^3

25. A building takes up a space of $20,000 \text{ m}^3$.

What is this volume in cm^3 ?

26. What is 1 ft^3 in cm^3 ?

27. What is 1 m^3 in in^3 ?

Check it out:

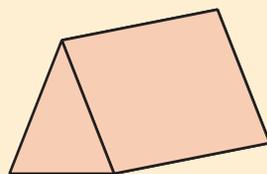
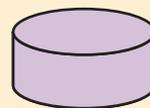
There are two ways you could do Exercises 26 and 27.

For Exercise 26, either convert from ft^3 to in^3 and then to cm^3 , or do ft^3 to m^3 and then to cm^3 . You'll get different answers depending on which way around you do it — due to rounding errors.

✓ Independent Practice

A cylinder has a surface area of $126\pi \text{ cm}^2$.

1. What is this surface area in m^2 ?
2. What is this surface area in in^2 ?



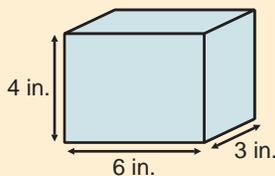
A prism has a volume of 2.85 ft^3 .

3. What is this volume in in^3 ?
4. What is this volume in m^3 ?

Karen makes a scale model of her school for a project.

The model has a surface area of 376 in^2 and a volume of 480 in^3 .

5. What is the surface area of the model in ft^2 ?
6. What is the volume of the model in cm^3 ?
7. Julio measures the area of one wall of his bedroom as 35.25 ft^2 . What is the area of the wall in square inches?
8. The volume of Brandy's suitcase is $184,800 \text{ cm}^3$. What is this volume in m^3 ?



This rectangular prism is shown with measurements in inches.

9. What is its surface area in square feet?
10. What is its volume in cubic feet?
11. An acre is 4840 square yards. What is an acre in square feet?

Now try these:

Lesson 7.3.3 additional questions — p469

Don't forget:

A yard is 3 feet.

Round Up

Don't forget that when you deal with *conversion factors*, you should always *check* that your answer is *reasonable*. If you convert from *small units* to *large units*, the number of units will *decrease*. If you convert from *large units* to *small units*, the number of units will *increase*.

Chapter 7 Investigation

Set Design

In a play or musical, the actors act on a stage that is often decorated with furniture and scenery. This is called the *set*. The process of *designing a set* begins with an idea, and then the set designer creates a *scale model* of it for the director to see before the real set is built.

Here is a designer's sketch for the layout of a stage set. The stage has a table in the shape of a **rectangular prism** which also serves as a bench and a couch. There are also **two cylinders** that serve as stools or tables.

Part 1:

Make a scale drawing of the set on $\frac{1}{4}$ inch by $\frac{1}{4}$ inch grid paper. Use **1 inch to represent 2 feet** (four small squares are 1 inch).

What is the **scale factor** of the actual objects to the drawing?

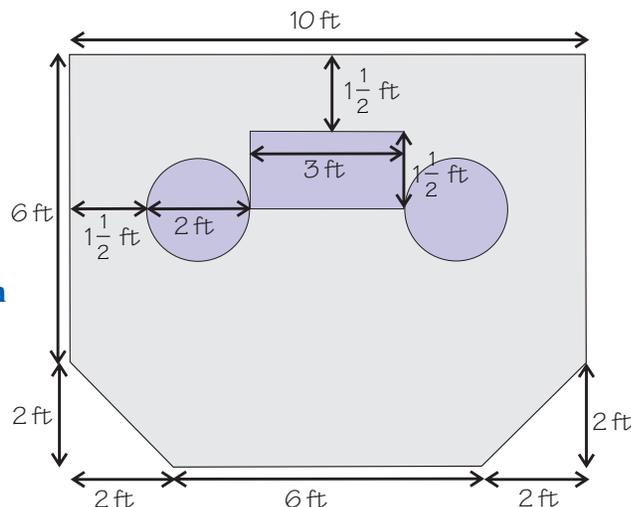
Part 2:

The rectangular prism and the cylinders each have a height of $1\frac{1}{2}$ feet.

- 1) Determine the **volumes** of the rectangular prism and the cylinders. Use $\pi = 3.14$.
- 2) Determine the **surface areas** of the rectangular prism and the cylinders.
- 3) Instead of a scale drawing, a set designer would show the director a three-dimensional model.

Suppose a model was made of this set using the scale factor in Part 1.

What would be the **surface areas** and **volumes** of the model rectangular prism and cylinders?



Extensions

- 1) Draw this set with a scale of 1 inch to 1 foot. How do your answers to Part 2 change?
- 2) The entire stage is raised by a platform that exactly fits the stage. The model of it is 3 inches high, when using a scale of 1 inch to 1 foot. What is the volume of the actual platform?

Open-ended Extensions

- 1) Your school probably has a stage for theatricals. Measure the dimensions of the stage and create a scale drawing of it.
- 2) Using your measurements of the stage platform, calculate its surface area and volume. Now create a model of the stage using the same scale as you did for your drawing. Calculate your model's surface area and volume, and use these measurements to calculate the volume and surface area of the actual stage. How do your answers compare?

Round Up

Scale models are used in real life to give you a good idea of what something will look like. It's a lot easier and less expensive to build a small model than to build the real thing. So if you don't like what you see, it's less of a problem to change it after only building a scale model.

Chapter 8

Percents, Rounding, and Accuracy

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Section 8.1 introduction — an exploration into: Photo Enlargements

You can *enlarge* or *reduce* photos — you have to increase or decrease the width and the length by the *same percent* though, or your image will be *stretched*. In this Exploration you'll figure out the *dimensions* a photo would have if it were enlarged or reduced by a given percent.

Example

A photograph that is 4 inches wide and 6 inches long is enlarged by 10%. What are its new dimensions?

Solution

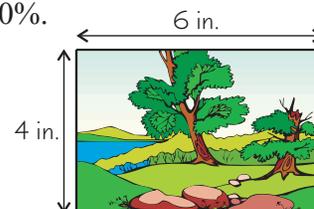
Find **10% of the length** and **10% of the width**, then add these to the original dimensions.

$$10\% \text{ of } 6 \text{ in} = \frac{10}{100} \times 6 = 0.6 \text{ in.}$$

$$10\% \text{ of } 4 \text{ in} = \frac{10}{100} \times 4 = 0.4 \text{ in.}$$

$$\text{New length} = 6 + 0.6 = 6.6 \text{ in.}$$

$$\text{New width} = 4 + 0.4 = 4.4 \text{ in.}$$



There's **another way** of doing this:

The original photo is 100%. Increasing it by 10% makes it **110%** of the original size. Find 110% of the length and 110% of the width. These are the new dimensions.

$$\text{New length} = 110\% \text{ of } 6 \text{ in} = \frac{110}{100} \times 6 = 6.6 \text{ in.}$$

$$\text{New width} = 110\% \text{ of } 4 \text{ in} = \frac{110}{100} \times 4 = 4.4 \text{ in.}$$

Exercises

Use the photograph that you have brought to class for the following exercises.

- Measure the length and width of the photograph in inches. Write the dimensions in the first row of a copy of the table below.
- Calculate the length and width of the photograph when it is enlarged by 20% and 50%, and reduced by 50% and 25%.

Write the dimensions in your copy of the table.

	Length	Width
100% — original size		
Enlarged by 20%		
Enlarged by 50%		
Reduced by 50%		
Reduced by 25%		

Round Up

When you *enlarge* or *reduce* a photo, you have to keep the dimensions in *proportion*. So if you increase the length by 20%, you have to increase the width by 20% too. The longer dimension will increase by *more inches* than the shorter one does. That's what *percent change* is all about.

Lesson 8.1.1

Section 8.1 Percents

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

What it means for you:

You'll see what percents are and how they're related to fractions and decimals.

Key words:

- percent
- fraction
- decimal
- hundredth

Check it out:

On a penny you'll see the words one cent because a penny is one-hundredth of a dollar.

You hear percents used a lot in everyday life. You might score 83% on a test, or a store might have a 20% off sale. A *percent* is really just a way to write a fraction — it tells you how many *hundredths* of a number you have.

Percents Tell You How Many Hundredths You Have

A percent is a way to write a **fraction** as a **single number**.

It tells you how many **hundredths** of something you have.

The word percent means **out of 100**.

Writing **one percent** or **1%** is the same as writing $\frac{1}{100}$,

and writing 10% is the same as writing $\frac{10}{100}$.

Decimals can also be written as percents. The decimal 0.01 means “1 hundredth,” so it's the same as 1%. There's more on converting decimals to percents next lesson.

Example 1

In a box of 100 pencils, 26 are blue. What percent of the pencils are blue?

Solution

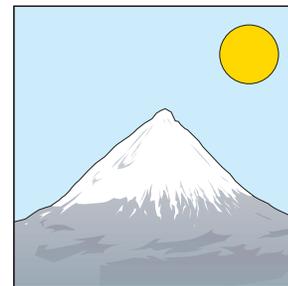
The fraction of pencils that are blue is $\frac{26}{100}$.

So you can say that **26% of the pencils are blue**.

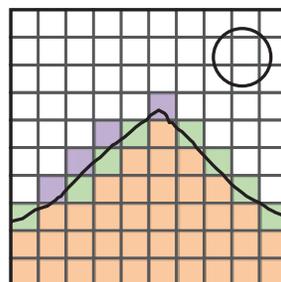
It's useful to be able to visually estimate a percent.

Example 2

Estimate what percent of the picture on the right is covered by the mountain.



Solution



Trace the outline of the picture onto tracing paper. Draw a 10×10 grid over the tracing.

Count the number of squares the mountain covers. It covers **37 whole squares**, **8 half squares** and **4 quarter squares**.

$$37 + (0.5 \cdot 8) + (0.25 \cdot 4) = 42 \text{ squares.}$$

The grid has 100 squares. So **the mountain covers about 42% of the picture**.

Don't forget:

For a reminder on how to turn fractions into decimals see Section 2.1.

✓ Guided Practice

In Exercises 1–3, write each fraction as a decimal and a percent.

1. $\frac{5}{100}$

2. $\frac{25}{100}$

3. $\frac{62}{100}$

In Exercises 4–6, write each percent as a fraction in its simplest form.

4. 1%

5. 50%

6. 20%

In Exercises 7–9, draw a 10 by 10 square. Shade in the given percent.

7. 8%

8. 27%

9. 100%

Percents Can Be Greater Than 100

You can also have percents that are **bigger than 100**.

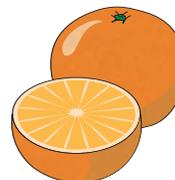
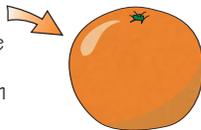
In the same way that $\frac{1}{100}$ is 1%, $\frac{150}{100}$ is 150%.

And just as **0.01** is the same as **1%**, **1.5** is the same as **150%**.

Percents bigger than 100 leave you with **more** than the **original number**.

Look at these oranges:

This is one whole orange. That's the same as $\frac{100}{100}$ of an orange, or 100% of an orange.



This is one and a half oranges. That's the same as $\frac{100}{100} + \frac{50}{100} = \frac{150}{100}$ of an orange, or 150% of an orange.

Guided Practice

In Exercises 10–12, write each fraction as a percent.

10. $\frac{120}{100}$

11. $\frac{200}{100}$

12. $\frac{1200}{100}$

In Exercises 13–15, write each decimal as a percent.

13. 1.4

14. 3.6

15. 22.0

To Find a Percent of a Number You Need to Multiply

You already know that to find a fraction of a number, you **multiply** the number by the **fraction**.

Finding a **percent** of a number means finding a fraction out of 100 of the number.

Example 3

What is 25% of 160?

Solution

Write out the percent as a fraction: $25\% = \frac{25}{100}$

$$\begin{aligned} \frac{25}{100} \times 160 &= \frac{4000}{100} \\ &= 40 \end{aligned}$$

Multiply the fraction by the number

Simplify the answer

Don't forget:

To multiply a fraction by an integer, just multiply the numerator of the fraction by the integer.

Finding the Original Amount — Write an Equation

Sometimes, you'll know how much a certain percentage of a number is and want to find the **original amount**.

Example 4

25% of a number is 40. What is the number?

Solution

Write out the percent as a fraction: $25\% = \frac{25}{100}$

Call the number that you're finding x .

$$\frac{25}{100} \times x = 40$$

Multiply both sides by 100.

$$25x = 4000$$

Divide both sides by 25.

$$x = 160$$

40 is 25% of 160.

Check it out:

$\frac{25}{100}$ is the same as $\frac{1}{4}$. If you used $\frac{1}{4}$ here, you'd get exactly the same answer.

Guided Practice

Find:

16. 10% of 40

17. 60% of 250

18. 64% of 800

In Exercises 19–21, find the value of x .

19. 50% of x is 30

20. 4% of x is 7

21. 65% of x is 130

22. Pepe was chosen as president of his class. He got 75% of the votes, and his class has 28 members. How many people voted for Pepe?

23. The school basketball team won 60% of their games this season. If they won 24 games, how many did they play altogether?

Independent Practice

In Exercises 1–4, write the fraction as a percent.

1. $\frac{10}{100}$

2. $\frac{50}{100}$

3. $\frac{23}{100}$

4. $\frac{156}{100}$

In Exercises 5–8, write the percent as a fraction in its simplest form.

5. 25%

6. 17%

7. 75%

8. 150%

9. Out of 6000 nails made, 2% were faulty. How many were faulty?

10. 150% of the people who were expected turned up at the school fair. If 340 people were expected, how many came?

11. 20% of the students riding a bus are from Town A. If 6 students on the bus are from Town A, how many students ride the bus in total?

12. 80 students auditioned for a play. After the audition, 20% were asked to come to a 2nd audition. 50% of those who came to the 2nd audition were cast. How many were cast? What percent of the original 80 is this?

Now try these:

Lesson 8.1.1 additional questions — p470

Round Up

Percents say how many hundredths of something you have. You find a percent of a number by converting the percent to a fraction, and then multiplying this fraction by the number.

Lesson 8.1.2

Changing Fractions and Decimals to Percents

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

What it means for you:

You'll see how to change fractions and decimals into percents.

Key words

- percent
- decimal
- fraction
- hundredth

Don't forget:

To multiply a decimal by 100 you just move the decimal point two places to the right.

Don't forget:

1 can be rewritten as 1.00.

Changing a *fraction* or a *decimal* into a percent is all about working out how many *hundredths* there are in it. The number of hundredths is always *the same* as the percent.

Changing Decimals to Percents

1% is the same as the **decimal 0.01** — which is one hundredth.

Each 1% is **one hundredth**. So the **number of hundredths** in the decimal is **the same** as the number of the percent.

$$0.\underline{04} = 4 \text{ hundredths} = 4\%$$

$$0.5 = 0.\underline{50} = 50 \text{ hundredths} = 50\% \quad \text{The first two digits after the decimal point tell you the number of hundredths.}$$

$$0.\underline{31} = 31 \text{ hundredths} = 31\%$$

$$0.\underline{025} = 2.5 \text{ hundredths} = 2.5\% \quad \text{Any numbers that come after that are parts of 1\%.}$$

So to rewrite any **decimal** as a **percent**, you need to **multiply** it by **100** and add a **percent symbol**.

Example 1

Write 0.25 and 3.12 as percents.

Solution

$$0.25 \cdot 100 = 25, \text{ so } 0.25 = \mathbf{25\%}$$

$$3.12 \cdot 100 = 312, \text{ so } 3.12 = \mathbf{312\%}$$

Multiply decimals by 100 to get the number of the percent

✓ Guided Practice

In Exercises 1–9 write each decimal as a percent.

1. 0.01

2. 0.5

3. 0.75

4. 0.23

5. 0.87

6. 1

7. 0

8. 2.5

9. 11.6

Changing Fractions to Percents

You've already seen that **1%** is the same as the fraction $\frac{1}{100}$. To change a **fraction** to a **percent** you need to work out how many **hundredths** are in it — because that's the same as the number of the percent.

Example 2

Write $\frac{3}{4}$ as a percent.

Solution

To write $\frac{3}{4}$ as a percent you need to change it to hundredths.

That means you want the denominator of the fraction to be 100.

So you need to multiply the top and bottom of the fraction by the number that will change the denominator to 100.

$$4 \times n = 100 \qquad n = \text{the number you need to multiply by}$$

$$n = 100 \div 4 = 25 \qquad \text{Divide both sides by 4}$$

Now turn the original fraction into a percent:

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\%$$

Example 3

Write $\frac{3}{8}$ as a percent.

Solution

This time the denominator of the fraction isn't a factor of 100.

So the fraction won't be a whole number of hundredths.

The most straightforward way of dealing with a fraction like this is to convert it into a decimal first.

$$\frac{3}{8} = 3 \div 8 = 0.375$$

Now change the decimal to a percent by multiplying it by 100:

$$0.375 \times 100 = 37.5, \text{ so } 0.375 = 37.5\%$$

✓ Guided Practice

In Exercises 10–18 write each fraction as a percent.

10. $\frac{1}{100}$

11. $\frac{100}{100}$

12. $\frac{27}{100}$

13. $\frac{1}{2}$

14. $\frac{2}{5}$

15. $\frac{3}{2}$

16. $\frac{7}{16}$

17. $\frac{1}{1000}$

18. $\frac{53}{400}$

Using Fractions in Real Life

Percentages are useful for reporting numbers. The meaning of 96% is usually easier for people to understand than the meaning of $\frac{585}{610}$.

Example 4

Tamika is a professional basketball player. She makes 552 out of 625 free throws in a season. What percent of her free throws does she make?

Solution

Tamika made 552 out of 625 free throws. Written as a fraction that's $\frac{552}{625}$.

625 isn't a factor of 100. So first turn the fraction into a decimal.

$$\frac{552}{625} = 552 \div 625 = 0.8832$$

Now change the decimal to a percent by multiplying it by 100:

$$0.8832 \times 100 = 88.32$$

Tamika made 88.32% of her free throws.

You could round this to **88%**.

Check it out:

In real-life questions, it's important to make sure that you get the right numbers in the numerator and the denominator of the fraction.

The phrase "out of" in the question often gives you a clue where the numbers should go — if it's "x out of y," then x is the numerator and y is the denominator. If there isn't an "out of" in the problem, try to reword it a bit so that there is. For example, in Guided practice Ex. 20, you could reword the problem to say, "15 out of 24 seventh graders go to camp."

✓ Guided Practice

19. In a set of napkins, 18 out of 24 are blue. What percent is this?
20. In a class of 24 7th graders, 15 go to camp. What percent is this?
21. James buys a ball of string that is 5 m long. Wrapping a parcel he uses a piece that is 0.8 m long. What percent of the string has he used?
22. A clothing manufacturer makes 3000 T-shirts. If 213 are returned because the color is wrong, what percent are the right color?

✓ Independent Practice

In Exercises 1–4, write each decimal as a percent.

1. 0.05 2. 0.2 3. 3.2 4. 0.235

5. Puebla and Mark are changing the decimal 0.5 into a percent.

Mark says $0.5 = 5\%$. Puebla says $0.5 = 50\%$. Who is correct?

In Exercises 6–9, write each fraction as a percent.

6. $\frac{6}{100}$ 7. $\frac{0}{100}$ 8. $\frac{1}{5}$ 9. $\frac{5}{32}$

10. Out of 50 dogs that are walked every day in the local park, 20 are labradors. What percent is this?

11. Mandy surveyed 96 seventh graders on their favorite cafeteria meal. 33 students responded "spaghetti bolognese." What percent is this?

12. Tyrone is saving up \$80 to buy some hockey skates. He has already saved \$47.20. What percent is this of the total that he needs?

Now try these:

Lesson 8.1.2 additional questions — p470

Round Up

Knowing how to change a *decimal* or a *fraction* to a *percent* will often come in handy. Percents are far easier to *compare* than fractions and decimals because they're all measured out of *one hundred*.

Lesson 8.1.3

Percent Increases and Decreases

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and **use these representations in estimations, computations, and applications.**

Number Sense 1.6

Calculate the percentage of increases and decreases of a quantity.

What it means for you:

You'll see how to use percents to show how much a quantity has gone up or down by.

Key words:

- percent
- increase
- decrease
- compare

Check it out:

"50 increased by 20%" is the same as "120% of 50." Both are 100% of 50, plus 20% of 50. So you could do this calculation by multiplying 50 by 1.2 — because 1.2 is the decimal equivalent of 120%.

Check it out:

Percents don't have any units. To find a percent you will always be dividing a number by another number that has the same units. So they cancel out.

When a number goes up or down, you can use percents to describe how much it has changed by. This can come in useful in real-life situations like comparing price rises or working out sale discounts.

You Can Increase a Number by a Given Percent

You can increase a number by a certain **percent** of itself. So, say if you want to increase a number by 10%, you have to **work out what 10% is**, then **add this to the original number**.

Example 1

Increase 50 by 20%.

Solution

First work out 20% of 50:

$$20\% \text{ of } 50 = \frac{20}{100} \times 50 = 0.2 \times 50 = 10$$

This is the amount that you need to **increase** 50 by:

$$50 + 10 = 60$$

So, 50 increased by 20% is 60.

Example 2

A photograph with a length of 14 cm is enlarged. This increases its length by 8%. What is the final length of the enlarged photograph?

Solution

First work out 8% of 14 cm:

$$8\% \text{ of } 14 \text{ cm} = \frac{8}{100} \times 14 \text{ cm} = 0.08 \times 14 \text{ cm} = 1.12 \text{ cm}$$

This is the amount that you need to **increase** 14 cm by:

$$14 \text{ cm} + 1.12 \text{ cm} = 15.12 \text{ cm}$$

The length of the enlarged photograph is 15.12 cm.

✓ Guided Practice

In Exercises 1–4, find the total after the increase.

1. 100 is increased by 10%
2. 20 is increased by 5%
3. 165 is increased by 103%
4. 40 is increased by 20.5%
5. Sarah goes out for lunch. Her bill comes to \$15. She wants to leave an extra 17% as a tip for the server. How much should Sarah leave in total?

You Can Describe an Increase as a Percent

When a number goes up, you can give the increase as a percent of the **original number**.

Example 3

A loaf of bread has 24 slices. As a special buy, a larger loaf is sold, which contains 27 slices. What is the percent increase in the number of slices?

Solution

First find the **increase** in the number of slices:

$$27 - 24 = 3$$

Call x the percent increase and write an equation.

$x\%$ of 24 is 3

$$\Rightarrow \frac{x}{100} \times 24 = 3$$

$$x \times 24 = 300$$

Multiply both sides by 100.

$$x = 12.5$$

Divide both sides by 24.

The number of slices has increased by 12.5%.

✓ Guided Practice

6. Reynaldo has 140 marbles. He buys 63 more. By what percent has he increased the size of his marble collection?

7. A company increases its number of staff from 1665 to 1998. What is this as a percent increase?

You Can Decrease a Number by a Given Percent Too

You can also **decrease** a number by a **percent** of itself.

Example 4

Decrease 80 by 15%.

Solution

First work out 15% of 80:

$$15\% \text{ of } 80 = \frac{15}{100} \times 80 = 0.15 \times 80 = 12$$

This is the amount you **decrease** 80:

$$80 - 12 = 68$$

So, 80 decreased by 15% is 68.

Check it out:

You could also do this by multiplying 80 by 0.85. 0.85 is the decimal equivalent of 85% — and finding 85% of a number is the same as decreasing it by 15%.

✓ Guided Practice

In Exercises 8–11, find the total after the decrease.

8. 100 is decreased by 15%

9. 40 is decreased by 35%

10. 37 is decreased by 8%

11. 10 is decreased by 3.9%

12. Tandi has saved \$152. She spends 25% of her savings on a shirt. How much does Tandi have left?

You Can Describe a Decrease as a Percent

When a number goes **down**, you can use a **percent** to describe how much it has changed by. The decrease is described as a percent of the **original number**.

Example 5

A river is 12.8 feet deep on January 1. By September 1, the depth has fallen to 9.6 feet. Find the percent decrease in the river depth.

Solution

First find the **amount** that the depth is **decreased** by:

$$12.8 \text{ feet} - 9.6 \text{ feet} = 3.2 \text{ feet}$$

Call x the percent decrease and write an equation.

$$x\% \text{ of } 12.8 \text{ feet is } 3.2 \text{ feet}$$

$$\Rightarrow \frac{x}{100} \times 12.8 \text{ feet} = 3.2 \text{ feet}$$

$$x \times 12.8 \text{ feet} = 320 \text{ feet} \quad \text{Multiply both sides by 100.}$$

$$x = 25$$

Divide both sides by 12.8 feet.

The river depth has decreased by 25%.

Check it out:

Make sure you find the percent of the **original amount**. 12.8 feet is decreased to 9.6 feet, so you find the change as a percent of 12.8 feet, which was the original amount.

Check it out:

There's a formula you can use to work out the percent increase or decrease:

$$\text{Percent change} = \frac{|\text{original amount} - \text{new amount}|}{\text{original amount}} \times 100$$

Absolute value is used so you can use the formula for either a percent increase or decrease.

✓ Guided Practice

Find the percent decreases in Exercises 13–14.

13. 90 is reduced to 81.

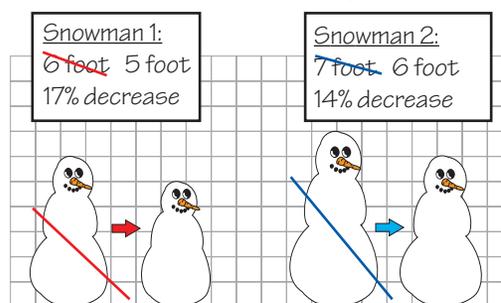
14. 4 is reduced to 3.5

15. Jon is selling buttons for a fund-raiser. He starts with 280 buttons and sells all but 21. What percent of his stock has Jon sold?

Use Percents to Compare Changes

You can use **percent increases** and **decreases** to **compare** how much two numbers have changed **relative to each other**. For example:

Snowman 1 and Snowman 2 have both lost the same amount in height as they've melted — 1 ft. But the **percent decrease** is greater for Snowman 1 — 1 ft is a bigger change **relative** to 6 ft than to 7 ft.



Example 6

In a store, a bagel is 40¢ and a loaf of bread is \$1.60. The store raises the price of both items by 5¢. Which has the larger percent increase in cost?

Solution

The price of both items is increased by 5¢.

So the percent increase in the cost of a bagel is:

$$\frac{x}{100} \times 40¢ = 5¢ \Rightarrow (5¢ \times 100) \div 40¢ = 12.5, \text{ so a } \underline{12.5\%} \text{ increase.}$$

And the percent increase in the cost of a loaf is:

$$\frac{x}{100} \times 160¢ = 5¢ \Rightarrow (5¢ \times 100) \div 160¢ = 3.125, \text{ so a } \underline{3.125\%} \text{ increase.}$$

 You need to have the original value and the increase in the same units. \$1.60 has been converted to 160¢ here.

The bagel shows the larger percent increase in cost.

Check it out:

In terms of actual cost both items increased by the same amount, 5¢. But the bagel has increased by a greater proportion of its original cost than the loaf.

✓ Guided Practice

16. Cindy has 250 baseball cards. Jim has 200 baseball cards. Both buy 50 extra cards. Whose collection increased by the larger percent?

17. Ava and Ian have a contest to see whose sunflower will increase in height by the greatest percent. Ava's starts 10 cm high and grows to 100 cm. Ian's starts 20 cm high and grows to 110 cm. Who won?

✓ Independent Practice

In Exercises 1–6, find the amount after the percent change.

1. Increase 200 by 25%
2. Decrease 200 by 75%
3. Increase 49 by 7%
4. Decrease 82 by 56%
5. Increase 50 by 142.6%
6. Decrease 80 by 33.2%

7. At store A, apples used to cost \$1.50 a pound. Then the price rose by 6%. What is the new cost of a pound of apples?

8. Kiona's brother Otis is 115.5 cm tall. The last time he was measured, his height was 110 cm. Find the percent increase in his height.

9. Last year, School C had 120 6th grade students. This year they have 5% fewer 6th graders. How many fewer students is this?

10. Mr. Hill's house rental costs \$900 a month. He moves to a house with a rental of \$828 a month. Find the percent decrease in his rental.

11. Duena collects comic books. 10 years ago, Comic A was worth \$70 and Comic B was worth \$40. Now Comic A is worth \$84 and Comic B is worth \$49. Which has shown the greater percent increase in value?

Now try these:

Lesson 8.1.3 additional questions — p470

Round Up

Percent increases and decreases tell you how big a change in a number is when you compare it to the original amount. It's useful to be able to work them out in real-life situations, especially when you're thinking about tips and discounts — and you'll learn more about them in Section 8.2.

Section 8.2 introduction — an exploration into: **What's the Best Deal?**

Discounts on sale items in stores are sometimes advertised as *percents off the original prices* and sometimes as *dollar amounts off the original prices*. By working out how much the percent discount is, you can find out which is the better deal among different sales.

You often won't have a calculator or pen and paper to hand when you want to work things like this out. So it's a good idea to develop strategies for calculating percent discounts **in your head**.

The easiest percents to find are **10%** and **50%**. From these you can find pretty much all the percent discounts that are commonly used in sales.

Example

A shirt is originally priced at \$20 and is on sale with 40% off. The shirt is also on sale for \$20 at a different store. You have a store coupon that you could use to save \$7.50 at this store. Which store will the shirt cost less at?



Solution

To answer this question, you can find 40% of \$20 and see if it's a bigger saving than \$7.50.

$$10\% \text{ of } \$20 = \$20 \div 10 = \$2$$

$$40\% = 4 \times 10\% = 4 \times \$2 = \$8 \quad \Rightarrow \text{ This is a bigger saving than } \$7.50.$$

So the shirt costs less at the first store (with 40% off).

Another way of doing this is to find 50% and 10%, then find 40% by subtracting the 10% amount from the 50% amount.

$$50\% \text{ of } \$20 = \$20 \div 2 = \$10 \text{ and } 10\% \text{ of } \$20 = \$20 \div 10 = \$2$$

$$\text{So } 40\% = 50\% - 10\% = \$10 - \$2 = \mathbf{\$8}$$

✓ Exercises

For each of the following Exercises explain how you found the percent discount.

1. A pair of sunglasses is originally priced at \$15 and is on sale in a store with 15% off. They are also on sale on an internet site for \$13. Which is the best deal?
2. Two stores are having a sale on the same video game originally priced at \$20. Store A has the game on sale for \$5 off. Store B has the game on sale for 30% off. Which store has the better deal?
3. A \$70 DVD player is on sale in two different stores. One store is selling the DVD player for 40% off. A second store is selling the player for \$35 off the original price. Which is the better deal?
4. A store has a side table, originally priced at \$55, with \$10 off. The same table can also be found on the internet for \$60 and is on sale for 25% off. Which is the better deal?

Round Up

\$20 off might sound like a bigger saving than 5%, and often will be. But not if you're buying something very expensive. So it definitely pays to be able to find percents without a calculator.

Lesson 8.2.1

Section 8.2 Discounts and Markups

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and **use these representations** in estimations, **computations**, and **applications**.

Number Sense 1.6

Calculate the percentage of increases and decreases of a quantity.

Number Sense 1.7

Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

What it means for you:

You'll see how percents are used in real life to figure out discounts and markups on things that are being sold.

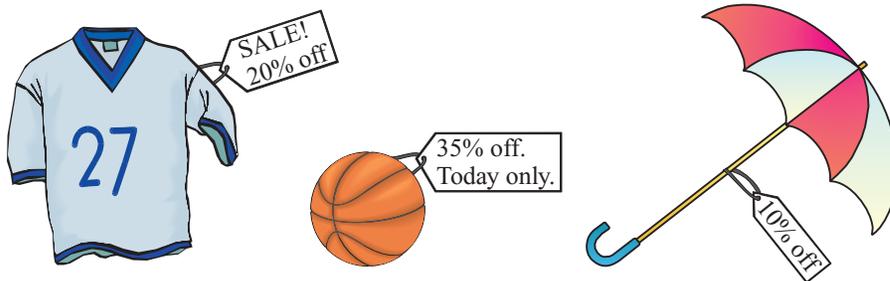
Key words:

- percent
- discount
- markup

In the last Section, you learned all about *percent increases and decreases*. In real life they're used all the time. One thing that they're used for is working out how much items will *cost* in stores — these price changes are known as *discounts* and *markups*. And that's what this Lesson is all about.

A Discount is a Percent Decrease

When you go shopping, you might see items that are **on sale** — they cost less than their **regular price**.



The **difference** between the **regular price** and the **sale price** is called the **discount**. Discounts are often given as **percents** of the **original value** — so they're examples of **percent decrease**.

Example 1

A skirt costing \$28 is on sale at 20% off. What is its sale price?

Solution

Write the percent of the discount as a fraction: $20\% = \frac{20}{100}$

Work out the amount of the discount:

$$\frac{20}{100} \times \$28 = 0.2 \times \$28 = \$5.60$$

Now subtract the amount of the discount from the original price:

$$\$28 - \$5.60 = \mathbf{\$22.40}$$

OR

The skirt has been discounted by 20%. This means that its sale price is $(100 - 20)\% = 80\%$ of the original price.

Write the percent as a fraction: $80\% = \frac{80}{100}$

Find the reduced price:

$$\frac{80}{100} \times \$28 = 0.8 \times \$28 = \mathbf{\$22.40}$$

Both methods give the **same answer**. You can use whichever one you find easier to remember.

✓ Guided Practice

1. A CD costing \$16 goes on sale at 25% off. What is its sale price?
2. A wheelbarrow has been marked at a discount of 35%. What percent of the original price is it on sale for?
3. An MP3 player retailing for \$90 has been marked down at 15% off. What is the sale price of the MP3 player?
4. A power tool that usually retails at \$52 is being sold for \$38.74. What is the percent discount on the power tool?

Work Out Two Discounts in a Row Separately

Sometimes the same item might be discounted twice. You have to work out each discount separately, one after the other.

Example 2

A shirt that usually costs \$50 is on sale at 10% off. The store then takes an extra 15% off the discounted price. What is the shirt's new sale price?

Solution

First work out the price after the original discount:

$$\frac{10}{100} \times \$50 = \$5 \quad \text{first discount} \quad \$50 - \$5 = \$45$$

Then work out the price after the second discount:

$$\frac{15}{100} \times \$45 = \$6.75 \quad \text{second discount} \quad \$45 - \$6.75 = \mathbf{\$38.25}$$

The new sale price is \$38.25.

Check it out:

To find the amount of two discounts in a row you can't add the percents. The 2nd percent is taken after the 1st one has already been applied. So in Example 2 to find the new sale price, you found 10% of 50, and then 15% of 45. This isn't the same as finding 25% of 50.

✓ Guided Practice

5. A pair of sneakers that usually costs \$100 is on sale at 50% off. The store takes another 20% off. What is the new sale price?
6. A computer costing \$976 goes on sale at 25% off. The store offers an extra 15% discount for students. What would the student price be?
7. In a store, two sweaters both costing \$60 go on sale. Sweater A is put on sale with 20% off, then another 10% is taken off. Sweater B is put on sale with 10% off, and then another 20% is taken off. Which is the least expensive sweater?

A Markup is a Percent Increase

Stores buy goods at **wholesale prices**. Before selling them, they **increase** the **prices** of the goods in order to cover their expenses and make a profit. The prices that stores sell goods for are called the **retail prices**. The **difference** between the **wholesale** and **retail** price is called the **markup**.

Example 3

The wholesale price of plain paper is \$3.20 a ream. If the markup is 75%, what is the retail price of a ream of plain paper?

Solution

Write the percent of the markup as a fraction: $75\% = \frac{75}{100}$

Work out the amount of the markup: $\frac{75}{100} \times \$3.20 = 0.75 \times \$3.20 = \$2.40$

Add the markup to the original price: $\$3.20 + \$2.40 = \mathbf{\$5.60}$

OR

The markup is 75%. So the retail price is 175% of the wholesale price.

Write the percent as a fraction: $175\% = \frac{175}{100}$

Find the increased price: $\frac{175}{100} \times \$3.20 = 1.75 \times \$3.20 = \mathbf{\$5.60}$

Don't forget:

Both methods will give you the same answer, so use whichever one you feel most comfortable with.

Check it out:

The method for finding a change as a percent was covered in the last Lesson.

Now try these:

Lesson 8.2.1 additional questions — p471

Guided Practice

8. The wholesale price of a case of oranges is \$13.50. If a retailer has an 80% markup, what will the retail price of a case of oranges be?
9. A \$125 wholesale price chair is marked up 62%. Find its retail price.
10. An item is marked up 50% from the wholesale price. What percent of the wholesale price is the retail price?
11. A \$9.20 wholesale price toy retails at \$14.72. Find the percent markup.

Independent Practice

1. A hat worth \$70 is on sale at 25% off. What is its sale price?
2. A kettle costing \$34 is put on sale at 10% off. The store then offers another 25% off the discounted price. What is the new sale price?
3. In a sale you buy a basketball with 20% off a retail price of \$20, sneakers with 40% off a retail price of \$80, and a tennis racket with 20% off a retail price of \$100. What is the total? What percent discount is this on the full amount?
4. A \$12 wholesale price bag is marked up 40%. Find its retail price.
5. The wholesale price of a sweater is \$35. If the markup is 55% what is the retail price of the sweater?
6. A shirt with a wholesale price of \$36 is marked up 40%. In store it is put on sale at 20% off its retail price. What is the shirt's sale price?
7. A store buys 100 kg of pears for \$1.20/kg. They mark them up 50%. Half sell at retail price and half at 25% off. How much profit does the store make?

Round Up

Discounts and markups are real-life examples of percent increase and decrease problems. Whether it's a discount or markup, you need to take care that you find the percent of the original price.

Lesson 8.2.2

Tips, Tax, and Commission

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and **use these representations in estimations, computations, and applications.**

Number Sense 1.6

Calculate the percentage of increases and decreases of a quantity.

Number Sense 1.7

Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

What it means for you:

You'll learn about more real-life uses of percent increase.

Key words:

- tip
- tax
- commission
- percent

Check it out:

There are other people who you might tip for their service — a taxi driver, a hairstylist, or a parking valet for instance.

Check it out:

If you rounded down to \$50 you'd leave a tip of \$5 — and that would be less than 10%.

Check it out:

It would also be reasonable here to round to \$18 and leave a tip of \$1.80.

*This lesson is about some more **real-life** uses of **percent increase**. You'll come across them in a lot of everyday situations, so they're definitely worth knowing about.*

A Tip is Calculated as a Percent of a Bill

When you eat at a restaurant you would usually leave a **tip** for the person who waited on you. The standard amount to leave is **15% of your bill** — though you might **vary** this **percent** depending on the **quality** of the service.

Example 1

Finn's restaurant bill comes to \$16. He wants to leave a 15% tip for the server. How much tip should he leave?

Solution

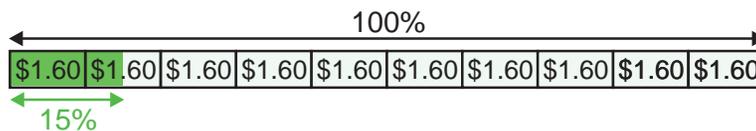
To find how much to leave for a **15%** tip, Finn should multiply his bill by $\frac{15}{100}$ or **0.15**.

$$0.15 \times \$16 = \mathbf{\$2.40} \quad \text{So Finn should leave a } \mathbf{\$2.40} \text{ tip.}$$

You Might Need to Work Out a Tip Mentally

Using **mental math**, 10% is an easier percent to work out than 15%. So find **10%** of the bill and leave that **plus half** as much again.

In Example 1, Finn might first work out that, as his bill is \$16, 10% is \$1.60.



$$\text{So his tip should be } \$1.60 + \left(\frac{1}{2} \times \$1.60\right) = \$1.60 + \$0.80 = \mathbf{\$2.40}.$$

You might sometimes **estimate** a tip, but you should usually **round up** and not down. For example, if your bill was **\$54.40** and you wanted to leave a **10%** tip, you could round the bill to **\$60**, and leave a **\$6 tip**.

Example 2

Raina's taxi fare is \$17.61. She wants to give the driver a tip of about 10%. Estimate how much she should give as a tip.

Solution

To estimate the tip needed, **round up** Raina's \$17.61 fare to \$20.

$$0.1 \times \$20 = \mathbf{\$2} \quad \text{So, Raina should give a } \mathbf{\$2} \text{ tip.}$$

$$10\% = \frac{10}{100} = 0.1$$

✓ Guided Practice

1. Vance's restaurant bill comes to \$40. He leaves a 15% tip. How much is the tip? How much does he leave altogether?

In Exercises 2–5, use mental math to find 15% of each amount.

2. \$10 3. \$4 4. \$7 5. \$12.60

6. Mrs. Clark's haircut costs \$48.59. She wants to leave a 20% tip. Estimate what amount would be sensible for her to leave as a tip.

Sales Tax is a Percent Increase on an Item's Cost

When you buy certain items, you pay a **sales tax** on them — an **extra** amount of money on top of the cost of the item that goes to the government. A sales tax is calculated as a **percent** of the cost of the item.

Tax rates are set by local governments — so they **vary** from place to place.

Check it out:

Like a tip, a tax is an extra amount paid on top of the basic cost of the item. It's paid by the buyer.

Example 3

In Fort Bragg, sales tax is 7.75%. Pacho buys a book costing \$12 before tax from a bookstore in Fort Bragg. How much sales tax will he pay?

Solution

To find the sales tax Pacho paid, find **7.75%** of the selling price.

$$7.75\% = \frac{7.75}{100} = 0.0775 \quad \rightarrow \quad 0.0775 \times \$12 = \mathbf{\$0.93}$$

Pacho pays **\$0.93** sales tax on his book.

Example 4

On a vacation, you buy a souvenir that was \$3.50 before tax. You were charged \$3.71. What is the rate of sales tax here?

Solution

The amount of tax paid was $\$3.71 - \$3.50 = \$0.21$

Let x = the rate of sales tax.

$$\frac{x}{100} \times \$3.50 = \$0.21$$

$$\$3.50x = \$0.21$$

$$x = 6$$

Multiply both sides by 100

Divide both sides by \$3.50

So the rate of sales tax is 6%.

Check it out:

You're working out what percent \$0.21 is of \$3.50. So you could write the fraction

$\frac{\$0.21}{\$3.50}$, then convert it to a decimal: $\$0.21 \div \$3.50 = 0.06$. This is 6 hundredths, which is 6%.

✓ Guided Practice

7. Hazel bought a calculator costing \$29.50 (before tax) in Santa Rosa, where the sales tax is 8%. How much sales tax did she have to pay?

8. The sales tax in San Francisco is 8.5%, while in Oakland it is 8.75%. What is the price difference in buying a \$21,000 (before tax) car in Oakland and San Francisco?

9. Dale bought a table costing \$520 (before tax). He paid \$37.70 sales tax on it. What was the rate of sales tax where he bought the table?

Commission is Paid to a Sales Agent

Commission is sometimes paid to **sales agents** — like realtors, or car salespeople. Realtors may get an amount of money for each property they sell — how much they get is calculated as a **percent** of the **selling price**.

Example 5

Althea is a realtor. She gets 6% commission on the sale price of a house. If a house sells for \$210,000 how much commission will she receive?

Solution

To find the commission that Althea gets, find **6%** of the selling price.

$$\$210,000 \times 0.06 = \mathbf{\$12,600}$$

The realtor will receive **\$12,600** commission.

✓ Guided Practice

10. A shoe salesman receives a 10% commission on each pair of shoes he sells. What commission will he get on a pair costing \$89?
11. A travel agent receives an 8% commission on all cruise sales. If a cruise ticket costs \$1689 how much commission will the agent get?
12. An auctioneer takes a commission on all items sold. A lamp sells for \$80, and the auctioneer gets \$9.60. What percent commission does the auctioneer take? How much does the seller receive?

✓ Independent Practice

1. Shakia wants to leave her hairstylist a 25% tip. If her haircut cost \$42, what tip should she leave?
2. In a restaurant, Mr. Baker's bill comes to \$76.32. He wants to leave a tip of about 15%. Using mental math, estimate what tip he should leave.
3. In Santa Clara, the sales tax rate is 8.25%. If Nina buys a radio costing \$40 before tax, how much sales tax will she pay?
4. Brad's restaurant bill comes to \$25. He leaves a tip of \$4. What percent of the bill has he left as a tip?
5. Leah buys a pair of jeans in Clearlake, where the sales tax rate is 7.75%. If the jeans cost \$40 before tax, how much does she pay in total?
6. A salesperson gets 7.5% commission on each car sold. How much commission will the salesperson earn on a car costing \$18,600?
7. The sales tax rate in Roseland is 8%. Daniel eats in a restaurant in Roseland. The bill is \$100 before tax. After the tax has been added he works out 25% of the total to leave as a tip. What tip does he leave?

Now try these:

Lesson 8.2.2 additional questions — p471

Round Up

Tips, tax, and commission are all just types of percent increases. Don't forget though — tips and tax are paid by the buyer and commission is paid by the seller. So you need to think carefully about what it is that you're being asked to find.

Lesson 8.2.3

Profit

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and **use these representations in** estimations, **computations, and applications.**

Number Sense 1.7

Solve problems that involve discounts, markups, commissions, and **profit** and compute simple and compound interest.

What it means for you:

You'll learn what profit is and how to find profit as a percent of a company's sales.

Key words:

- profit
- revenue
- expenses
- percent
- sales

Check it out:

You know that
 $\text{Profit} = \text{Revenue} - \text{Expenses}$.
Now add "Expenses" to both sides of the equation:
 $\text{Profit} + \text{Expenses} = \text{Revenue}$.

If you buy something and then sell it for more than the amount that it cost you, the extra money that you get is called profit. Because you end up with more money than you started with, you can think about profit as a percent increase.

Profit is the Amount of Money that a Business Makes

A business has to **spend money** buying stock and paying staff. The amount of money that a business spends is called its **expenses**. A business also **has an income** from selling its products or services. The total amount of money that a business brings in is called its **revenue**. The **profit** that a business makes is just the **difference between** its **revenue** and its **expenses**.

$$\text{Profit} = \text{Revenue} - \text{Expenses}$$

Example 1

A film had a revenue of \$55 million in ticket sales and \$35 million in licensing agreements. It had expenses of \$4 million in advertising and \$48 million in production costs. What profit did the film make?

Solution

The film's total **revenue** = $\$55,000,000 + \$35,000,000 = \mathbf{\$90,000,000}$

The film's total **expenses** = $\$4,000,000 + \$48,000,000 = \mathbf{\$52,000,000}$

$$\begin{aligned} \text{Profit} &= \text{Revenue} - \text{Expenses} \\ &= \$90,000,000 - \$52,000,000 = \mathbf{\$38,000,000} \end{aligned}$$

Guided Practice

1. Janet buys a rare baseball card for \$15. She later sells it to another collector for \$18. What profit has she made?
2. This year a company had a revenue of \$500,000 and \$356,000 of expenses. What profit did the company make this year?
3. A school held a fund-raiser. They paid \$200 to hire a band, and \$400 for food. They took \$1000 in ticket sales. How much profit did the event make?
4. A bookstore's total expenses in one year consisted of \$300,000 to buy stock, and \$150,000 to pay staff and cover other expenses. Their profit was \$40,000. What was their total revenue?

Profits are Often Given as Percents

You can also work out a **percent profit**. This compares the amount of **profit** to the amount of **sales revenue**.

Example 2

A company makes a profit of \$90,000 on total sales of \$720,000. What is their profit as a percent of sales?

Solution

The company made \$90,000 **profit** on sales of \$720,000. Write this as a **fraction**, and convert it to a **decimal**.

$$\frac{\$90,000}{\$720,000} = 0.125$$

Now change the decimal to a **percent** by **multiplying by 100**.
 $0.125 \times 100 = 12.5$, so **their profit is 12.5% of their sales**.

Guided Practice

Don't forget:

To find $x\%$ of a number, just multiply the number by $\frac{x}{100}$.

5. Sayon's lemonade stand made a \$20 profit. He sold \$80 worth of lemonade. What profit did he make as a percent of sales?
6. A company made a profit of \$6000 on total sales of \$40,000. What was their profit as a percent of sales?
7. Sophia buys a set of books for \$75. She later sells the books to a collector for \$90. What percent profit has she made?
8. Company A made a 12% profit on sales of \$295,000. How much profit did they make?

You Can Compare Profits Using Percents

Businesses often use **percents** to **compare** the **profits** that they have made in **consecutive years**. This shows how the company is performing over time.

Example 3

This year, Company B increased its profits by 5% over the previous year. If last year's profit was \$43,900, what was this year's profit?

Solution

Write the **percent** of the **increase** as a **fraction**: $5\% = \frac{5}{100}$

Work out the **amount** of the increase:

$$\frac{5}{100} \times \$43,900 = 0.05 \times \$43,900 = \$2195$$

Now **add** the amount of the increase to the original profit:
 $\$43,900 + \$2195 = \mathbf{\$46,095}$

Example 4

Last year, Company C made profits of \$40,000. This year, they made profits of \$28,000. What was the percent decrease in their profits?

Solution

Find the **amount** of the **profit decrease**: $\$40,000 - \$28,000 = \$12,000$

Now **divide** the amount of the decrease by the first year's profits:

$$\frac{\$12,000}{\$40,000} = 0.3$$

Change the **decimal** to a **percent** by **multiplying by 100**:

$0.3 \times 100 = 30$, so they had a **30% decrease in profit**.

✓ Guided Practice

9. This year, Company D increased its profits by 10% over last year. If last year's profits were \$12,000, what was this year's profit?
10. Company E's profits fell by 4% this year compared to last year. If last year's profits were \$29,500, what were this year's profits?
11. Last month, Company F made profits of \$1250. This month, they made profits of \$1500. Find the percent increase in their profits.
12. Last year, Company G made profits of \$200,000. This year, they made profits of \$192,000. Find the percent decrease in their profits.

✓ Independent Practice

1. In one year, a company has a total revenue of \$185,000 and total expenses of \$155,000. What were the company's profits that year?
2. A website selling clothes made a profit of \$7890 in a month. In the same month its revenue was \$12,390. Find its expenses for that month.
3. A toy store makes \$12,000 profit on sales of \$300,000. What percent profit has the store made?
4. A grocer buys \$270 of fruit. He sells it for \$283.50. What is his profit? What is his percent profit?
5. This year, Company H's profits fell by 7% compared to the previous year. If last year's profit was \$22,500, what was this year's profit?
6. Last month, a store made profits of \$4800. This month, they made profits of \$5400. What was the percent increase in their profits?
7. Your class organizes a dance as a fund-raiser. You spend \$100 hiring a DJ, \$180 on food, and \$40 on tickets and fliers. You have 50 tickets — if they all sell, what will you need to price them at to make a 25% profit?

Now try these:

Lesson 8.2.3 additional questions — p471

Round Up

Profit is the money that a business is left with when you take away what it spends from what it takes in sales. Percent change in profit is a way of measuring the performance of a business over time.

Lesson 8.2.4

Simple Interest

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and **use these representations** in estimations, **computations**, and **applications**.

Number Sense 1.6

Calculate the percentage of increases and decreases of a quantity.

Number Sense 1.7

Solve problems that involve discounts, markups, commissions, and profit and **compute simple** and compound **interest**.

What it means for you:

You'll see what interest is and how to work out how much simple interest you could earn over time.

Key words:

- interest
- simple interest
- principal
- interest rate

Check it out:

To invest just means to put money into something.

*Interest is an important real-life topic because it's all about **saving** and **borrowing** money. If you keep your money in a **savings** account, the bank will **pay** you something just for keeping it there. The **interest** that you **gain** will be **based on** how much you **put in** — and that means it's another use of **percent increase**.*

Interest is a Fee Paid For the Use of Money

When you keep money in a savings account, the bank **pays you interest** for the privilege of using your money.

When you **borrow** money from a bank, the bank **charges you interest** for the privilege of using their money.

Interest is a **fee** that you pay for using someone else's money.

The interest to be paid is worked out as a **percent** of the money invested or loaned. The **percent** that is paid over a given **time** is called the **interest rate**.

Simple Interest is Paid Only on the Principal

The amount of money you **put into** or **borrow from** a bank is called the **principal**. **Interest** that is paid **only** on the principal is called **simple interest**.

With simple interest, the **interest rate** tells you how much money you will get back every year as a **percent** of the **principal**.

For example: think about depositing **\$100** in a savings account with a simple interest rate of **5%** per year. For **each year** you leave your money in the account, you will get **5%** of **\$100** back.

Number of years	Interest earned	Total
0	\$0	\$100
1	\$5	\$105
2	\$5	\$110
3	\$5	\$115

Example 1

You deposit \$50 in a savings account that pays a simple interest rate of 2% per year. How much interest will you get over 3 years? How much will be in the account after 3 years?

Solution

First find 2% of \$50: $\$50 \times \frac{2}{100} = \1 . This is the amount of interest you will get each year.

So over 3 years you will earn: $3 \times \$1 = \3

After 3 years you will have: $\$50 + (3 \times \$1) = \$53$ in the account.

✓ Guided Practice

1. If you put money into a savings account which pays simple interest, will the amount of interest you get in the first year be the same as in the second year? Explain your answer.
2. You borrow \$150 from a bank at a simple interest rate of 8% per year. How much interest will you pay in one year?
3. You deposit \$200 in a savings account that pays a simple interest rate of 5% per year. How much interest will you get over 4 years?
4. You deposit \$65 in a savings account that pays a simple interest rate of 4% per year. How much will be in your account after 4 years?

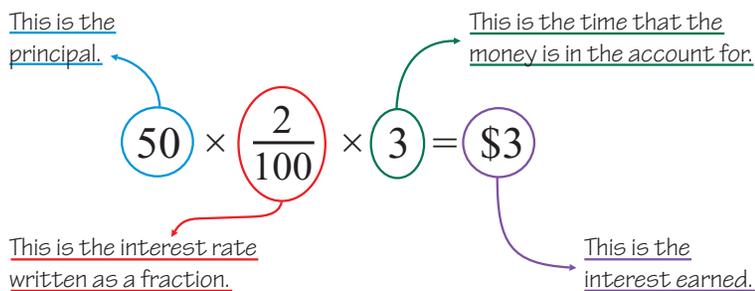
Use the Simple Interest Formula to Calculate Interest

Look back at **Example 1**. To work out **how much interest** you got over 3 years, you worked out the **percent** of the **principal** that you would get **each year** and **multiplied it by 3**.

So the calculation you did was:

$$\left(50 \times \frac{2}{100}\right) \times 3 = \$3$$

Now think about what each part of that equation **represents**.



Don't forget:

You can remove the parentheses from this equation because of the associative property of multiplication — see Lesson 1.1.5.

You can use this to figure out a **general formula** for finding **simple interest**. First assign a **variable** to stand for each part of the equation:

- ***P*** stands for the **principal**.
- ***r*** stands for the **interest rate (in % per year)**, written as a fraction or a decimal.
- ***t*** stands for **time (in years)**.
- ***I*** stands for the **amount of interest** that has built up.

To find the **amount of interest** that you got, you **multiplied** together the **principal**, the **interest rate**, and the **time** the money was in the account for.

Written as a **formula** this is:

$$I = Prt$$

Example 2

You deposit \$276 in a savings account that has a simple interest rate of 6% per year. How much interest will you get over 5 years?

Solution

$$I = Prt$$

$$I = \$276 \times 0.06 \times 5 \quad \text{Substitute the values into the formula}$$

$$I = \mathbf{\$82.80}$$

Over 5 years you'll earn \$82.80 interest.

Don't forget:

When you write the interest rate, you can write 6% as either the fraction $\frac{6}{100}$ or the decimal 0.06.

Check it out:

Read each question carefully. Sometimes you'll be asked to just find the amount of interest, other times you'll be asked to find the total amount in the account.

To find the total amount that an account will contain, first calculate the interest. Then add the interest to the principal.

Guided Practice

- You borrow \$57 from a bank at a simple interest rate of 9% per year. How much interest will you pay in one year?
- You deposit \$354 in a savings account that pays a simple interest rate of 2.5% a year. How much interest will you get over 7 years?
- You deposit \$190 in a savings account that pays a simple interest rate of 4% a year. How much will be in your account after 4 years?
- You put \$520 in a savings account with a simple interest rate of 6% a year. You take it out after 6 months. How much interest will you get?

Independent Practice

- You borrow \$75 from a bank at a simple rate of 9% per year. How much interest will you pay over 7 years?
- You deposit \$64 in a savings account that pays a simple interest rate of 2.5% a year. How much will be in your account after 17 years?
- Ian put \$4000 into a short-term investment for 3 months. The simple interest rate was 5.2% per year. How much interest did Ian earn?
- Luz borrows money from a bank at a simple interest rate of 5% a year. After 4 years she has paid \$50 interest. How much did she borrow?
- Ty puts \$50 in a savings account with a simple interest rate of 3% a year. He works out what interest he will get in 5 years. His calculation is shown on the right. What error has he made? How much interest will he get?
- Anna puts \$50 in a savings account that pays a simple interest rate of 5% a year. After 4 years she takes out all the money, and puts it in a new account that pays a simple interest rate of 6% a year. She leaves it there for 5 years. How much will Anna have in total at the end of this time?

$I = Prt$
$I = \$50 \times 3 \times 5$
$I = \$2250$

Now try these:

Lesson 8.2.4 additional questions — p472

Round Up

Interest is money that is paid as a fee for using someone else's money. Simple interest means that each year you get back a fixed percent of the initial amount you invested. Make sure you understand how simple interest works. You'll use a lot of the same math in the next lesson on compound interest.

Lesson 8.2.5

Compound Interest

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and **use these representations in** estimations, **computations,** and **applications.**

Number Sense 1.6

Calculate the percentage of increases and decreases of a quantity.

Number Sense 1.7

Solve problems that involve discounts, markups, commissions, and profit and **compute** simple and **compound interest.**

What it means for you:

You'll learn about compound interest, and how to work out how much compound interest you could earn over time.

Key words:

- compound interest
- principal
- interest rate
- annually
- quarterly
- monthly

Check it out:

Quarterly means every three months — this is a quarter of the year.

Check it out:

0.25 is used here because the interest is compounded quarterly. If it were compounded yearly, the multiplication factor would be 1. And if it were monthly, the multiplication factor would be one-twelfth.

In the last Lesson, you saw what *interest* was and how to work out *simple interest*. There's another type of interest that you need to know about called *compound interest*. And that's what this Lesson is about.

Compound Interest is Paid on an Entire Balance

Simple interest is **only** paid on the **principal**. So although the **balance** of your account **rises**, the **amount** of interest you get is the **same each year**.

Compound interest is paid on the **principal** and on any **interest you've already earned**. Interest is added (or compounded) at regular intervals — and the amount paid is a percent of **everything** in the account.

Think about putting \$100 in an account with an interest rate of 5% compounded yearly. **Each year** you leave your money in the account you will get **5% of the account's balance** paid into your account.

Number of years	Interest earned	Total
0	\$0	\$10,000
1	$(\$10,000 \times 0.05) = \500	\$10,500
2	$(\$10,500 \times 0.05) = \525	\$11,025
3	$(\$11,025 \times 0.05) = \551.25	\$11,576.25

Interest can also be worked out **daily**, **monthly**, or **quarterly**. It often isn't an exact number of cents — so the bank **rounds it** to the nearest cent.

Example 1

You put \$80 into an account with an interest rate of 5% per year, compounded quarterly. What is the account balance after 6 months?

Solution

After the first 3 months you'll get:

$$I = Prt = \$80 \times 0.05 \times 0.25 = \mathbf{\$1 \text{ interest.}}$$

So you'll have **\$81** in the account.

Over the next 3 months, you'll get:

$$I = Prt = \$81 \times 0.05 \times 0.25 \approx \mathbf{\$1.01 \text{ interest.}}$$

So you'll have **\$82.01** in the account.

In 6 months you earned \$2.01 interest and have **\$82.01** in the account.

Guided Practice

1. If you put money into a savings account which pays compound interest, will the amount of interest you get in the first year be the same as in the second year? Explain your answer.
2. You borrow \$100 from a bank at an interest rate of 5% a year compounded annually. How much interest do you pay in 2 years?

Calculate Compound Interest Using the Formula

There's a formula for calculating the **amount in an account** (A) that has been earning compound interest:

$$A = P(1 + rt)^n$$

- **P is the principal.** This is the amount that is put into the account or loaned in the first place.
- **r is the interest rate**, written as a fraction or a decimal. So an interest rate of 6% could be written as $\frac{6}{100}$ or 0.06.
- **t is the time between each interest payment** in years. In Example 1 this was 0.25 because interest was paid quarterly.
- **n is the number of interest payments made.** In Example 1 this was 2 — in 6 months quarterly interest was paid twice.

Example 2

You put \$80 into an account that pays an interest rate of 6% per year compounded quarterly. Use the compound interest formula to find the account balance after 6 months.

Solution

$$A = P(1 + rt)^n$$

$$A = 80(1 + (0.06 \times 0.25))^2 \quad \text{Substitute the values into the formula}$$

$$A = 80 \times 1.015^2$$

$$A = 82.418 \quad \text{Evaluate}$$

The account balance is **\$82.42** to the nearest cent.

Don't forget:

On bank statements, interest payments are usually rounded to the nearest cent.

✓ Guided Practice

3. You put \$100 into an account with a compound interest rate of 10% per year, compounded annually. What's the account balance after 4 years?
4. You put \$150 into an account with a compound interest rate of 4% per year, compounded quarterly. What's the account balance after 6 months?
5. You put \$88 into an account with a compound interest rate of 1% per year, compounded quarterly. What's the account balance after 9 months?
6. You put \$200 into an account with a compound interest rate of 2% per year compounded monthly. What's the account balance after 7 months?

Comparing Simple and Compound Interest

Imagine you have \$10,000 to invest for three years, and you intend to make no transactions during that time. You can choose from two accounts: one pays 5% **simple interest** per year, the other 5% **compound interest** per year, compounded annually.

This table shows the amount of interest the two accounts would build up:

	SIMPLE INTEREST		COMPOUND INTEREST	
Number of years	Interest earned	Total	Interest earned	Total
0	\$0	\$10,000	\$0	\$10,000
1	\$500	\$10,500	\$500	\$10,500
2	\$500	\$11,000	\$525	\$11,025
3	\$500	\$11,500	\$551.25	\$11,576.25

The account with compound interest would earn you **an extra \$76.25**.

Comparing two accounts with **the same** annual interest rate:

- If you are **SAVING** a fixed sum of money, the account with **compound interest** will be a better choice because it will earn **MORE** interest.
- If you are **BORROWING** a fixed sum of money, **simple interest** will be a better choice because you'll be charged **LESS** interest overall.

✓ Guided Practice

7. On a loan of \$100, Bank A charges simple interest at 6% a year. Bank B charges 6% a year, compounded annually. Neither loan offers repayment in installments. Which bank has the better deal?
8. Myra has \$100 to invest for 6 years. She can pick from 2% simple interest a year, or 2% compound interest a year, compounded annually. How much more will be in her account if she picks compound interest?
9. Rai has \$500 to invest for 3 years. He can pick from 5% simple interest a year or 4% compound interest a year, compounded quarterly. Which will leave him with the greater account balance?

Check it out:

To compare simple and compound interest when the interest rates are different you'll have to work out the account balances.

✓ Independent Practice

- In Exercises 1–3, work out the account balance using the formula.
1. \$100 is invested for 5 years at 5% a year, compounded annually.
 2. \$50 is invested for 2 years at 3% a year, compounded quarterly.
 3. \$800 is invested for 8 months at 5% a year, compounded monthly.
 4. Ezola borrows \$200 at 7% a year compounded quarterly. She makes no repayments in the 1st year. What does she owe at the end of it?
 5. Ben puts \$2000 in an account that pays 5% a year simple interest. Dia puts \$2000 in an account that pays 5% a year compounded annually. What's the difference between their balances after 6 years?
 6. Geroy has \$1000 to invest for 2 years. He can pick from 3.5% simple interest a year or 3.4% compound interest a year, compounded monthly. Which will leave him with the greater account balance?
 7. Kim puts \$10,000 in an account that pays a rate of 4% interest a year compounded annually. After 2 years the rate goes up to 5% a year compounded quarterly. What is her account balance after 30 months?

Now try these:

Lesson 8.2.5 additional questions — p472

Round Up

Compound interest is when you're paid interest on the whole account balance and not just on the money you first put in. It's a great way to save — but not always such a good way to borrow.

Section 8.3 introduction — an exploration into: Estimating Length

An *estimate* is an *educated guess* about something — such as the size of a measurement. In this Exploration, you'll test your estimation skills by estimating the *length* of different objects in the classroom. You'll then *test* your estimates by measuring, and finding your *percent error*.

If you estimated the **length of a field** and were only 2 inches away from the actual measurement, it would be much more impressive than if you estimated the **length of a pencil** and were 2 inches out. That's why you use **percent error** — 2 inches as a percent of the length of a field, would be **tiny**, whereas 2 inches as a percent of the length of a pencil would be **much bigger**.

Example

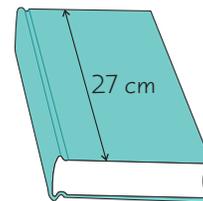
A student estimates that the length of a math textbook is 30 centimeters. She measures it, and finds that it's actually only 27 centimeters long. What is her percent error?

Solution

Her error is $30 - 27 = 3$ centimeters.

You have to find 3 centimeters out of 27 centimeters as a percent:

$$\frac{3}{27} \times 100 = 11.1 \quad \Rightarrow \quad \text{So her percent error was 11.1\%}.$$



Exercises

1. Make a copy of the table below. Complete it by estimating the things listed, measuring them, and then calculating the percent error.

Item	Estimate (cm)	Actual measurement (cm)	Error (cm)	Percent error
Length of student desk				
Width of door				
Diameter of clock				
Width of light switch				

Pick two other items in your classroom.



2. How did your accuracy change over the course of the Exploration? Did your estimation skills improve? Explain your answer.

Round Up

Percent error tells you how big your error is compared to the size of the thing you are measuring. You normally estimate with *"easy" numbers* — like whole numbers, or to the nearest 10 or 100. For instance, you'd estimate something as *"about a meter"* rather than *"about 102.3 centimeters."*

Lesson 8.3.1

Section 8.3 Rounding

California Standard:
Mathematical Reasoning 2.7
Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

What it means for you:

You'll see how to round numbers, and how to describe how you've rounded them.

Key words:

- rounding
- decimal places
- hundredth
- thousandth

Check it out:

Rounding is all about figuring out which of two numbers your answer is closer to.

Check it out:

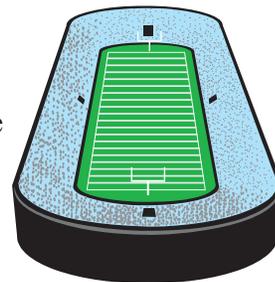
72.5 lies exactly half way between 72 and 73. The rule is to round it up to 73 though.

Often, it's fine to give an *approximate* answer. For instance, if you calculate the length of a yard as 11.583679 meters, then it'd probably be most sensible to say that it's approximately 11.58 meters. Also, *rounding* numbers that have lots of digits makes them easier to handle. There's a set of rules to follow to help you round any number.

Rounding Makes Numbers Easier to Work With

Sometimes, using **exact numbers** isn't necessary.

For example: the exact number of people who came to a football game might be 65,327. But most people who want to know what the attendance was will be happy with the answer "about 65,000."



Rounding reduces the **number of nonzero digits** in a number while keeping its value similar. Rounded numbers are **less accurate**, but **easier to work with**, than unrounded numbers.

There are Rules to Follow When You Round

Think about rounding 65.3 to the **nearest whole number**.

"To the nearest whole number" means that the units column is the last one that you want to keep. So look at the digit to the right of that:

65.3
You're rounding to this place... ...so look at this place.

Because this digit is less than 5, it means that the number is closer to 65 than to 66. So you can round it down to **65**.

It might help to think about where the number is on a number line:



You can see that 65.3 is closer to 65 than to 66.

Rules of rounding:

- Look at the digit to the right of the place you're rounding to.
- If it's 0, 1, 2, 3, or 4, then round the number down.
- If it's 5, 6, 7, 8, or 9, then round the number up.

Example 1

Round 57.51 to the nearest whole number.

Solution

You're rounding to the nearest whole number, so look at the units column.

$57.\overline{5}1$ You're rounding to this place...
 $57.\overline{5}1$...so look at this place.

This digit is 5 — so you can round 57.51 up to **58**.

Guided Practice

In Exercises 1–9, round the number to the nearest whole number.

- | | | |
|---------|-----------|-------------|
| 1. 3.1 | 2. 4.8 | 3. 2.5 |
| 4. 21.6 | 5. 7.01 | 6. 43.19 |
| 7. 0.61 | 8. 127.20 | 9. 1849.271 |

You Need to Say What You're Rounding To

When you round, you need to say in your work what you've rounded your answer to. That might be...

- | | |
|----------------------------------|------------------------------|
| ... to the nearest whole number | $17.23 \longrightarrow 17$ |
| ... to the nearest 10 | $285 \longrightarrow 290$ |
| ... to the nearest 100 | $1243 \longrightarrow 1200$ |
| ... to the nearest one-hundredth | $1.379 \longrightarrow 1.38$ |

Check it out:

To help you figure out where to round a number, you can circle the digit you're rounding to.

For example: if you're rounding 1872 to the nearest hundred, ring the digit that represents hundreds.

$1\textcircled{8}72$

Now look at the digit to the right of that. In this case, as it is 7, you round up to 1900.

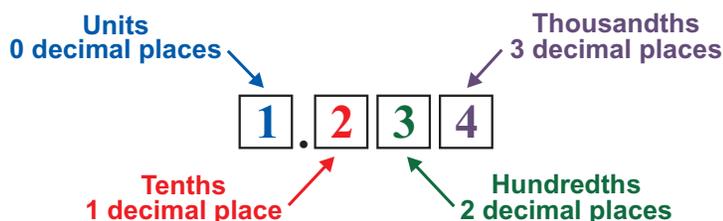
Guided Practice

In Exercises 10–15, round the number to the size given.

- | | |
|---------------------------------|-----------------------------------|
| 10. 726 to the nearest 10 | 11. 1851 to the nearest 100 |
| 12. 21241 to the nearest 1000 | 13. 0.15 to the nearest 10th |
| 14. 0.2149 to the nearest 100th | 15. 0.00827 to the nearest 1000th |

You Can Round to Decimal Places

Another way of rounding numbers is to round to **decimal places**.



Check it out:

Rounding to the nearest hundredth is the same as rounding to 2 decimal places. Rounding to the nearest whole number is the same as rounding to 0 decimal places.

The number of **decimal places** that have been used is just the **number of digits** there are after the **decimal point**.

Example 2

Round 1.48934 to 3 decimal places.

Solution

You're rounding to **3 decimal places**, so look at the number to the right of the third digit after the decimal point.

1.48934
You're rounding to this place...
...so look at this place.

This digit is 3 — so you can round 1.48934 down to **1.489**.

✓ Guided Practice

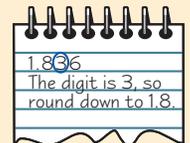
In Exercises 16–21 round to the number of decimal places given.

- | | |
|--------------------------------|----------------------------------|
| 16. 0.27 to 1 decimal place | 17. 2.237 to 1 decimal place |
| 18. 4.118 to 2 decimal places | 19. 1.4619 to 2 decimal places |
| 20. 0.6249 to 3 decimal places | 21. 0.012419 to 4 decimal places |

✓ Independent Practice

In Exercises 1–8, round the number to the size given.

- 7.8 to the nearest whole number
- 4.23 to the nearest 10
- 19410 to the nearest 100
- 1.205 to the nearest 100th
- 5.63 to 1 decimal place
- 0.74 to 0 decimal places
- 1.118 to 2 decimal places
- 7.2462 to 3 decimal places
- Duenna's school has 1249 pupils on its roll. How many pupils does it have to the nearest hundred? What about to the nearest 10?
- Multiply 1501 by 8. Give your answer to the nearest 1000.
- Divide 150 by 31. Give your answer to 1 decimal place.
- Kelvin is asked to round 1.836 to 2 decimal places. His work is shown on the right. What mistake has he made? What answer should he have gotten?
- The local news reports that, in a survey of 3000 local families, 1000 had 3 or more children below the age of 18. The actual number was 583. Do you think it was sensible to round to the nearest 1000 here? What would you have rounded to?



Now try these:

Lesson 8.3.1 additional questions — p472

Round Up

When you don't need to use an *exact number* you can *round*. *Rounding* makes numbers with a lot of *digits* easier to handle. Use the *digit to the right* of the one you're *rounding to* to decide whether you need to *round up* or *down*. And don't forget to always say *how* you've rounded a number — whether it's to the *nearest 100*, the *nearest hundredth*, or to a certain number of *decimal places*.

Lesson 8.3.2

Rounding Reasonably

California Standards:

Mathematical Reasoning 2.7

Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Mathematical Reasoning 3.1

Evaluate the reasonableness of the solution in the context of the original situation.

What it means for you:

You'll see how to round numbers in situations where the normal rules of rounding don't apply.

Key words:

- rounding
- decimal places
- round up
- round down

Check it out:

Whenever you're solving a real-life problem, you have to check that your answer is a reasonable one for that particular problem.

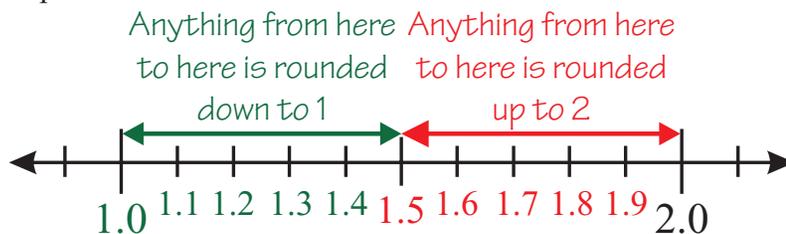
If the question asked how many cans Latoria would use, the answer could be 5.2. But she couldn't buy this number of cans.

There are times when the rules about *rounding up and down* that you learned in the last Lesson *don't apply*. In some real-life situations it *isn't reasonable* to round an answer up, and in others it *isn't reasonable* to round it down. This Lesson is all about being able to spot them.

Ordinary Rounding is Rounding to the Nearest

In the last Lesson, you learned about the ordinary rules of rounding.

For example:



- If the digit to the right of the place you're rounding to is 0, 1, 2, 3, or 4 you should round down.
- If the digit to the right of the place you're rounding to is 5, 6, 7, 8, or 9 you should round up.

Rounding with these rules is called “**rounding to the nearest**” because whether you round up or down depends which number the digit is **closest** to.

Sometimes It's Sensible to Round a Number Up

There are real-life situations when it's sensible to round an answer **up** — even though it's actually closer to the lower number.

Example 1

Latoria is decorating. She has to paint a total wall area of 130 m^2 . A can of paint covers 25 m^2 of wall. How many cans of paint should Latoria buy?

Solution

To find exactly how many cans of paint Latoria will need, **divide** the total **area of wall** by the area covered by **one paint can**.

$$130 \text{ m}^2 \div 25 \text{ m}^2 = 5.2$$

But Latoria can only buy a whole number of cans. So you need to round your answer to a whole number.

- Conventional rounding rules would say that the digit to the right of the units column is a **2**. So the answer would round to **5 cans**.
- But if Latoria only buys 5 cans, she **won't have enough** paint to cover the whole wall. So you need to round the answer up to **6 cans**.

Real-life situations where you need to **round up** instead of down include:

- Working out **how many** of something you need for a task — it's better to have a bit left over than not have enough. Example 1 was a good illustration of this.
- Figuring how much to leave for a **tip** — it's fine to leave a little over the percent tip you intended, but you wouldn't want to leave any less.
- Working out how much **money** you need to buy an item — if you give **too much** you get change, but if you don't have enough you can't pay.

✓ Guided Practice

1. A large cake contains 5 eggs. You're baking a small birthday cake that is half the size. How many eggs should you buy?
2. Reece is laying a path that is 76 m long. Each bag of gravel will cover 3 m of path. How many bags should Reece buy?
3. To get a grade A on a math test, Kate needs to score 80% or higher. If the test has a possible total of 74 points, how many points does Kate need to score an A?
4. Emilio's taxi fare comes to \$17.42. He wants to leave a tip of at least 10%. What is the amount of the smallest tip he can leave?
5. At Store A, a can of tuna costs \$1.77. Tess is going to the store to buy 3 cans for a recipe. If she only has dollar bills, how many should she take?

Sometimes It's Sensible to Round a Number Down

There are real-life situations when it's sensible to round the answer **down** — even though you'd round it up according to the rounding rules.

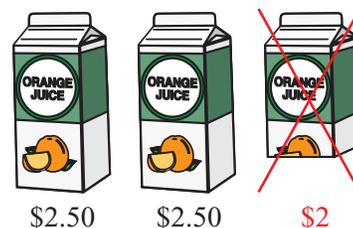
Example 2

A store charges \$2.50 for a carton of orange juice. If you have \$7, how many cartons of orange juice can you buy?

Solution

To find exactly how many cartons you can buy, **divide** the money that you have by the price of **one carton**.

$$\$7 \div \$2.50 = 2.8 \text{ cartons}$$



But you can only buy a whole number of cartons. So you need to round your answer to a whole number.

- Conventional rounding rules would say that because the digit to the right of the units column is an **8**, the answer would round to **3 cartons**.
- But you can't buy 3 cartons because you don't have **enough money** to pay for them. So you need to round the answer down to **2 cartons**.

Don't forget:

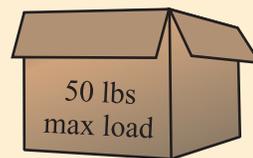
There are some things that you can usually buy fractions of. These will mostly be products that are sold by weight or length. For example, you could buy part of a pound of fruit, or part of a yard of fabric.

Real-life situations where you need to **round down** instead of up include:

- Working out how many **whole items** you can **make** from an amount of material. For instance, if it takes 4 balls of yarn to knit a sweater, and you have 10 balls, you might calculate that you can knit 2.5 sweaters. This isn't a reasonable answer — you can only knit 2.
- Working out **how many items** you can **buy** with a certain amount of money — you can't buy part of an item.

✓ Guided Practice

6. At a local store, pens cost \$2 each. If you go in with \$13.30, how many pens can you buy?
7. You are making up bags of marbles to sell at a fund-raiser. Each bag contains 24 marbles. How many bags can you make from 306 marbles?
8. A room has 4 walls, each with an area of 22 m^2 . One can of paint covers 30 m^2 . How many whole walls can Trayvon paint with 2 cans?
9. Blanca is packing books into a box that supports a maximum weight of 50 pounds. Each book weighs 2.2 pounds. How many books can Blanca put in the box?



✓ Independent Practice

1. Joel has 26 yards of material. He needs 3 yards to make one cushion. How many cushions can he make?
2. A bread recipe calls for 5 cups of flour. How many loaves can be made from 64 cups of flour?
3. Lydia is making gift tags. One sheet of card makes 4 tags. How many sheets of card will she need to make 57 tags?
4. A class is planning to buy their teacher a going-away present. The vase they want to buy costs \$50 and there are 23 people in the class. How much should they each contribute?
5. Patrick's restaurant bill came to \$22.92. He wants to leave a tip of at least 15%. What is the amount of the smallest tip he can leave?
6. You have a 1 kg bag of flour. You want to use it to make 7 cakes for a bake sale. How many whole grams of flour will go into each cake?
7. You use a payphone to make a call. Calls are charged at \$0.32/minute. If you have \$3 change, how many full minutes can you talk for?
8. Hannah is saving to buy an MP3 player costing \$80. Each week she gets an allowance of \$6.20, which she saves toward it. How many weeks will she need to save for?

Now try these:

Lesson 8.3.2 additional questions — p473

Round Up

Usually, when you round a number you use the “*rounding to the nearest*” method. But in some situations you might need to **round a number up or down** that you'd usually round the **other way**. It's all about making sure your answer is **reasonable** — there's more on that in the next two Lessons.

Lesson 8.3.3

Exact and Approximate Answers

California Standards:

Mathematical Reasoning 2.7

Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Mathematical Reasoning 2.8

Make precise calculations and check the validity of the results from the context of the problem.

Mathematical Reasoning 3.1

Evaluate the reasonableness of the solution in the context of the original situation.

What it means for you:

You'll think about how accurate answers to questions need to be, and when it's a good idea to round them.

Key words:

- exact
- approximate
- rounding
- round-off error

Check it out:

A wavy equals sign " \approx " means "is approximately equal to."

Don't forget:

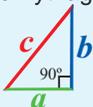
An irrational number is a decimal that carries on forever without repeating.

Don't forget:

Circumference = $\pi \times$ diameter

Don't forget:

The Pythagorean Theorem:

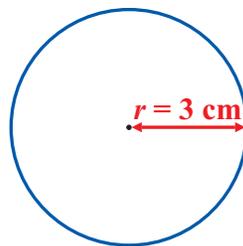


$$a^2 + b^2 = c^2$$

When you're figuring out the answer to a math question, it's important to think about *how precise* your answer needs to be. You have to decide if it's *sensible* to *round* an answer or not, and *how much* to round it by. And that's what this Lesson is about.

Leave π and $\sqrt{\quad}$ In For a Completely Accurate Answer

Sometimes in math you'll need to give very **exact** answers, and sometimes you'll only be able to give an **approximate** answer.



Think about finding the **area** of this **circle**. The formula is: **Area = πr^2** .

To find the area, you would do the calculation: $\text{Area} = \pi \times 3^2$. But there are different ways that you could write your answer.

- π is an **irrational number**, so the only way to write the answer absolutely **accurately** would be: **Area = $9\pi \text{ cm}^2$**
- If you're asked for an **approximate** answer then round the number off: **Area $\approx 28.3 \text{ cm}^2$ (to 1 decimal place)**

If the question doesn't tell you **how precise** your answer needs to be then make it **as accurate as possible**. That means leaving **irrational numbers**, like π or square roots, and **non-terminating decimals** in your answer.

Guided Practice

1. What is the area of a circle with a radius of 5 feet?
2. If the radius of planet Earth at the equator is 6380 km, what is its circumference at the equator? Give your answer to the nearest 100 km.
3. You are asked to do the calculation $\frac{1}{3} \times 4$. Think of two ways that you could write your answer exactly.
4. A square has a side length of 10 cm. What is the length of its diagonal? What is the length of its diagonal to 2 decimal places?
5. 6 people share 10 pears equally. How many pears will each person get to 1 decimal place? How many thirds of a pear will each person get?

The Data in the Question Decides the Accuracy

In **real-life problems**, approximate answers often make more sense than exact ones. There are two things to think about when deciding **whether to round** your answer, and **how to round it**:

1) The context of the question.

As you saw in the last Lesson, how you round may be affected by what the question is asking you to find.

Example 1

Lupe is making buttons. It cost her \$15 to make 13. What is the lowest price she can sell each one for and make at least as much as she spent?

Solution

Each button cost Lupe exactly $\$ \frac{15}{13}$ to make. $\frac{15}{13} = 1.\overline{153846}$.

- As she can't charge less than a cent, you should round to 2 decimal places.
- And as she needs to make at least what she spent, round up not down.

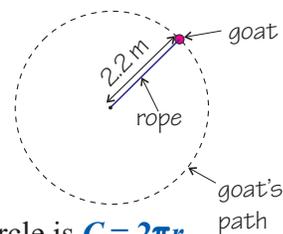
So Lupe needs to charge **\$1.16** for each button.

2) The accuracy of the data in the question.

Sometimes data you are given to use in a question will be approximate. If it is, then your answer depends on how precise the data is.

Example 2

A goat is tied to a length of rope, which is measured as 2.2 m long. If the goat walks a complete circle as shown, how far has it walked?



Solution

The formula for finding the **circumference** of a circle is $C = 2\pi r$.

Using $\pi = 3.142$, the goat has walked $2 \times 3.142 \times 2.2 = 13.8248$ m.

But the rope's length is **approximate** — it could be a little more or less than 2.2 m. You are told the rope's length **to the nearest 0.1 m**, so it's sensible to give your final answer to the nearest meter.

The goat has walked 14 m to the nearest meter.

Check it out:

Measurements always introduce some error — you can never measure anything completely accurately.

Although the rope in Example 2 is 2.2 m long to the nearest 0.1 m, you don't know the exact length. For example, it could be 2.21 m or 2.18 m.

Check it out:

When you have approximate data in a calculation involving multiplication, any rounding errors are multiplied, making your answer less precise than the data you began with.

In Example 2, the least precise measurement was given to 1 decimal place. This was then multiplied by 2π , giving a bigger rounding error. This makes it unreasonable to give the answer to 1 decimal place — it's more sensible to give the answer to the nearest whole number instead.

The reason for this is demonstrated in more detail in Example 3.

Guided Practice

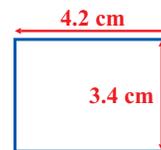
6. Lee measures the legs of a right triangle as 6.2 in. and 8.3 in., to the nearest tenth of an inch. He calculates the hypotenuse as 10.36 in. Is this an appropriate level of accuracy? Explain your answer.
7. La-trice completes a motor race of 190 miles, to the nearest ten miles. She then drives the car a further 0.92 miles back to the pit lane. Should the total distance she traveled be given to the nearest 10 miles, to the nearest mile, or to the nearest hundredth of a mile?
8. Eli wants to make a tablecloth that overhangs by 10 cm for his rectangular table. To what level of accuracy should he measure the length and width of his table?

Rounding Makes Your Answer Slightly Inaccurate

Rounding numbers creates small inaccuracies called **round-off errors**.

The length of this rectangle's sides have been measured to the nearest tenth of a centimeter. Think about finding its **area**:

$$\text{Area} = \text{length} \times \text{width} = 4.2 \times 3.4 = \mathbf{14.28 \text{ cm}^2} \approx \mathbf{14.3 \text{ cm}^2}$$



Check it out:

The length could be anything from 4.15 cm to just less than 4.25 cm. All the values in this range would round to 4.2 cm.

The width could be anything from 3.35 cm to just less than 3.45 cm. All the values in this range would round to 3.4 cm.

But the measurements are **rounded** to the nearest tenth of a centimeter.

So actually: $\mathbf{4.15 \text{ cm} \leq \text{length} < 4.25 \text{ cm}}$ and $\mathbf{3.35 \text{ cm} \leq \text{width} < 3.45 \text{ cm}}$.

The **minimum area** of the rectangle is found by multiplying the smallest possible length and the smallest possible width, so:

$$\mathbf{\text{Minimum area}} = 4.15 \times 3.35 = \mathbf{13.9025 \text{ cm}^2}$$

And the rectangle's **maximum area** = $4.25 \times 3.45 = \mathbf{14.6625 \text{ cm}^2}$

The **actual value** could be anywhere between these two. The **difference** between the true value and your calculated value is a **round-off error**.

✓ Guided Practice

9. Daisy measures the lengths of 2 planks as 10.2 m and 5.6 m to the nearest 10 cm. She adds them to give a total of 15.8 m. Find the greatest and least possible sums of the lengths.

10. Rey measures a triangle's base as 10 mm, and its height as 6 mm to the nearest mm. With round-off error, what is its minimum area?

11. Shantel is finding the product of 1.86 and 0.55. She rounds both numbers to 1 decimal place, multiplies them and gives her answer to 1 decimal place. What round-off error has she introduced?

Check it out:

The earlier you round, the bigger your round-off error tends to be. If Shantel had multiplied 1.86 and 0.55, then rounded her solution, her round-off error would have been smaller.

✓ Independent Practice

1. Liam measures the base of a triangle as 2.34 m and its height as 1.69 m. What is the triangle's area to the nearest m^2 ?

2. A square has a side length of one seventh of a meter. What is its exact area? What is its area to 2 decimal places?

3. Zoe and Tion both add a third to a seventh and give the answer to 2 decimal places. Their work is below. Which answer is most accurate?

Zoe

$$\frac{1}{3} + \frac{1}{7} = \frac{7}{21} + \frac{3}{21} = \frac{10}{21}$$

$$10 \div 21 = 0.48 \text{ (2 decimal places)}$$

Tion

$$\frac{1}{3} \approx 0.33 \quad \frac{1}{7} \approx 0.14$$

$$0.33 + 0.14 = 0.47 \text{ (2 decimal places)}$$

4. Kelly measures the side length of a cube as being 10.1 cm to the nearest mm. With round-off error, what is its minimum volume?

5. Inez adds the areas of a circle with a 2 cm radius, and a triangle with a base of 6 cm and a height of $\sqrt{2}$ cm. What is her exact answer?

Now try these:

Lesson 8.3.3 additional questions — p473

Round Up

Sometimes in math you'll be asked to give an **approximate answer**. Always think carefully about **how much to round your answer**. And don't forget that rounding always introduces **round-off errors**.

Lesson 8.3.4

Reasonableness and Estimation

California Standards:

Mathematical Reasoning 2.1

Use estimation to verify the reasonableness of calculated results.

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

Mathematical Reasoning 3.1

Evaluate the reasonableness of the solution in the context of the original situation.

Number Sense 1.3

Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

What it means for you:

You'll think about ways to check whether the answer to a question is sensible and of roughly the right size.

Key words:

- reasonable
- sensible
- estimate

Check it out:

Think about:

- Whether your answer ought to be a whole number.
- Whether your answer should be negative or positive.
- Whether to round an answer up or down.

When you answer a math question, you need to be sure your answer makes sense and is about the right size. Making an *estimate* before doing a calculation is a good way to check your answer is sensible — if your estimate is very *different* from your answer, you'll know there's an *error* somewhere.

Think About Whether Your Answer is Sensible

The first thing to look at is whether your answer is a **sensible** answer to the **particular question** you've been asked.

Example 1

Mrs. Moore is splitting students into teams. She needs to split 59 students into 4 teams, as equal in size as possible. What would be a reasonable way to split the students up?

Solution

If Mrs. Moore split the class **equally** there would be $59 \div 4 = 14.75$ people on a team. This isn't reasonable. You can't put part of a person on a team.

Rounding up doesn't work — 15 people on each of 4 teams needs 60 people. And **rounding down** to 14 means some people are left out.

The most reasonable thing to do would be to split the students into **almost equal teams** of **15, 15, 15, and 14**.

✓ Guided Practice

In Exercises 1–3, say whether the answer given is reasonable.

1. A camp has 4 empty tents and 18 new visitors. So the camp supervisor decides to put 4.5 people in each tent.
2. Kea has \$7. 1 kg of plums costs \$4. Kea says she can buy 1.75 kg.
3. The area of a square is 36 cm^2 . Alan finds its side length by taking the square root of 36. He says the side length of the square is ± 6 cm.

Look at Whether Your Answer is the Right Size

Another thing to think about is whether your answer is about the **right size**. Sometimes it's quite clear that your answer is the wrong size.

Example 2

Rocio wants to find out how far 2 miles is in meters. She does a calculation and gets the answer 3.2 meters. Is she likely to be correct?

Solution

2 miles is a fairly long walk, but 3.2 meters is only about as big as two adults lying head to toe. Her answer isn't likely to be correct.

Use an Estimate to Check Your Answer

Sometimes it's not quite so obvious that an answer is the wrong size. So, it's a good idea to **estimate** the answer to a problem before you solve it.

You do this by **rounding** the numbers and using **mental math** to do a simple calculation. If the **estimate** is **about the same** as the **answer**, you'll know it's probably right.

Think about finding the **product** of **41** and **29**. This is what Ralph did:

- 1) He estimated the answer first by **rounding** both numbers to the nearest 10.

- 2) Ralph's answer of 451 is **very different** from his estimate of 1200. He thinks there may be an error in his work.

- 3) He **checks his work** and finds his error. With the error corrected, his worked answer is close to his estimate — so he can be more confident his answer is right.

Estimate: $40 \times 30 = 1200$	
41	41
29	29
369	369
82	820
451	1189
x	✓

The error Ralph made was a missing 0 in the tens column.

Whenever you think your answer to a question is **not reasonable**, you should go back and **check your work** to find the **error**.

Estimation can be very useful in **real-life problems** too.

Example 3

Ceria buys a sweater for \$51.99 and jeans for \$39.50. Sales tax is charged at 8.75%. She pays \$99.50. Use estimation to check if the total cost is about right.

Solution

First round the costs of the items and add them:

$$\$51.99 \approx \$50 \quad \$39.50 \approx \$40$$

$$\$50 + \$40 = \mathbf{\$90}$$

Now round the sales tax rate, and apply it to the total cost:

$$8.75\% \approx 10\%$$

$$\$90 \times 0.1 = \mathbf{\$9}$$

Add the tax to the item's cost to find the final price: $\$90 + \$9 = \mathbf{\$99}$

This is very close to the total cost, so it is probably correct.

Don't forget:

The symbol \approx means "is approximately equal to."

✓ Guided Practice

4. Find the product of 51 and 68. Check your answer using estimation.
5. Karl put \$1021 into a savings account paying 6% simple interest per year. Estimate roughly how much interest Karl will earn in a year.
6. Ruby's meal cost \$39.95. She wants to tip the waiter 15%. She says she should leave about \$4. Is she right? Estimate what tip to leave.
7. The school council sold 197 tickets to a dance. A ticket entitles you to 2 cartons of juice. If juice cartons come in boxes of 52, estimate how many boxes the school council should buy.

✓ Independent Practice

In Exercises 1–4, say whether the answer given is reasonable.

1. Umar buys lettuce for \$2.10, some bananas for \$2.05, and a melon for \$4. He estimates that his bill will be about \$80.
2. A shirt selling for \$51.30 is discounted by 22%. Clare says this is a reduction of about \$10. Is her estimate reasonable?
3. In winter, the temperature outside Iago's house is 23 °F. He converts it to °C, and says it is –5 °C.
4. Lashona measures the legs of a right triangle as 7 in. and 9 in. Using the Pythagorean theorem, she says the hypotenuse is 11.4018 in. long.
5. Rachel's cab fare is \$32. She wants to give a 10% tip. Rachel says this is \$10. Does this seem reasonable?
6. Jeron and Ann are painting. Jeron paints 1.8 walls/hour, and Ann paints 2 walls/hour. Ann says it will them take less than 2 hours to paint 7 walls. Is this a reasonable thing to say?
7. Felix finds the length of a rectangle with a 10 mm² area and a 3 mm width. He says its length is 3.333333 mm. Is this a sensible answer?
8. Tandi is finding the circumference of a circular trampoline. She measures its radius as 3 m, and says its circumference is 19 m. Does this seem reasonable?
9. Ben is saving up to buy a \$98 camera. To earn money he washes cars, charging \$12 per car. Estimate the number of cars he will have to wash to earn \$98.
10. Xenia uses the Pythagorean theorem to find the hypotenuse of a right triangle. Its legs are 36 cm and 60 cm. She gets 48 cm. Is this sensible?
11. Divide 9.88 by 5.2. Check your answer using estimation.
12. Evan walked for 1.9 km at 2.8 km/hr, and then for 5.9 km at 3.2 km/hr. Estimate how long his walk took in hours and minutes.

Don't forget:

$$^{\circ}\text{C} = \frac{5}{9} (^{\circ}\text{F} - 32).$$

Now try these:

Lesson 8.3.4 additional questions — p473

Round Up

*It's always important in math to think about whether your answers are **reasonable** or not — there's no point in giving an answer that **doesn't make sense**. Remember that you can always **estimate** before finding an **exact answer**. Then you'll have an idea whether your answer is right or not.*

Chapter 8 Investigation

Nutrition Facts

Percents are used a lot in real life, so you really need to get confident working with them. In this Investigation, you'll see how percents are used to provide information about foods.

On the right is a **nutrition facts label** from a packet of crackers. The table below shows the recommended daily values you should eat if you need 2000 calories or 2500 calories a day.

	2000 calories per day	2500 calories per day
Total fat	65 g	80 g
Saturated fat	20 g	25 g
Cholesterol	300 mg	300 mg
Sodium	2400 mg	2400 mg
Total carbohydrates	300 g	375 g
Dietary fiber	25 g	30 g

NUTRITION FACTS		
Serving size = 30 g Servings per box = 5		
Amount Per Serving		
Calories	120	Calories from Fat 40
% Daily Value		
Total Fat	5.5 g	8%
Saturated Fat	0.6 g	3%
Trans Fat	0 g	
Cholesterol	0 mg	
Sodium	180 mg	7%
Total Carbohydrate	21 g	7%
Dietary Fiber	3 g	12%
Sugars	0 g	
Protein	4 g	

The **Percent Daily Value** figures on the label show what percent of the recommended daily value each serving contains.

The number of calories you need depends on things like your gender, and the exercise you do.

- 1) How many **calories per day** are the **percent daily values** on the nutrition facts label based on?
- 2) The label reads 7% for the percent daily value of total carbohydrate. What **fraction** was **converted** to report this percent?
- 3) Suppose there were 8 mg of cholesterol in a serving size. What would you report as the percent daily value for this amount? Explain the calculations and rounding technique you used.
- 4) The percent daily value of sodium is found by dividing the 180 milligrams of sodium in the crackers by the 2400 milligrams of sodium recommended. What **rounding technique** did the company use to post a percent daily value of 7%? Suggest a reason why this was.

Extension

- 1) Compute the percent daily values for the crackers above using the 2500 calorie diet.
- 2) The same company put out a reduced fat version of the same cracker. The nutrition facts label is shown here. Calculate the **percent increase** or **decrease** in the actual amounts in each category (NOT the percent daily values), going from the original to the reduced fat cracker. Round your answers to the nearest whole percent.

Open-ended Extension

Find food labels for two similar products. Calculate the percent differences between the products. Then make a poster comparing the products.

NUTRITION FACTS		
Serving size = 30 g Servings per box = 5		
Amount Per Serving		
Calories	120	Calories from Fat 25
% Daily Value		
Total Fat	3 g	5%
Saturated Fat	0 g	0%
Trans Fat	0 g	
Cholesterol	0 mg	
Sodium	150 mg	6%
Total Carbohydrate	23 g	8%
Dietary Fiber	3.5 g	14%
Sugars	0 g	
Protein	3 g	

Round Up

Percent daily values make things easier to interpret. It'd be hard to remember how many grams of different things you should have — using percent daily values mean you don't need to.

✓ Lesson 1.1.1 — Variables and Expressions

Write the variable expressions in exercises 1 – 4 as word expressions.

- $10(b + 8)$
- $2p + 7$
- $3r - 4s$
- $5(2x + 6)$

Evaluate the expressions in exercises 5 – 8 when $r = 4$ and $s = 7$

- $2(rs - r^2)$
- $s^2 - 7r - 3s$
- $(2rs - 2) \div 6$
- $5s - 3r + 30$

Mike and Abdul collect model cars. Mike has m cars and Abdul has $10m$ cars.

- Write a sentence that describes how many cars Abdul has compared to Mike.
- If Mike has 6 cars in his collection how many does Abdul have?
- A company uses the formula $25 + 13.50h$ to calculate the daily cost in dollars for renting lawn equipment. The variable h represents the number of hours the equipment is rented for. How much would it cost to rent a piece of equipment for 4 hours?
- The formula for the perimeter, P , of a rectangle is given by the formula $P = 2l + 2w$ where l is the length and w is the width. Find the perimeter of a room whose length is 10 feet and width is 14 feet.

✓ Lesson 1.1.2 — Simplifying Expressions

Simplify the expressions in exercises 1 – 6 by expanding parentheses and collecting like terms.

- $3x + 5y + 8x - 3y + 2 + 4$
- $-5(7c - 8)$
- $-e(2f - 7)$
- $2x - 5 - 8x + 11$
- $5(3r + 6) - 23$
- $5(7a + 6) + 4(5 - 3a)$

7. Hector is 3 years older than Ami. Kim is twice as old as Hector. Toni is 5 years younger than Kim. If Hector is x years old, write an expression for the combined age of Hector, Ami, Kim and Toni, then simplify your answer as much as possible.

Lisa earns \$8 per hour in her job. She works a fixed 20 hours between Monday and Friday and sometimes works extra hours at the weekend.

- Write an expression to describe how much money she earns in a week if she works an extra h hours over the weekend.
- Use your expression to find how much Lisa earns in a week if she works 6 hours over the weekend.
- Kendra, Shawn, and Mario are collecting bottles for a recycling project at their school. Kendra collected 5 times as many bottles as Shawn. Mario collected 15 bottles. Let s represent the number of bottles Shawn collected. Write down and simplify an expression for the total number of bottles collected.

✓ Lesson 1.1.3 — Order of Operations

Evaluate.

- $16 \div 2 \cdot 4 + 5$
- $24 + 4 \cdot 3 - 8^2$
- $(2 + 3)^2 \cdot (11 - 8)$

Simplify.

- $k \cdot (8 + 2) - 12$
- $4 + t^2 \cdot (12 \div 3 + 6)$
- $y + 4^2 - (9 - 2^3) \cdot y + 25$

7. Insert parentheses into the expression $3 + 8 \cdot 4^2 - 10 \div 2$ to make it equal 27.

The local brake repair shop charges \$65 per hour for labor plus cost of parts.

- Write a calculation to describe the cost of a 3-hour repair if parts cost \$54.
- Evaluate your expression to find the cost of the job.

✓ Lesson 1.1.4 — The Identity and Inverse Properties

1. What is the multiplicative inverse of $\frac{2}{3}$?
2. What is the additive inverse of $(x + y)$?
3. Does zero have a multiplicative inverse? Explain your answer.

Simplify the expressions in Exercises 4 – 10. Justify each step.

4. $2 + x - x$
5. $6 - a \cdot 1$
6. $-12a + 12a + 5$
7. $\frac{1}{2}(4b + 2) + b$
8. $d \cdot 1 + 2 - d$
9. $8(p - \frac{1}{8}) + 1$
10. $2(3y + \frac{1}{2} + 0) + (-8 + 8 - 6y)$

Determine if the following statements are true or false.

11. Any whole number can be written as a fraction.
12. Any fraction can be written as a whole number.
13. A number multiplied by its reciprocal is always 1.
14. A number divided by itself is zero.

✓ Lesson 1.1.5 — The Associative and Commutative Properties

Identify the property used in Exercises 1 – 3.

1. $2 + x = x + 2$
2. $(2 \cdot 3) \cdot 7 = 2 \cdot (3 \cdot 7)$
3. $(2x + 5)8 = 16x + 40$

Simplify the expressions in Exercises 4 – 9. Justify your working.

4. $(5 + 2x) + 3x$
5. $2v + 7 + 8v$
6. $12(7g)$
7. $-3y + (7y + 2 - 3)$
8. $7r + 6 + 5r + 4$
9. $5 \cdot w \cdot 7$

Determine if statements 10 – 14 are true or false.

10. Subtraction is commutative.
11. Division is commutative.
12. Subtraction can be rewritten as addition by adding the opposite.
13. $5 - (a - 3) = (5 - a) - 3$
14. $a \div b = a \times \frac{1}{b}$

✓ Lesson 1.2.1 — Writing Expressions

Write the variable expressions to describe the word expressions in exercises 1 – 6.

1. the product of 5 and a number, x
2. the quotient of a number, y , and 10
3. 12 less than a number, c
4. 2 increased by twice a number, f
5. the product of 8 and the sum of a number, r , and 5
6. 15 decreased by twice the quotient of a number, p , and 4

Are these statements true or false? If false, rewrite the statement so that it is correct.

7. To triple a number means to add 3 to the number.
8. To quadruple a number means to multiply the number by 4.
9. Twice a number means to raise a number to the second power.

Write variable expressions for the following. Use x as the variable in each case and say what it represents.

10. The height of a triangle is 8 cm. What is the area of the triangle?
11. The width of a rectangle is 9 meters. What is the area of the rectangle?
12. William has \$25 more than Luke. How much money does William have?
13. The video store charges a monthly membership fee of \$15 plus \$2.50 per movie rental. What is the total cost of per month?

✓ Lesson 1.2.2 — Variables and Expressions

Prove the equations are true in Exercises 1 – 3.

- $2(10 - 7) + 4 = 55 \div 11 \cdot 2$
- $8 \cdot 3 \div 6 \cdot 9 = 6^2$
- $7 - 4 - 2 \cdot \frac{1}{2} + 5 = 5(2^3 + 1) - (3 \cdot 13 - 1)$

Say whether each of the following is an expressions or an equation.

- $3b$
- $4x = 12x + 5$
- $2 + 7 = 32$
- $2a - 4$

Write an equation to describe each of the sentences in Exercises 8 – 14.

- Three more than the product of five and b is equal to 40.
- Twenty decreased by the quotient of four and h is equal to 10.
- Eight increased by the product of three and c is equal to the difference of five and c .
- Three less than the quotient of y and 2 is equal to the sum of 11 and y .
- Jessica earns \$8.50 per hour. She earned \$306 for working h hours.
- A cell phone company charges a \$10 monthly fee plus \$0.05 a minute for phone calls. Denise's monthly bill for m minutes of phone calls was \$14.75.
- Jose had \$75. He bought 3 movie tickets at \$ d a ticket and spent \$25 on food, leaving \$24.50.

✓ Lesson 1.2.3 — Solving One-Step Equations

Name the operation that is appropriate for solving each of the equations in Exercises 1 – 3.

- $5t = 45$
- $b - 7 = 3$
- $w \div 12 = 4$

Find the values of the variables in Exercises 4 – 9.

- $3k = 39$
- $b - 5 = 12$
- $-45 = x - 40$
- $-18 = -3p$
- $h \div -8 = 11$
- $a - 24 = -60$

- The Spring Hill city council is planning to construct a new courthouse that, at 970 feet, will be twice as tall as the existing courthouse. Use the equation $2x = 970$ to find the height of the existing courthouse.
- Marcus purchased an outfit for \$54 after receiving a \$15 markdown. This can be described by the equation $x - \$15 = \54 . Solve the equation and say what x represents.
- The height of the Washington Monument is 152 meters. The combined height of the Washington Monument and the Statue of Liberty is 245 meters. Write an equation to find the height of the Statue of Liberty and then solve for the height.

✓ Lesson 1.2.4 — Solving Two-Step Equations

In exercises 1 – 4 say which order you should undo the operations in.

- $8x - 2 = 22$
- $w \div 12 + 8 = 12$
- $3 \cdot (d - 4) = 24$
- $b \div 2 - 4 = 6$

Find the values of the variables in Exercises 5 – 10.

- $3b + 4 = 16$
- $30 = 12 + 9x$
- $y \div 8 + 10 = 15$
- $-45 = x \div 9 - 48$
- $18 = -3p + 9$
- $8h - 12 = -76$

- A decorator charges \$17 an hour plus a \$25 fee for each job. In one job the decorator made \$620. Write an equation using this information and solve it to find how many hours the job took.
- The video store charges a monthly membership fee of \$15 plus \$2.50 per movie rental. Amanda's monthly bill was \$30. Write an equation and solve it to find the number of movies she rented.

Lesson 1.2.5 — More Two-Step Equations

Find the values of the variables in Exercises 1 – 6.

1. $\frac{1}{3}c = 8$

2. $6 = \frac{3}{4}a$

3. $-\frac{4}{3}b = -8$

4. $-\frac{5}{6} \cdot w = 10$

5. $\frac{3b}{5} = -9$

6. $\frac{y+7}{4} = 8$

Solve the equations in exercises 7 – 10 and check your solution.

7. $3x - 5 = 7$

8. $-18 = 2y - 6$

9. $n \div 8 + 2 = -7$

10. $\frac{5}{6}t = -10$

In Exercises 11 – 14 say which order you should undo the operations.

11. $\frac{2}{3}x = 8$

12. $-\frac{4x}{7} = 3$

13. $\frac{4c+5}{9} = 3$

14. $-5 + \frac{3a+2}{8} = -12$

Lesson 1.2.6 — Applications of Equations

In Exercises 1 – 2 use estimation to pick the answer that is reasonable.

- Joan's annual income is \$16,276. Which of the following is her weekly income?
a. \$846,352 b. \$313 c. \$1600
- A car travels at a speed of 45 mph. Which of these is the distance it would travel in $\frac{1}{2}$ an hour?
a. 22.5 miles b. 90 miles c. 15 miles
- Tonya is 10 years younger than her brother. The sum of their ages is 32. How old is Tonya?
- A decorating firm needs to order 362 gallons of paint. Paint is sold in 5 gallon containers. Write an equation to describe the number of containers, n , they must order. Solve the equation and say if your answer is reasonable in the context of the situation.
- Lana got her car repaired at the local garage. She paid \$585 for parts and labor. The parts cost \$225. She was billed for 8 hours of labor. Solve an equation to find the hourly labor charge, $\$d$.
- A parking garage charges \$3 for the first 2 hours, then \$2.50 for each additional hour. How many hours, h , after the first two hours can you keep your car in the garage if you have \$15?
- Joyce spent \$205 on supplies for her art work. She paid \$175 for the easel and canvas. She also bought two sets of paint brushes which cost $\$d$ each. How much did each paint brush set cost?

Lesson 1.2.7 — Understanding Problems

In Exercises 1 – 3, say what missing piece of information is needed to solve each problem.

- The length of a rectangle is 5 cm. What is the area?
- Neal has \$75. How long is it going to take him to save \$200?
- Greg's car repair bill was \$225. Parts cost \$85. What was the hourly labor charge?

In Exercises 4 – 5, solve the problem and state what information is not relevant.

- Taylor's cell phone plan charges a monthly fee of \$15 plus \$0.05 a minute for each call. Taylor's cell phone costs \$65. How much was last month's bill if she used 90 minutes?
- Kenneth makes \$12 per hour in his job. Last week he worked 5 days. How many hours did he work if his paycheck was \$420?

In Exercises 6 – 7, fill in the blanks to make the statements true.

- _____ miles \div 2 hours = 48 _____
- 15 meters/second \cdot _____ = 45 meters
- _____ Newtons \cdot 6 _____ = 48 Newton-meters

Lesson 1.3.1 — Inequalities

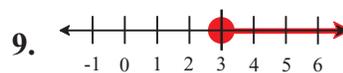
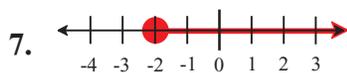
Fill in the blanks in Exercises 1 – 3.

1. If $x > y$ then y ___ x .
2. If $m \leq n$ then n ___ m .
3. If $a \geq b$ then ___ \leq ___.

In Exercises 4 – 6 give a number that is part of the solution set of the inequality.

4. $x > -4$
5. $-5 \geq m$
6. $x \geq 2$

Write the inequality expression for the given number line in Exercises 7 – 9.



Determine if the statements in Exercises 10 – 13 are true or false.

10. Inequalities have an infinite number of solutions.
11. -4 is in the solution set of $x > -3$.
12. 5 is in the solution set of $y \geq 5$.
13. $t > 2$ means the same thing as $2 < t$.

Lesson 1.3.2 — Writing Inequalities

In Exercises 1 – 3, insert an inequality symbol to make a true statement.

1. 24 inches ___ 3 feet
2. 1 century ___ 6 decades
3. 5 days ___ 50 hours

Write an inequality to describe the sentences in Exercises 4 – 7.

4. A number, p , decreased by four is more than fifteen.
5. Twenty-five times a number, m , is less than or equal to seven.
6. The quotient of a number, y , and three is less than ten.
7. Eight increased by a number, c , is at least twelve.

Write an inequality to describe the situations in exercises 8 – 11.

You will need to use and define a suitable variable in each case.

8. The maximum weight permitted on the bridge is ten tons.
9. You must be at least twenty-five years old to run for an elective position on the city council.
10. The minimum allowed height for entry to a rollercoaster ride is 120 cm.
11. Sarah and Miguel have over \$350 in savings combined. Sarah has twice as much as Miguel.

Lesson 1.3.3 — Two-Step Inequalities

Write an inequality to describe the sentences in Exercises 1 – 4.

1. Seven more than the quotient of a number, y , and three is less than twenty.
2. Eight increased by the product of a number, w , and five is at least twelve.
3. Nine is less than or equal to the difference of five and the product of two and a number, h .
4. Sixteen less than the quotient of a number, k , and five is at most twenty-four.
5. Danielle ordered a meal that cost under \$20. She had a \$12 main course and a drink and desert each costing \$ d . Write an inequality to describe how much Danielle spent.
6. Pam is making a rectangular fenced area in her back yard. She has 100 meters of fencing. The width of the fenced area must be 20 meters. Write an inequality to describe the possible lengths, l .
7. Roberto needs an average score of at least 90 from four algebra tests to gain a grade A. His first three test scores are 98, 97, and 82. Write an inequality to describe what Roberto's final test score, x , must be in order to get an A.

Lesson 2.1.1 — Rational Numbers

Show that the numbers in Exercises 1–4 are rational.

1. -6
2. 4
3. -4.5
4. 0.75

Convert the fractions in Exercises 5–8 into decimals without using a calculator.

5. $\frac{3}{5}$
6. $\frac{3}{8}$
7. $\frac{84}{5}$
8. $\frac{5}{6}$

Say whether the statements in Exercises 9–14 are true or false.

9. All terminating decimals are rational numbers.
10. All decimals can be written as the quotient of two integers.
11. All rational numbers can be written as a decimal.
12. Terminating and repeating decimals are rational numbers.
13. π is a rational number because you can write it as a fraction by putting it over 1.
14. $\frac{1}{6}$ is a terminating decimal because when you do $1 \div 6$ on a calculator you get 0.16666667.

Lesson 2.1.2 — Converting Terminating Decimals to Fractions

Convert the decimals in Exercises 1–3 into fractions without using a calculator.

1. 0.37
2. -0.103
3. 0.023

Convert the decimals in Exercises 4–6 into fractions and simplify them if possible.

4. 0.208
5. -12.84
6. -4.005

Say whether the statements in Exercises 7–10 are true or false.

7. $0.23 = 0.2300$
8. $2.05 = 2.50$
9. Dividing the numerator and denominator of a fraction by the same number makes another fraction with the same value as the original.
10. Any decimal greater than 1 can be written as a mixed number or an improper fraction.
11. Mario noticed that 8 out of 10 people on a particular team were female. Sarah looked at the same team and said 4 out of 5 people were female. How can they both be right?

Lesson 2.1.3 — Converting Repeating Decimals to Fractions

In Exercises 1–3, use $x = 0.\overline{5}$.

1. Find $10x$.
2. Use your answer to Exercise 1 to find $9x$.
3. Write x as a fraction in its simplest form.

In Exercises 4–6, use $x = 1.\overline{67}$.

4. Find $100x$.
5. Use your answer to Exercise 4 to find $99x$.
6. Write x as a fraction in its simplest form.

Say whether the statements in Exercises 7–9 are true or false.

7. $1.\overline{235} = 1.235\overline{35}$
8. $32.\overline{34} = 323.\overline{4}$
9. Every rational number is a terminating or repeating decimal.

Find whether each number in Exercises 10–12 is equal to $3.\overline{8}$.

10. $\frac{35}{9}$
11. $3\frac{8}{10}$
12. $3\frac{8}{9}$

✓ Lesson 2.3.2 — Multiplying and Dividing Integers

Evaluate Exercises 1–4 using the number line.

- 3×2
- -2×4
- $12 \div 4$
- $-10 \div 5$

Evaluate Exercises 5–6 by drawing a rectangle and breaking the numbers into tens and units.

- 15×14
- 24×18

Evaluate Exercises 7–8 using long multiplication.

- 15×14
- 24×18

Evaluate Exercises 9–17 without using a calculator.

- -12×84
- -62×-34
- 14×-36
- $816 \div 3$
- $448 \div 7$
- $736 \div 8$
- $1455 \div -5$
- $-3258 \div 9$
- $728 \div 14$

Say whether each statement in Exercises 18–19 is true or false.

- The product of two negative numbers is positive.
- The quotient of a positive number and a negative number is negative.

✓ Lesson 2.3.3 — Multiplying Fractions

Use the area model to evaluate the fraction multiplications in Exercises 1–2.

- $\frac{1}{2} \times \frac{1}{4}$
- $\frac{3}{4} \times \frac{1}{2}$

Find the product in Exercises 3–5.

- $\frac{2}{3} \times \frac{5}{7}$
- $\frac{4}{5} \times \frac{6}{7}$
- $\frac{3}{8} \times \frac{4}{5}$

Find the product in Exercises 6–11. Simplify the results as much as possible.

- $\frac{3}{10} \times \frac{8}{15}$
- $1\frac{1}{2} \times \frac{2}{3}$
- $4\frac{1}{6} \times 3\frac{2}{5}$
- $-8 \times \frac{3}{4}$
- $\frac{-6}{25} \times \frac{-5}{12}$
- $\frac{2}{3} \times \frac{6}{15}$

- A rectangular patio measures $4\frac{3}{8}$ meters long and $5\frac{1}{4}$ meters wide. What is the area of the patio as a mixed number?

✓ Lesson 2.3.4 — Dividing Fractions

Find the reciprocals of the numbers in Exercises 1–3.

- $\frac{3}{4}$
- $\frac{5}{6}$
- 2

Calculate the divisions in Exercises 4–9. Give your answers as fractions in their simplest form.

- $\frac{3}{4} \div 3$
- $\frac{5}{8} \div \frac{3}{7}$
- $\frac{3}{8} \div \frac{5}{6}$
- $-\frac{2}{3} \div \frac{4}{9}$
- $-\frac{11}{12} \div -\frac{25}{24}$
- $-5 \div -\frac{5}{4}$

Evaluate the expressions in Exercises 10–12 and express the solutions as mixed numbers or integers.

- $-8\frac{1}{2} \div 6$
- $-5\frac{3}{5} \div -\frac{2}{25}$
- $1\frac{1}{7} \div \frac{-4}{5}$

- What is the product of a number and its reciprocal?

- The area of a room is $140\frac{1}{4}$ square feet, and its length is $14\frac{1}{2}$ feet. What is the width of the room?

Lesson 2.3.5 — Common Denominators

Find the prime factorization of the numbers given in Exercises 1–6.

- 18
- 24
- 56
- 11
- 120
- 150

Find the least common multiple of the pairs of numbers in Exercises 7–12.

- 2 and 3
- 5 and 8
- 4 and 6
- 8 and 12
- 3 and 9
- 12 and 18

In Exercises 13–16 put the fractions in each pair over a common denominator to show which is larger.

- $\frac{3}{4}$ and $\frac{1}{2}$
- $\frac{5}{6}$ and $\frac{3}{8}$
- $\frac{4}{5}$ and $\frac{7}{8}$
- $\frac{11}{12}$ and $\frac{9}{10}$

17. Put the fractions $\frac{3}{8}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{9}{20}$, and $\frac{3}{4}$ in order.

18. Three-fifths of the senior class voted to have the prom at the same place as last year's prom. One-third voted to change locations. Which option won the vote?

Lesson 2.3.6 — Adding and Subtracting Fractions

Evaluate the expressions in Exercises 1–9. Give your answers as fractions in their simplest form.

- $\frac{2}{3} - \frac{5}{3}$
- $\frac{4}{5} - \frac{3}{5}$
- $\frac{3}{4} + \frac{4}{9}$
- $-\frac{7}{2} - \frac{11}{12}$
- $\frac{7}{10} - \frac{9}{2}$
- $\frac{2}{3} + \frac{1}{6}$
- $-\frac{2}{3} + \frac{1}{2}$
- $-\frac{2}{3} - \frac{5}{6}$
- $-\frac{11}{15} - \left(-\frac{3}{20}\right)$

Say whether the statements in Exercises 10–11 are true or false.

10. Subtracting a negative number is the same as adding a positive number.

11. $-\left(\frac{3}{4}\right) = \frac{-3}{-4}$

12. Jennifer left home and jogged $\frac{5}{8}$ of a mile. She got tired, walked back $\frac{1}{4}$ of a mile, and then took a break. How far away from home was she when she took the break?

Lesson 2.3.7 — Adding and Subtracting Mixed Numbers

Evaluate the expressions in Exercises 1–12. Give your answers in their simplest form.

- $3\frac{1}{5} + 4\frac{2}{5}$
- $8\frac{5}{6} - 3\frac{1}{6}$
- $-4\frac{3}{8} - 2\frac{5}{8}$
- $10\frac{3}{5} - 6\frac{1}{2}$
- $5\frac{1}{8} + 4\frac{1}{3}$
- $4\frac{2}{9} - 3\frac{1}{6}$
- $4\frac{3}{4} - \left(-2\frac{5}{6}\right)$
- $-3\frac{5}{6} + 6\frac{1}{4}$
- $-3\frac{1}{4} - 1\frac{1}{2}$
- $-5\frac{2}{9} - \left(-4\frac{5}{6}\right)$
- $3 - 6\frac{1}{2} + 8\frac{3}{4}$
- $4\frac{1}{3} + 7\frac{3}{4} - 7\frac{5}{6}$

13. The length of a rectangular room is $3\frac{1}{2}$ feet, and its width is $5\frac{3}{4}$ feet.

What is the perimeter of the room?

14. In the morning, Jacob drank a glass and a half of fruit juice. That evening he drank another three and two-thirds glasses. How much juice did Jacob drink in total?

Lesson 2.4.1 — Further Operations With Fractions

Do the calculations in Exercises 1–12 and simplify your answers where possible.

1. $\frac{3}{5} + \frac{1}{5} \times \frac{3}{4}$

2. $\frac{3}{8} - \frac{5}{8} \div \frac{7}{16}$

3. $2\frac{1}{4} \div \frac{3}{8} \times \frac{1}{5}$

4. $\left(4 - \frac{3}{8}\right) \times \frac{2}{3}$

5. $3\frac{1}{4} \div 2\frac{1}{2} + 4$

6. $\frac{5}{6} \times 8 - 2\frac{3}{5} \times \frac{10}{3}$

7. $\left(2\frac{1}{2} - 6\frac{3}{4}\right) + 5 \times \frac{3}{8}$

8. $\frac{1}{8} \times \frac{2}{3} + \frac{-7}{12} - \frac{1}{6}$

9. $\left(-2\frac{3}{8} + \frac{1}{12}\right) \div 4\frac{3}{5}$

10. $\frac{4\frac{2}{3}}{\frac{5}{6}} + \frac{1}{\frac{8}{4}}$

11. $\frac{\left(\frac{5}{6} - \frac{1}{6}\right)}{\frac{5}{12}} - \frac{3}{4}$

12. $\frac{-2}{1\frac{3}{4}} + 2 \times \frac{4}{5}$

Exercises 13–14 are about a room that is $8\frac{1}{2}$ feet wide and $10\frac{3}{4}$ feet long.

13. What is the area of the room?

14. What is the perimeter of the room?

Lesson 2.4.2 — Multiplying and Dividing Decimals

Use the area model to solve the multiplications in Exercises 1–2.

1. 0.6×0.1

2. 0.8×0.1

Calculate the products in Exercises 3–5 by rewriting the decimals as fractions.

3. 0.6×0.5

4. 0.4×0.05

5. 1.2×2.3

Calculate the quotients in Exercises 6–8 by rewriting the decimals as fractions.

6. $0.3 \div 0.1$

7. $0.05 \div 0.1$

8. $15.96 \div 4.2$

Find the products in Exercises 9–11.

9. -1.23×-0.006

10. -2.04×-0.008

11. 3.5×-0.4

Say whether each statement given in Exercises 12–13 is true or false.

12. Dividing a number by 100 moves the decimal point two places to the right.

13. If you multiply together three decimals, each with two decimal places, then the answer can have up to six decimal places.

14. If $54 \div 18 = 3$, what is $0.54 \div 0.018$?

Lesson 2.4.3 — Operations With Fractions and Decimals

Calculate the value of each expression given in Exercises 1–9.

1. $\frac{3}{4} \times 0.05$

2. $-0.28 \times \frac{1}{2}$

3. $\frac{1}{4} \times \left(2.8 + \frac{1}{2}\right)$

4. $0.6 \times \left(\frac{1}{2} + 1.4\right)$

5. $\frac{3}{8} \div (4 \times 1.2)$

6. $\frac{2}{3} - 0.5$

7. $\left(\frac{5}{6} + 0.25\right) \times 0.8$

8. $\frac{-2.4 + 5.6}{\frac{3}{4}}$

9. $\frac{\frac{4}{5}}{0.5 + \frac{3}{4}}$

10. What is the area of a rectangular room that is $16\frac{3}{4}$ feet long and 10.5 feet wide?

11. What is the length of a rectangle with an area of $183\frac{2}{3}$ square feet and a width of 14.5 feet?

Lesson 2.4.4 — Problems Involving Fractions and Decimals

1. While cooking, Jose used $6\frac{1}{2}$ cups of flour, $2\frac{3}{4}$ cups of sugar, $5\frac{1}{2}$ cups of water, and $\frac{1}{4}$ cup of chocolate chips. How many cups of ingredients did he use in total?

Exercises 2–4 use the table on the right, which shows what proportion of 850 high school students use various methods of transport to get to school.

2. How many students walk or take the school bus to school?
3. How many students use a car to get to school?
4. How many students use public or “Other” transportation to get to school?

Walk	$\frac{4}{17}$
School bus	$\frac{5}{34}$
Public transportation	$\frac{9}{34}$
Car	$\frac{7}{34}$
Other	$\frac{5}{34}$

A neighborhood has a meeting hall with a floorspace of 206.4 square yards. On Wednesdays half of the floorspace is used by the knitting club.

5. How many square yards do the knitting club use?
6. How much would it cost to carpet the meeting hall if carpet costs \$8.25 per square yard?
7. A shop sells wire in spools and each spool has $9\frac{3}{4}$ yards of wire on it. Jeanie needs nineteen pieces of wire, each 1.5 yards in length. How many spools of wire must Jeanie buy?

Lesson 2.5.1 — Powers of Integers

Write each of the expressions in Exercises 1–9 as a power in base and exponent form.

1. $4 \cdot 4$ 2. $6 \cdot 6 \cdot 6$ 3. $9 \cdot 9 \cdot 9 \cdot 9$
4. $-6 \cdot -6$ 5. $-5 \cdot -5 \cdot -5$ 6. 7
7. $3 \cdot 3 \cdot 4 \cdot 4 \cdot 5$ 8. $-4 \cdot -4 \cdot 6 \cdot 6 \cdot 6$ 9. $-(-4 \cdot -4)$

Evaluate the expressions in Exercises 10–13.

10. $(-3)^2$ 11. $-(3^2)$
12. $(-2)^3$ 13. $4^2 \cdot 3^1$

Say whether each of the statements in Exercises 14–17 is true or false.

14. A negative number raised to an even power always gives a positive answer.
15. $(-2)^4 = -(2^4)$
16. $2 + 2 + 2 + 2 + 2 = 2^5$ 17. $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$

Lesson 2.5.2 — Powers of Rational Numbers

Evaluate each of the expressions in Exercises 1–9.

1. $\left(\frac{1}{3}\right)^2$ 2. $\left(\frac{2}{5}\right)^3$ 3. $\left(\frac{2}{3}\right)^4$
4. $\left(-\frac{5}{7}\right)^2$ 5. $\left(-\frac{3}{4}\right)^3$ 6. $(0.3)^2$
7. $(0.13)^2$ 8. $(-0.4)^3$ 9. $(-0.06)^2$

Write each of the expressions in Exercises 10–12 in base and exponent form.

10. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$ 11. $(-0.04)(-0.04)$ 12. $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{-1}{2} \cdot \frac{-1}{2} \cdot \frac{-1}{2}$

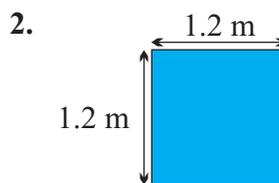
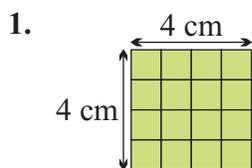
Say whether each of the statements in Exercises 13–15 is true or false.

13. If $\frac{a}{b}$ is between 0 and 1 then $\left(\frac{a}{b}\right)^2 > \frac{a}{b}$. 14. $\left(\frac{-a}{b}\right)^2 = \left(\frac{a}{-b}\right)^2$

15. Raising a decimal to a power is like repeatedly multiplying the decimal by itself.

Lesson 2.5.3 — Uses of Powers

Find the area of the squares in Exercises 1–2.



3. What is the area of a square with a side length of $\frac{1}{2}$ a foot?

4. What is the volume of a cube of side length 2.4 feet?

Write the numbers in Exercises 5–8 in scientific notation.

5. 218,534

6. -32,400,000

7. 5,183,000,000

8. 500

The numbers in Exercises 9–11 are written in scientific notation. Write them out in full.

9. 7.36×10^4

10. -8.1×10^6

11. 6.123929×10^7

Lesson 2.5.4 — More on the Order of Operations

Evaluate each expression in Exercises 1–12.

1. $12 \div 3 \cdot 4$

2. $10 - 6 + 3$

3. $8 \cdot 6 \div 12$

4. $2^3 \cdot 4 - 20$

5. $(8 - 5)^3 \div 9$

6. $2^4 + 3^2 \cdot (25 - 3^2)$

7. $-5^2 - 8^2$

8. $(-5)^2 - (8)^2$

9. $(3^4 - 2 \cdot 3) \div 5^2$

10. $\left(\frac{3}{4} \div \frac{5}{8}\right) \cdot \frac{10}{3}$

11. $(0.5)^2 \cdot 8(5) + 3^2 \cdot 8$

12. $\frac{4^3 - 6^2}{5 - 3 \cdot 10}$

Say whether each of the statements in Exercises 13–18 is true or false.

13. $(5 + 3)^2 = 5^2 + 3^2$

14. $2 \cdot 3^2 = (2 \cdot 3)^2$

15. $(a \cdot b)^3 = a^3 \cdot b^3$

16. In the order of operations, division comes before subtraction.

17. You should always do divisions before multiplications.

18. It doesn't matter which order you do additions and parentheses in.

Lesson 2.6.1 — Perfect Squares and Their Roots

Give the perfect square of each of the numbers in Exercises 1–6.

1. 3

2. 0

3. 5

4. -6

5. 13

6. -11

Evaluate the expressions in Exercises 7–12.

7. $\sqrt{225}$

8. $-\sqrt{100}$

9. $9^{\frac{1}{2}}$

10. $121^{\frac{1}{2}}$

11. $-36^{\frac{1}{2}}$

12. $-25^{\frac{1}{2}}$

13. There are 400 people in a marching band. How many people should be in each row if the band want to march in a square formation?

A square shaped room has an area of 576 square feet.

14. What is the length of the room?

15. As part of a renovation, each wall is being extended by 5 feet. What will be the area of the newly renovated room?

✓ Lesson 2.6.2 — Irrational Numbers

In Exercises 1–6, prove that each number is rational by writing each one as a fraction in its simplest form.

- | | | |
|---------|----------------|----------------|
| 1. 10 | 2. 0.6 | 3. 0.375 |
| 4. -7 | 5. $\sqrt{25}$ | 6. $3.\bar{2}$ |

Classify each of the numbers in Exercises 7–15 as rational or irrational.

- | | | |
|------------------------|--------------------------|-------------------------|
| 7. 2π | 8. 2.756 | 9. 8 |
| 10. $\sqrt{49}$ | 11. $\sqrt{6}$ | 12. $-16^{\frac{1}{2}}$ |
| 13. $-8^{\frac{1}{2}}$ | 14. $2.24\overline{635}$ | 15. -1.2 |

Say whether each of the statements in Exercises 16–22 is true or false.

16. Irrational numbers can be written as the quotient of two integers.
17. The square root of a number that is not a perfect square is irrational.
18. The square root of any integer other than a perfect square is irrational.
19. Irrational numbers can be displayed in full on calculators.
20. Terminating and repeating decimals are rational numbers.
21. All integers are rational numbers.
22. All rational numbers are integers.

✓ Lesson 2.6.3 — Estimating Irrational Roots

Say whether each of the numbers in Exercises 1–4 is rational or irrational.

- | | |
|-----------------|-----------------|
| 1. $\sqrt{12}$ | 2. $\sqrt{25}$ |
| 3. $-\sqrt{36}$ | 4. $-\sqrt{14}$ |

Use your calculator to approximate the square roots in Exercises 5–8.

Give your answers to six decimal places.

- | | |
|-----------------|-----------------|
| 5. $\sqrt{8}$ | 6. $\sqrt{23}$ |
| 7. $\sqrt{129}$ | 8. $\sqrt{520}$ |

In Exercises 9–12, say which two perfect squares each number lies between.

- | | |
|--------|--------|
| 9. 8 | 10. 17 |
| 11. 55 | 12. 2 |

In Exercises 13–16, find the whole numbers that each root lies between.

- | | |
|-----------------|------------------|
| 13. $\sqrt{18}$ | 14. $\sqrt{2}$ |
| 15. $\sqrt{33}$ | 16. $\sqrt{112}$ |

In Exercises 17–19, say whether each statement is true or false.

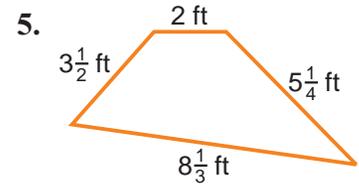
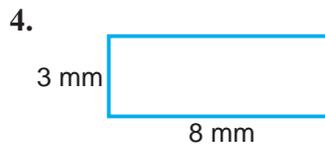
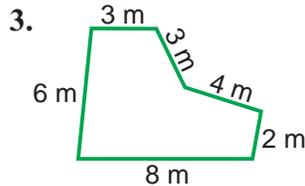
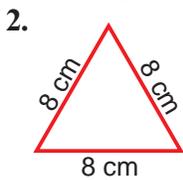
- | | |
|--|---------------------------------|
| 17. $\sqrt{7} = 2.64575131106$ | 18. $\sqrt{11} \approx 3.31662$ |
| 19. The square root of a perfect square is irrational. | |

20. Will a 13 foot long bookcase fit along the wall of a square room with a floor area of 140 ft^2 ? Explain your answer.

Lesson 3.1.1 — Polygons and Perimeter

- Describe how to find the perimeter of a square.

Find the perimeter of the figure in Exercises 2–5.



Karl is putting up a fence around an 8 foot square plot.

- What is the perimeter of the plot?
- The fence is going to be 4 rails high. How many feet of railing will Karl need to complete the fence?

Alejandra is walking around the edge of a rectangular field measuring 23 meters by 14 meters.

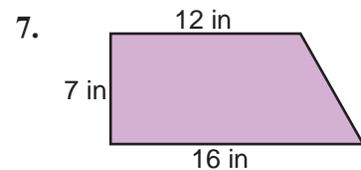
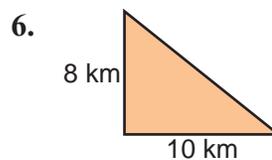
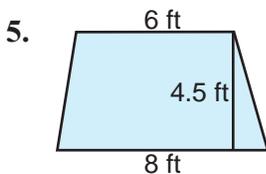
- What is the perimeter of the field?
- Alejandra walked a total of 407 meters. How many times did she walk around the field?

Lesson 3.1.2 — Areas of Polygons

Find the area of each figure described in Exercises 1–2.

- A triangle with base of 3 m and height of 4 m.
- A parallelogram with base of $3\frac{2}{3}$ in. and height of $2\frac{1}{2}$ in.
- Copy the sentence below, and fill in the blank with the term that best completes the statement.
The area of a triangle is _____ the area of a parallelogram that has the same base and vertical height.
- Ramon is tiling a square kitchen that is 20 feet on each side.
If each tile is a 1 ft square, how many tiles will he need?

Find the area of each of the shapes in Exercises 5–7.



Lesson 3.1.3 — Circles

In Exercises 1–2, determine the missing measure.

- Radius = _____ cm, diameter = 15 cm.
- Radius = $18\frac{3}{4}$ in, diameter = _____ in.
- Explain the difference between the radius and the diameter of a circle.
- Explain how to find the circumference of a circle when given the diameter.

In Exercises 5–12, use 3.14 for π in calculations involving whole numbers and decimals and $\frac{22}{7}$ for those involving fractions.

Find the circumference of each circle described in Exercises 5–7.

- Radius = 3.3 in.
- Radius = 2.52 yds.
- Diameter = $6\frac{1}{2}$ mm

Leilani is designing a circular medal with a 3 inch radius. Find:

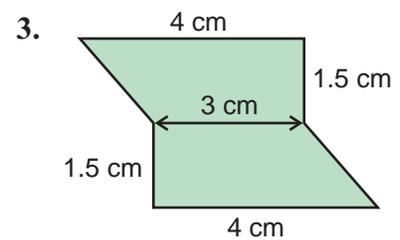
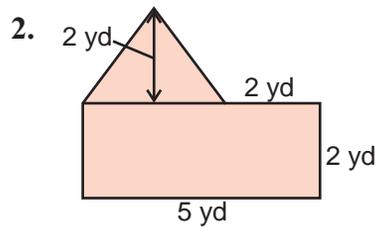
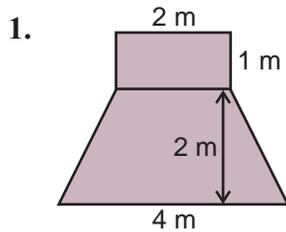
- The circumference of the medal.
- The area of the circular surface of the medal.

Find the area of each circle described in Exercises 10–12.

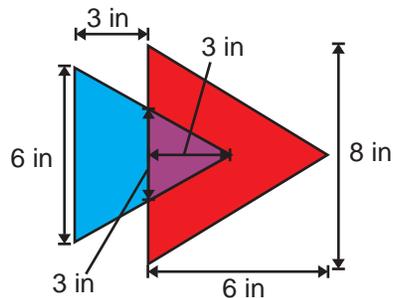
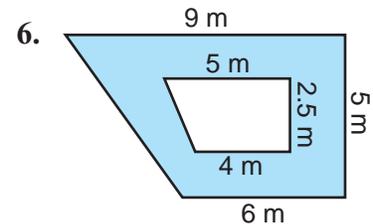
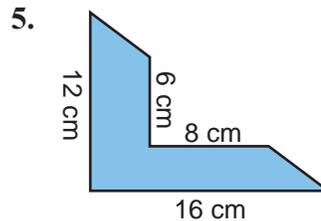
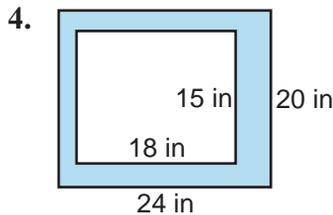
- Radius = 19 cm
- Radius = $\frac{1}{8}$ in
- Diameter = $6\frac{2}{3}$ m

Lesson 3.1.4 — Areas of Complex Shapes

In Exercises 1–3, find the area of each complex shape.



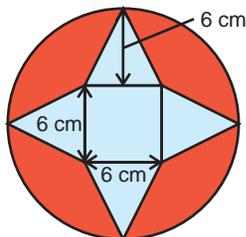
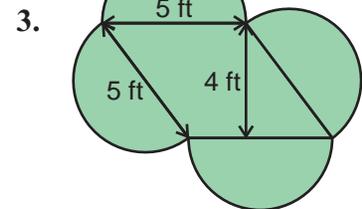
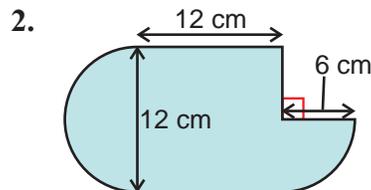
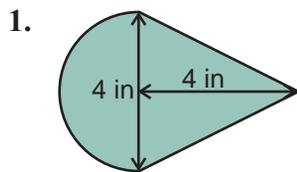
In Exercises 4–6, find the blue area.



7. The math club created a new logo for their t-shirts. The new logo is shown on the left. What is the area covered by the logo?

Lesson 3.1.5 — More Complex Shapes

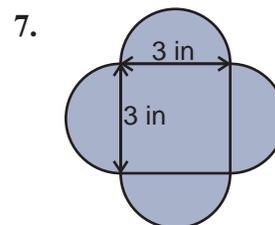
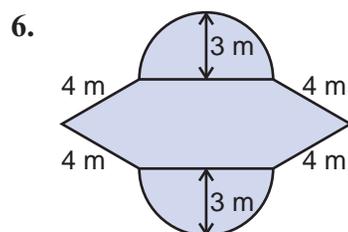
In Exercises 1–3, find the area of each complex shape. Use 3.14 for π .



The shape on the left is part of a design for a quilt Regina is making. The four triangles are all the same size.

4. What is the area of the blue part of the design?
5. What is the area of the red part of the design?

In Exercises 6–7, find the perimeter of each shape.



Lesson 3.2.1 — Plotting Points

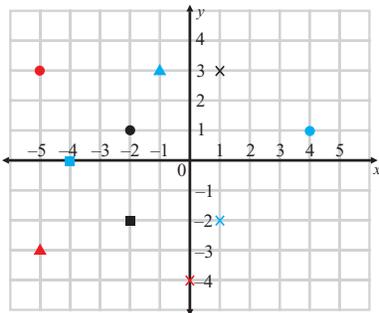
In Exercises 1–3, plot each pair of coordinates on the coordinate plane.

- (4, -1)
- (-2, -3)
- (-3, 2)
- On the coordinate plane, which axis is the horizontal axis?
- What are the coordinates of the origin on the coordinate plane?

Use the grid below to answer Exercises 6–13.

Identify which shapes are at the following coordinates:

- (1, 3)
- (-5, -3)
- (0, -4)



Find the coordinates of:

- The black square
- The blue triangle
- The red circle

One of the following pairs of coordinates on this grid does not belong with the others: (4, 1), (-1, 3), (-4, 0), (-5, -3)

- Say which pair of coordinates does not belong. Explain your answer.
- Which pair of coordinates could put instead of your answer to Exercise 15 that would match the others in the set? Explain your answer.

Lesson 3.2.2 — Drawing Shapes in the Coordinate Plane

In Exercises 1–4, plot the points given to find the missing coordinates.

- Square ABCD: A(?, ?) B(4, 4) C(4, 0) D(0, 0)
- Rectangle EFGH: E(-2, 4) F(1, 4) G(1, -1) H(?, ?)
- Rhombus KLMN: K(-4, 1) L(?, ?) M(0, -1) N(-4, -4)
- Parallelogram QRST: Q(-3, 4) R(4, 4) S(?, ?) T(-5, 2)

Exercises 5–8 are about the shapes from Exercises 1–4.

- Find the perimeter and area of square ABCD
- Find the perimeter and area of rectangle EFGH
- Find the area of rhombus KLMN
- Find the area of parallelogram QRST

9. Alyssa, the yearbook editor, has mapped out the layout of each yearbook page using a grid. If the photo of the volleyball team is placed with the edges at (1,2), (1, 8), (8, 2), and (8, 8), what area will the photo cover?

Lesson 3.3.1 — The Pythagorean Theorem

In Exercises 1–3, copy and complete the following sentences:

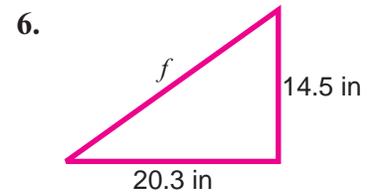
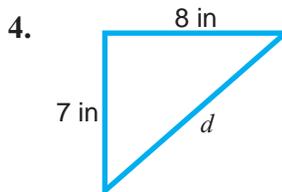
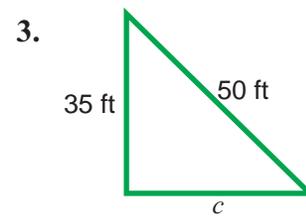
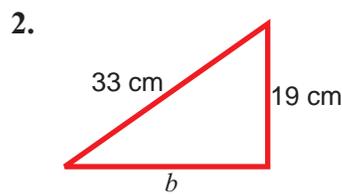
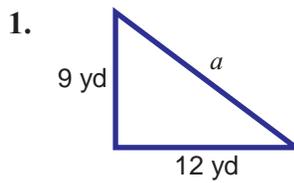
- For any right triangle, $c^2 = a^2 + b^2$, where c is the length of the _____ and a and b are the lengths of the _____.
- The hypotenuse is always the _____ side of a right triangle.
- In a right triangle, the hypotenuse is always opposite an angle that measures _____°.
- The Pythagorean Theorem is only true for right triangles. If you know the lengths of the three sides of a triangle, how can you use the Pythagorean Theorem to find out if it is a right triangle?

In Exercises 6–8, use the Pythagorean Theorem to decide whether a triangle with the given side lengths is a right triangle or not.

- 4 cm, 9 cm, 12 cm
- 10 ft, 6 ft, 8 ft
- 5 yd, 7 yd, 5 yd

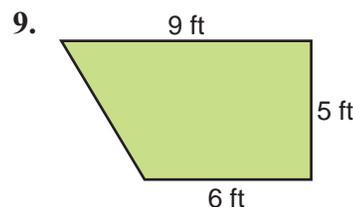
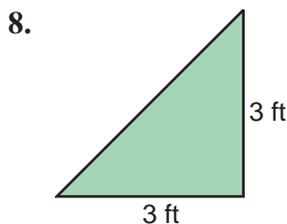
Lesson 3.3.2 — Using the Pythagorean Theorem

In Exercises 1-8, use the Pythagorean Theorem to find the missing length. Round decimals to the nearest hundredth.



7. Lana has a triangular corner bookshelf. She wants to add rope edging along the hypotenuse. If each of the leg sides is 2.5 feet long, how much rope edging will she need?

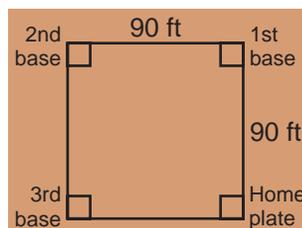
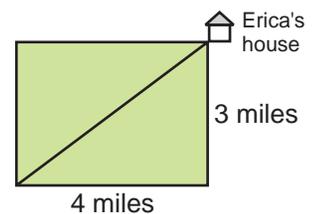
Eduardo and Destiny are planting a vegetable garden. Each plot needs to be fenced in. In Exercises 8 and 9, determine how much fencing is needed for the plot shown.



Lesson 3.3.3 — Applications of the Pythagorean Theorem

1. Mrs. Lopez is decorating the class bulletin board. She wants to place decorative trim around the perimeter and along each diagonal. The board is 8 ft. long and 4 ft. wide. How much trim will Mrs. Lopez need?

2. Erica usually runs the distance around the park shown on the right each day. When it is raining, she ends the run early by returning home along the diagonal. How much further does Erica run on a dry day compared to a rainy day?



3. The school yard has a baseball diamond that is really a 90 foot square as shown on the left. If the catcher throws from home plate to 2nd base, what is the distance thrown?

A quilt square is stitched along each diagonal to make 4 right triangles. Each diagonal is 12 inches long.

- What is the perimeter of the square?
- What is the area of the quilt square?
- How many quilt squares from can be cut from a piece of fabric that is 8 feet long and 2 feet wide?

Lesson 3.3.4 — Pythagorean Triples and the Converse of the Theorem

Tell whether the side lengths given in Exercises 1–5 indicate a right, obtuse or acute triangle.

- 13, 13, 20
 - 8, 9, 11
 - 45, 60, 75
 - 4, 7.5, 8.5
 - 1.2, 1.5, 1.7
6. A blanket has length of 80 inches, width of 60 inches and a diagonal of 100 inches. Is the blanket a perfect rectangle? Explain your answer.

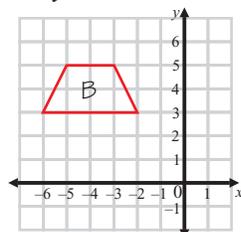
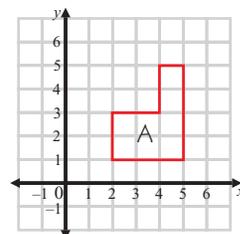
Exercises 7–9 are about a triangle, XYZ. Side XY is 10.5 cm long. Side YZ is 17.5 cm long.

- If side XZ was 13 cm long, would XYZ be right, acute or obtuse?
- If XYZ was obtuse, and XZ was the longest side, what could you say about the length of XZ?
- Tammy claims that there is only one possible length for XZ that would make XYZ a right triangle. Is this true? Explain your answer.

Lesson 3.4.1 — Reflections

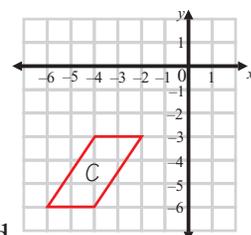
Copy shape A on to a set of axes numbered –6 to 6 in both directions.

- Draw the image A', made by reflecting A over the x -axis.
- Write the coordinates of the vertices of the image A'', made by reflecting A over the y -axis.



Copy shape B on to a set of axes numbered –6 to 6 in both directions.

- Draw the image B', made by reflecting B over the y -axis.
- Draw the image B'', made by reflecting B over the x -axis.
- Write the coordinates of the vertices of the image B''', made by reflecting the image B' over the x -axis.



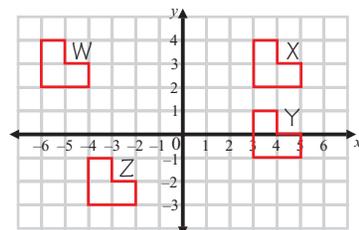
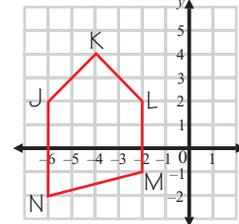
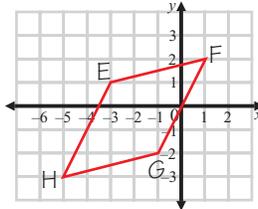
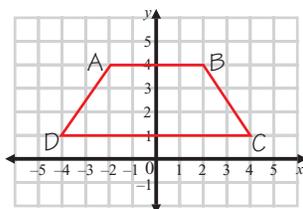
Copy shape C on to a set of axes numbered –6 to 6 in both directions.

- Draw the image C', made by reflecting C over the x -axis.
- Write the coordinates of the image C'', made by reflecting C over the y -axis.
- The vertices of the image C''' have the coordinates $(-2, -3)$, $(-4, -6)$, $(-6, -6)$, and $(-4, -3)$. Describe in words the transformation used to create C''' from the image C'.

Lesson 3.4.2 — Translations

In Exercises 1–3, copy the shapes shown on to grid paper, and graph the indicated translations.

- $(x, y) \rightarrow (x - 2, y - 5)$
- $(x, y) \rightarrow (x + 3, y + 2)$
- $(x, y) \rightarrow (x + 7, y - 2)$



Use the grid shown on the left to answer Exercises 4–9

In Exercises 4–7, describe the indicated translations in coordinates.

- X to W
- W to Y
- X to Z
- Z to Y

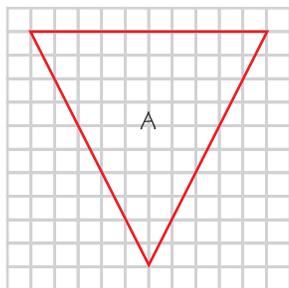
Exercises 8–9 each describe a translation between two shapes on the grid. Write the translation described in the form “A to B”.

- $(x, y) \rightarrow (x, y + 3)$
- $(x, y) \rightarrow (x - 2, y + 5)$

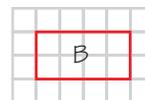
Lesson 3.4.3 — Scale Factor

In Exercises 1–3, draw an image of each figure using the given scale factor.

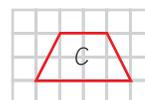
1. Scale factor 0.4



2. Scale Factor 1.25



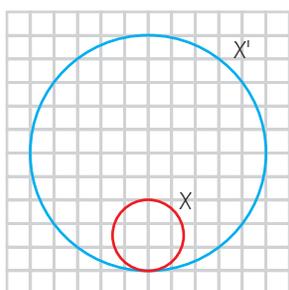
3. Scale Factor 1.5



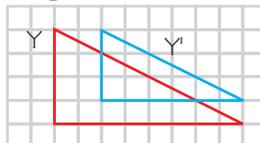
4. Explain what happens when a scale factor of 1 is applied to a figure.

In Exercises 5–7, find the scale factor that produced each transformation.

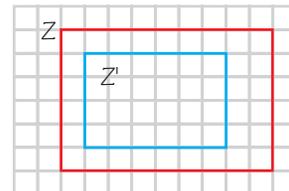
- 5.



- 6.



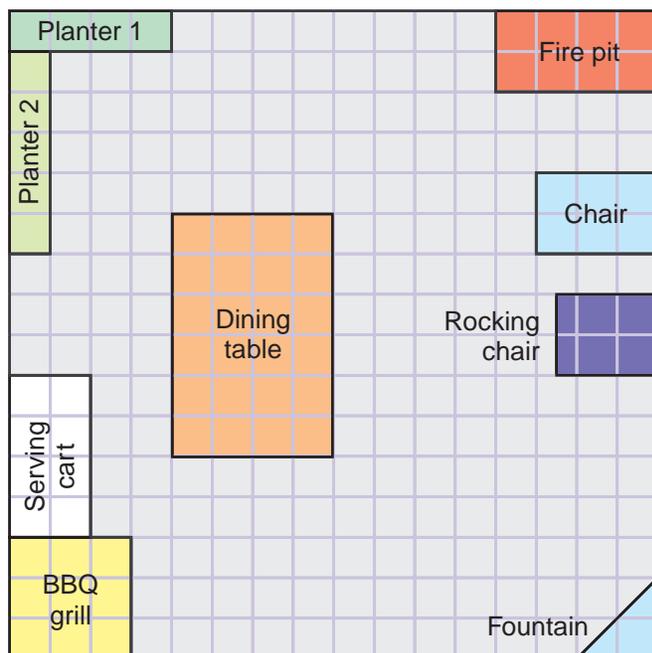
- 7.



Lesson 3.4.4 — Scale Drawing

In Exercises 1–5, make the following scale drawings

- A rectangular 20 × 40 ft. swimming pool, using the scale 1 in = 5 ft
- A square play ground with sides of 60 yds using a scale 1 cm = 10 yds
- A rectangular 28 × 32 ft classroom using a scale 1 cm = 4 ft
- A circular spa tub with a 9 ft diameter using a scale of 2 in = 3 ft



This is a scale drawing of Michael's patio, using the scale 1 grid square = 2 ft.

In Exercises 5–12, find the real life dimensions of the following objects.

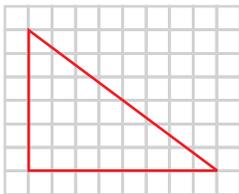
- Planter 1
- Planter 2
- Fire pit
- Dining Table
- Serving Cart
- BBQ grill
- Rocking Chair
- Fountain

13. Ian made a miniature of a portrait he was painting. The miniature was 7 inches long. If the actual portrait is 42 inches long, what is the scale factor he used?

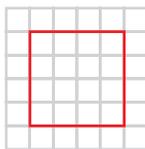
Lesson 3.4.5 — Perimeter, Area, and Scale

In Exercises 1–5, calculate the perimeter and area if the image if the figure shown is multiplied by the given scale factor.

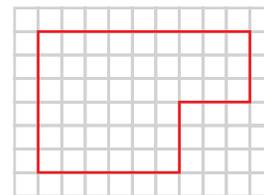
1. Scale factor 1



2. Scale factor 5



3. Scale factor $\frac{1}{3}$



4. Scale factor 1.5



5. Scale factor $\frac{1}{4}$



In Exercises 6–8, find the scale factor used in each transformation.

6. Perimeter of original = 35 in.; Perimeter of image = 50 in.

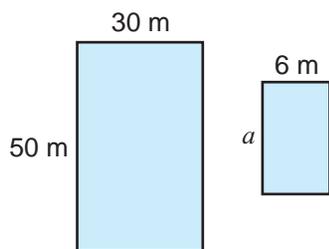
7. Area of original = 92 mm; Area of image = 23 mm

8. Area of original = 12 cm; Area of image = 72 cm

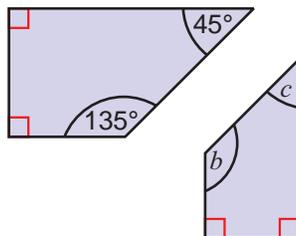
Lesson 3.4.6 — Congruence and Similarity

In Exercises 1–6, each pair of figures is similar.

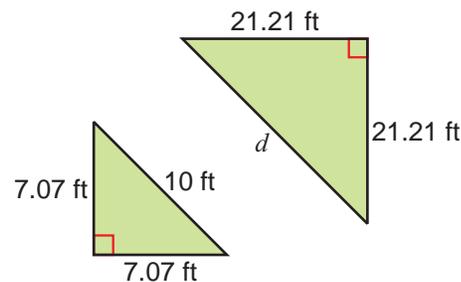
1. Find a .



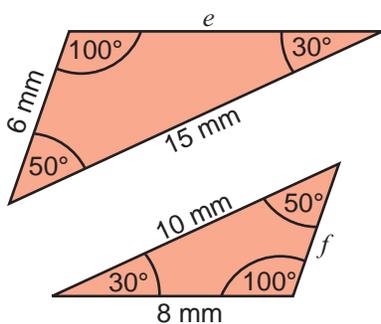
2. Find b and c .



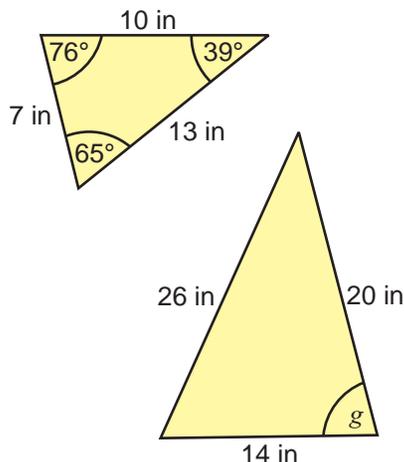
3. Find d .



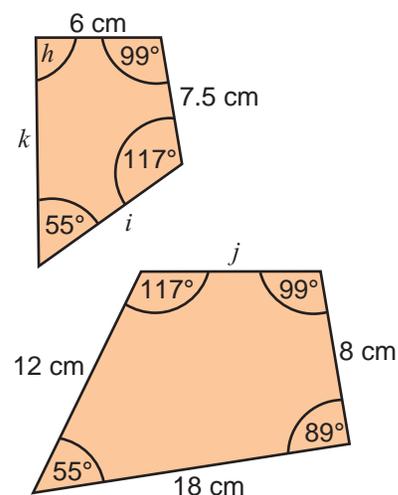
4. Find e and f .



5. Find g .



6. Find h , i , j and k .



✓ Lesson 3.5.1 — Constructing Circles

In Exercises 1–6, use a ruler and compass to construct circles with the following features.

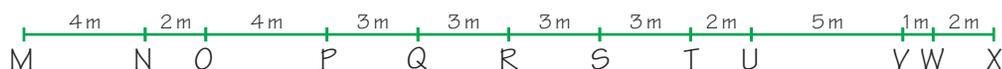
1. Circle of radius 2 in, with a chord of length 3 in. and a central angle of 120° .
2. Circle of radius 3 in, with a chord of length 3.5 in. and a central angle of 55° .
3. Circle of diameter 4.5 cm, with a chord of length 3 cm and a central angle of 160° .
4. Circle of diameter 5 in, with a chord of length 4 in. and a central angle of 40° .
5. Circle of radius 5 cm, with a chord of length 3.5 cm and a central angle of 145° .
6. Circle of radius 3.2 cm, with a chord of length 4.8 cm and a central angle of 70° .

In Exercises 7–8, copy and complete the sentences.

7. The _____ is the distance from a point on the circumference of a circle to the center.
A _____ is the distance from a point on the circumference to another point on the circumference.
8. A chord of a circle can never be _____ than the circle's diameter.

✓ Lesson 3.5.2 — Constructing Perpendicular Bisectors

Use the diagram below to find the midpoints of each segment in Exercises 1–6.



1. Segment NR
2. Segment PR
3. Segment PT
4. Segment NT
5. Segment SV
6. Segment MX

In Exercises 7–9, use a compass and straight edge to construct line segments of the following lengths, then construct their perpendicular bisectors.

7. $4\frac{1}{2}$ inches
8. 4.1 cm
9. $3\frac{1}{4}$ inches

✓ Lesson 3.5.3 — Perpendiculars, Altitudes, and Angle Bisectors

Draw a line segment, AB, that is 5 inches long. In Exercises 1–8, mark the following points on the line. Draw a perpendicular through each point.

1. Point C, 1 inch away from A.
2. Point D, $2\frac{1}{2}$ inches away from A.
3. Point E, $1\frac{3}{4}$ inches away from B.
4. Point F, $\frac{3}{4}$ inch away from B.
5. Point G, $1\frac{1}{2}$ inches below AB.
6. Point H, $2\frac{1}{2}$ inches below AB.
7. Point I, 1 inch above AB.
8. Point J, $\frac{1}{2}$ inch above AB.
9. Draw a line segment PQ that is 4 cm long. Use a protractor to draw an angle at Q measuring 140° . Mark a point R on the new ray that you have drawn, so that the distance QR is 3 cm.
10. Bisect angle PQR using a compass and straightedge.
Mark the point S on the angle bisector that is 5 cm away from Q.
11. What are the measures of the angles PQS and SQR?
12. Use a compass and straightedge to bisect the angle PQS.
Mark a point T on the new angle bisector that is 3.5 cm from Q.
13. What are the measures of angles PQT and TQS?

Lesson 3.6.1 — Geometrical Patterns and Conjectures

In Exercises 1–2, draw the next instance in the given sequence.



Exercises 3–7 are about the sequence of dots shown below.



Instance 1



Instance 2



Instance 3

- Make a specific conjecture about instance 4.
- Make a specific conjecture about instance 5.
- Make a general conjecture about the pattern.
- Draw the 6th instance in the sequence.
- How many dots are in the 10th instance?

Are the following conjectures true or false? Explain your answers.

- There were 365 people inside the school building during the last fire drill.
Conjecture: If John was inside the school at the time, John is a student.
- There is a pecan tree in Maria's yard. Yesterday Maria picked up nuts that had fallen in the yard.
Conjecture: The nuts Maria collected must be pecans.
- Marcus made a perfectly square table. One corner is a right angle.
Conjecture: All the corners of the table are right angles.
- Angela has a round cushion with a diameter of 1 yard.
Conjecture: The radius of the cushion is 18 inches.

In Exercises 12–14, find the next three numbers in the series.

12. 6, 17, 28, 39 ...

13. 3, 7, 12, 18 ...

14. 1, 1, 2, 3, 5, 8 ...

Lesson 3.6.2 — Expressions and Generalizations

In Exercises 1–2, find the next term in the following sequences. Explain your answer.

1. January, April, July ...

2. Sunday, Tuesday, Thursday ...

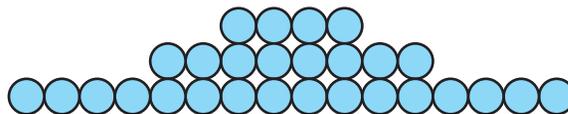
Exercises 3–5 are about the following number sequence: 3, 13, 23, 33 ...

- Find the next term in the sequence.
- Write an expression for the n th term in the sequence.
- Use your answer to Exercise 4 to find the 17th term in the sequence.

Exercises 6–8 are about the following number sequence: $-6, 2, 10, \dots$

- Find the next term in the sequence.
- Write an expression for the n th term in the sequence.
- Use your answer to Exercise 4 to find the 12th term in the sequence.

Use the diagram below to answer Exercises 9–12.



- How many circles are in the 4th row?
- How many circles are in the 5th row?
- How many circles are in the n th row?
- How many circles are in the 9th row?

Lesson 4.1.1 — Graphing Equations

In Exercises 1–4, say whether the equation given is a linear equation or not.

- $y = 2x + 4$
- $y - 5 = x$
- $y^2 + x^2 = 12$
- $2y - x = 8$

The diagram on the right is the graph of the equation $y = 2x + 1$.

Use the graph to explain whether the following are solutions to the equation $y = 2x + 1$.

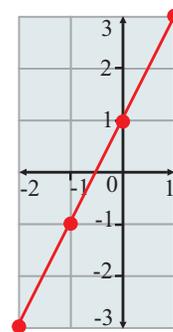
- $x = 1, y = 2$
- $x = -1, y = -1$

7. Show that $x = 2, y = 9$ is a solution of the equation $y = 12x - 15$.

8. Show that $x = -2, y = 1$ is a solution of the equation $y = 3x + 7$.

9. Find the solutions to the equation $y = \frac{1}{2}x + 1$ which have x values of $-2, -1, 0, 1,$ and 2 . Use the ordered pairs you have found to draw the graph of $y = \frac{1}{2}x + 1$.

10. Find the solutions to the equation $y = 2x - 2$ which have x values of $0, 0.5, 1, 1.5,$ and 2 . Use the ordered pairs you have found to draw the graph of $y = 2x - 2$.



Lesson 4.1.2 — Systems of Linear Equations

1. How many possible solutions are there to a system of linear equations in two variables?

In Exercises 2–3, write a system of linear equations to represent the statements.

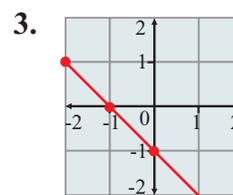
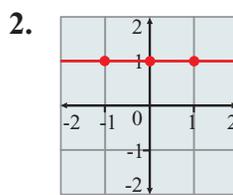
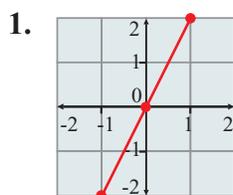
- Twice a number, y , is equal to a number, x , increased by 9.
Four less than the product of a number, x , and 3 is equal to a number, y .
- A number, q , increased by the product of a number, p , and 3 is equal to 10.
A number, p , is equal to the quotient of a number, q , and 2.
- Explain how to find the solution of a system of two linear equations by plotting them both on a graph.
- Check that the point $(-1, 1)$ is the solution to the system of equations $y = 2x + 3$ and $y - x = 2$.

Solve the systems of equations in Exercises 6–9 by graphing.

- $y = x + 1$ and $y = 2x + 1$
- $y = 1 - x$ and $2y = 2x + 2$
- $y = 2x$ and $4y = 2x$
- $y = 0.5x + 1$ and $2y = x$

Lesson 4.1.3 — Slope

In Exercises 1–3, say whether the slope of each line is positive, negative, or zero. Then find the slope.



4. Plot the graph of the equation $2y = 1 - x$ and find its slope.

5. Point A with coordinates $(-1, 4)$ lies on a line with a slope of 4. Give the coordinates of any other point that lies on the same line.

In Exercises 6–11, find the slope of the line that passes through the two points given.

- $(3, 1)$ and $(4, 2)$
- $(1, 1)$ and $(2, 3)$
- $(0, 2)$ and $(2, 0)$
- $(-2, -3)$ and $(-1, 0)$
- $(-1, 3)$ and $(3, 5)$
- $(2, 1)$ and $(-3, 0)$

✓ Lesson 4.2.1 — Ratios and Rates

In Exercises 1–2, express each statement as a ratio in its simplest form.

1. A store sells 2 rulers for every 1 eraser that is bought.
2. For every 2 lemons that I have, I also have 8 oranges.
3. Clayton is cooking breakfast for his family. He knows that he needs 18 pancakes to feed all 6 people. What is this written as a unit rate?

In Exercises 4–9 express the quantities as unit rates.

4. 60 apples in 10 pies.
5. \$4 for 2 meters of fabric.
6. 70 grams of food for 2 gerbils.
7. 120 miles in 3 hours.
8. 90 books for 15 students.
9. \$2.97 for 3 pens.

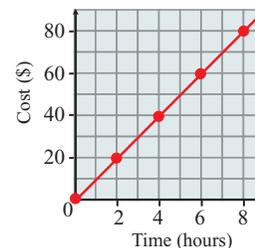
In Exercises 10–13 say which is the better buy.

10. 1 pen for \$2 or 5 pens for \$9
11. 2 kg of rice for \$6, or 3 kg for \$8.40
12. 300 ml of soda for \$1.29, or \$2.20 for 500 ml.
13. \$7 for 100 minutes of calls or \$7.20 for 2 hours.
14. A store sells 2kg bags of flour for \$3.20, and 500 g bags of flour for \$0.90. What is the price per kg for each size?

✓ Lesson 4.2.2 — Graphing Ratios and Rates

1. At a gas station the price of gas is \$2.40 a gallon. Draw a graph to represent the relationship between the cost of the gas and the volume purchased.
2. A doctor measured a patient's resting pulse rate at 80 beats per minute. Draw a graph to show the relationship between time and the number of times the patient's heart beats. Use it to estimate how many times the patient's heart will beat in 18 minutes.

The graph on the right shows the relationship between the cost of hiring a bike and the number of hours you hire it for. Use the graph to answer Exercises 3–5.



3. How much does it cost to hire a bike for 5 hours?
4. What is the slope of the graph?
5. How much does it cost to hire a bike for 1 hour?
6. Madre works as a waitress. Today she worked an 8 hour shift, and was paid \$92. Plot a graph of the amount Madre earns against the time she works for. Then find how much Madre is paid per hour.

✓ Lesson 4.2.3 — Distance, Speed, and Time

1. Dan walks 12 blocks to school. It takes him 6 minutes. What is his average speed?
2. If a tortoise is crawling along at a speed of 6 yards per minute, how far will it crawl in 4 minutes?
3. Mr Valdez drove 400 miles in 8 hours. What was his average speed?
4. A hiking club go on a camping expedition. They can cover a distance of 18 km each day. If they plan to go 90 km in total, how long will their expedition take?
5. A plane flies 2520 miles in 4 hours and 30 minutes. What is its average speed for the flight?
6. A machine can make 5 miles of silk ribbon in an hour. What length of ribbon can the machine make in an average 40 hour working week?
7. Each day Tya cycles to the bus stop to catch the school bus. She rides at an average speed of 10 mph, and the bus goes at an average speed of 40 mph. It takes her half an hour to do the 15 mile trip. Assuming she doesn't have to wait for the bus, find how long she spends riding her bike.

✓ Lesson 4.2.4 — Direct Variation

The numbers a and b are in direct variation, and $a = 3$ when $b = 4$. In Exercises 1–6, find the value of a when b equals the value given.

- 8
 - 12
 - 2
 - 7
 - 1
 - 34
7. It costs a school \$100 to take 20 students on a trip to the museum. Use direct variation to find how much it would cost to take 55 students.
8. What point on the coordinate plane do all graphs that show direct variation pass through?
9. It took Jesse 3 hours to drive the 159 miles to his Grandmother's house. If he was driving at a constant speed, how far would he have driven after 2 hours?
10. A saleswoman receives \$20 in commission for every \$150 worth of goods she sells. Show this relationship on a graph. Use your graph to find how much commission she receives from \$225 of sales.

The numbers x and y are in direct variation, and $x = -1$ when $y = 1$.

- Write an equation relating x and y , and graph it on the coordinate plane.
- What is the slope of your graph from Exercise 11? What does the slope represent?

✓ Lesson 4.3.1 — Converting Measures

In Exercises 1–6, give the ratio between the units.

- feet:inches
 - centimeters:millimeters
 - meters:kilometers
 - fluid ounces:cups
 - kilograms:grams
 - pint:quart
7. Complete this equation: $0.1 \text{ km} = ? \text{ m} = ? \text{ cm} = 100,000 \text{ mm}$
8. Complete this equation: $2 \text{ quarts} = ? \text{ pints} = ? \text{ cups} = 64 \text{ fluid ounces}$

In Exercises 9–16, set up and solve a proportion to find the missing value, x , in each case.

- $480 \text{ mm} = x \text{ cm}$
 - $15 \text{ kilometers} = x \text{ meters}$
 - $64 \text{ ounces} = x \text{ pounds}$
 - $5.5 \text{ pints} = x \text{ cups}$
 - $50 \text{ grams} = x \text{ kilograms}$
 - $300 \text{ pounds} = x \text{ tons}$
15. Nora is making soup. Her recipe calls for 3 quarts of water. How many one cup servings will it make?
16. Dell needs 70 feet of wallpaper border. If the border comes in 5 yard rolls, how many should he buy?

✓ Lesson 4.3.2 — Converting Between Unit Systems

In Exercises 1–4, give the ratio between the units

- inches:centimeters
- kilometers:miles
- liters:gallons
- kilograms:pounds

In Exercises 5–10 find the missing value, x , in each case. Give all your answers to 2 decimal places.

- $18 \text{ inches} = x \text{ cm}$
- $49 \text{ kg} = x \text{ pounds}$
- $840 \text{ yards} = x \text{ meters}$
- $31 \text{ km} = x \text{ miles}$
- $10 \text{ gallons} = x \text{ liters}$
- $14 \text{ kg} = x \text{ ounces}$

11. Salma's house is 3 km from the store. She cycles to the store to buy bread, and then rides on to the library. The library is a further 750 m from the store. How many miles has Salma cycled in total?

12. Bill is working on a science project. His task is to record the daily high temperature outside the school for a week. Bill's table of results is shown below. Fill in the missing temperatures.

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Temperature (°F)	86			86.9			85.1
Temperature (°C)		32	27		31	30	

Lesson 4.3.3 — Dimensional Analysis

1. Match the equations with their missing units.

i) $320 \text{ copies} \times 0.10 \frac{\text{dollars}}{\text{copy}} = 32 ?$

ii) $100 \text{ feet} \div 5 \text{ minutes} = 20 ?$

iii) $2 \text{ feet} \times 12 \frac{\text{inches}}{\text{foot}} = 24 ?$

A) inches

B) dollars

C) $\frac{\text{feet}}{\text{minute}}$

In Exercises 2–9, find the missing unit

2. $90 \text{ miles} \div 3 \text{ hours} = 30 \underline{\hspace{1cm}}$

4. $2 \text{ persons} \times 3 \text{ days} = 6 \underline{\hspace{1cm}}$

6. $1095 \text{ days} \times \frac{1 \text{ year}}{365 \text{ days}} = 3 \underline{\hspace{1cm}}$

8. $15 \text{ m/s} \div 5 \text{ s} = 3 \underline{\hspace{1cm}}$

3. $4 \text{ hours} \times 20 \frac{\text{dollars}}{\text{hours}} = 80 \underline{\hspace{1cm}}$

5. $10 \text{ inches} \times 5 \text{ inches} = 50 \underline{\hspace{1cm}}$

7. $8 \text{ miles} \div 2 \text{ miles per hour} = 4 \underline{\hspace{1cm}}$

9. $16 \text{ m}^3/\text{person-day} \div 4 \text{ m}^2/\text{person} = 4 \underline{\hspace{1cm}}$

10. The school dance team sell team pins during recess to fund travel to an out of town tournament. They earn \$15 each day. The cost of the trip is \$300. How many days do they need to sell for to cover the trip?

11. My go-kart can travel at a maximum speed of 10 miles/hour. How far can it go in 1800 seconds?

Lesson 4.3.4 — Converting Between Units of Speed

In Exercises 1–6 create a conversion factor equal to one for each pair of units.

1. Days and weeks

2. Meters and kilometers

3. Seconds and minutes

4. Miles and kilometers

5. Days and hours

6. Meters and yards

In Exercises 7–10 perform the conversions to find the missing numbers.

7. $32 \text{ miles per hour} = w \text{ km per hour}$

8. $5 \text{ yards per minute} = x \text{ meters per minute}$

9. $10 \text{ cm per second} = y \text{ cm per hour}$

10. $3 \text{ feet per minute} = z \text{ yards per hour}$

11. Which is faster, 85 miles per hour, or 120 km per hour?

12. Davina and Juan had a race over a course 1000 m long. Davina's average speed was 9 kilometers per hour. Juan's average speed was 3 meters per second. Who won the race?

Lesson 4.4.1 — Linear Inequalities

In Exercises 1–4, write the inequality in words.

1. $p > 5$

2. $q < -13$

3. $r \leq -2$

4. $s \geq 2.5$

In Exercises 5–8, plot the inequality on a number line.

5. $j > 1$

6. $k \leq -2$

7. $n \geq -1$

8. $m < 0$

9. A number, x , increased by twelve is at least nineteen. Write this statement as an inequality and solve it.

In Exercises 10–13, solve the inequality for the unknown.

10. $a + 2 \leq 10$

11. $b - 4 > 0$

12. $c + (-2) < -1$

13. $d - (-4) \geq 8$

14. Ula is painting a room. She needs at least 5 liters of paint to cover the walls. She already has a 1.5 liter can. Write and solve an inequality to show how many liters of paint, p , Ula needs to buy.

15. Mike is 8 cm taller than Darla. She is less than 160 cm tall. Write an inequality to show Mike's height.

Lesson 4.4.2 — More On Linear Inequalities

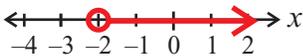
In Exercises 1–6, solve the inequality for the unknown.

- $2x > 18$
 - $3x < 48$
 - $x \div 8 \geq 8$
 - $x \div 12 \leq 11$
 - $86x \leq 1032$
 - $\frac{1}{2}x \geq 18$
7. Which of the following is the correct solution of the inequality $-2y < 4$?
- $y < -2$
 - $y > -2$
 - $y > 2$

In Exercises 8–13, solve the inequality for the unknown.

- $2x < -4$
- $x \div 3 > -1$
- $-4x \geq 8$
- $x \div -10 < 1$
- $-x < -7$
- $x \div -5 \leq -6$

In Exercises 14–17 say which inequality goes with which solution graphed on the number line.

- | | | |
|------------------------|--|---|
| 14. $2x < 4$ |  | A |
| 15. $\frac{x}{3} > 3$ |  | B |
| 16. $2x > 1$ |  | C |
| 17. $-\frac{x}{2} < 1$ |  | D |

Lesson 4.4.3 — Solving Two-Step Inequalities

1. Eva is saving money to buy a computer. She needs to save at least \$720. She already has \$200, and thinks that she can save \$40 more each month. Write and solve an inequality to find the number of months, m , that Eva will have to save for to get her computer.

In Exercises 2–7, solve the inequality for the unknown.

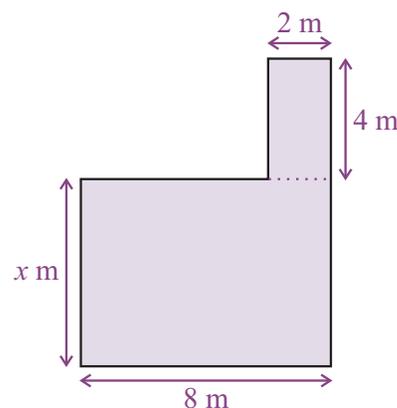
- $3x + 5 > 8$
- $2x - 9 < 13$
- $4x + 5 < -55$
- $13x - 6 > -58$
- $-18x + 3 \geq 39$
- $-15x - 3 \leq 72$

8. Sean is 6 years older than twice his cousin's age. Given that Sean is over 30, write and solve an inequality to describe his cousin's age, c .

In Exercises 9–14, solve the inequality for the unknown.

- $(x \div 2) + 4 > 7$
- $(x \div 4) - 5 \geq 1$
- $(x \div 3) + 9 < -2$
- $(x \div 7) - 4 > -8$
- $-\frac{x}{4} + 3 \leq 1$
- $-\frac{x}{5} - 7 < -11$

15. The diagram on the right shows the floor of a room in Felicia's house. Felicia has decided to lay a new carpet in the room. She hasn't measured the distance labeled x yet, but she knows that the total area of the floor is less than or equal to 80 m^2 . Write and solve an inequality for x .



✓ Lesson 5.1.1 — Multiplying With Powers

Write the expressions in Exercises 1–3 in base and exponent form.

1. $15 \cdot 15$

2. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

3. $w \cdot w \cdot w$

Evaluate the expressions in Exercises 4–9 using the multiplication of powers rule. Give your answers in base and exponent form.

4. $3^2 \cdot 3^3$

5. $6^1 \cdot 6^{12}$

6. $2^5 \cdot 2^7$

7. $a^5 \cdot a^1$

8. $(-5)^4 \cdot (-5)^9$

9. $x^8 \cdot x^9$

10. The distance from Anna's middle school to her home is 3^2 times the distance from the school to the library. If the distance from the school to the library is 3^5 yards, how many blocks away is Anna's middle school from her home?

Evaluate the expressions in Exercises 11–15 using the multiplication of powers rule. Give your answers in base and exponent form.

11. $4 \cdot 8$

12. $5 \cdot 125$

13. $3 \cdot 81$

14. $8 \cdot 16$

15. $1000 \cdot 100$

16. $36 \cdot 7776$

16. The 7th grade class has a display at the science fair which is a triangle-shaped board with a base of 3^2 inches and a height of 3^3 inches. What is its area?

✓ Lesson 5.1.2 — Dividing With Powers

Evaluate the expressions in Exercises 1–6 using the division of powers rule. Give your answers in base and exponent form.

1. $12^{10} \div 12^3$

2. $11^9 \div 11^7$

3. $g^{17} \div g^7$

4. $20^7 \div 20$

5. $15^{48} \div 15^{19}$

6. $(-8)^{15} \div (-8)$

7. The area of a rectangular game board is s^6 centimeters². The length of the board is s^2 centimeters. What is the width of the board?

Evaluate the expressions in Exercises 8–13 using the division of powers rule. Give your answers in base and exponent form.

8. $64 \div 2$

9. $128 \div 8$

10. $4096 \div 32$

11. $3125 \div 25$

12. $343 \div 49$

13. $12167 \div 529$

14. Each month, a company purchases 10^7 cell phone minutes and shares them equally among its 10^3 employees. How many minutes does each employee receive?

✓ Lesson 5.1.3 — Fractions With Powers

Simplify the expressions in Exercises 1–4. Give your answers in base and exponent form.

1. $\left(\frac{3}{8}\right)^2 \cdot \left(\frac{5}{8}\right)^3$

2. $\left(\frac{6}{5}\right)^{11} \cdot \left(\frac{6}{7}\right)^{12}$

3. $\left(\frac{2}{7}\right)^5 \cdot \left(\frac{2}{9}\right)^7$

4. $\left(\frac{a}{b}\right)^5 \cdot \left(\frac{a}{3}\right)^{10}$

5. $\left(\frac{-5}{13}\right)^{12} \div \left(\frac{-1}{10}\right)^9$

6. $\left(\frac{2}{z}\right)^8 \div \left(\frac{z}{2}\right)^6$

7. Can either the multiplication of powers rule or the division of powers rule be used to simplify the expression $\frac{3}{7} \div \frac{1}{14}$? Explain your answer.

8. Lisa studied $\left(\frac{3}{2}\right)^2$ hours for her math test and $\frac{3}{4}$ as long for her science test. How long did she study for her science test? Give your answer in base and exponent form.

9. Ashanti runs $\left(\frac{3}{x}\right)^2$ yards every day. Louise runs $\left(\frac{2}{x}\right)^3$ as far as Ashanti. How far does Louise run? Give your answer in base and exponent form.

Lesson 5.2.1 — Negative and Zero Exponents

Evaluate the expressions in Exercises 1–4.

- 21^0
- $(xy)^0, xy \neq 0$
- $38^0 + 38^0$
- $(9 - 5)^0$

Rewrite each of the expressions in Exercises 5–10 without a negative exponent.

- 3^{-2}
- 10^{-5}
- 5^{-13}
- 41^{-8}
- 99^{-2}
- x^{-y}

Rewrite each of the expressions in Exercises 11–14 using a negative exponent.

- $\frac{1}{10^4}$
- $\frac{1}{t^8}$
- $\frac{1}{55 \times 55 \times 55 \times 55 \times 55}$
- $\frac{1}{(-c) \times (-c) \times (-c)}$

15. Lisa's little sister is 1 year old and her big sister is 25. The ages of all three sisters are powers of the same base. How old is Lisa? Give your answer in base and exponent form.

Lesson 5.2.2 — Using Negative Exponents

Simplify the expressions in Exercises 1–4. Give your answers as powers in base and exponent form.

- $7^7 \times 7^{-5}$
- $29^8 \div 29^{-2}$
- $43^8 \div 43^{-21}$
- $12^{-12} \div 12^{-8}$

Simplify the expressions in Exercises 5–8 by first converting any negative exponents to positive exponents. Give your answers as powers in base and exponent form.

- $4^{15} \times 4^{-5}$
- $98^{-8} \times 98^{-5}$
- $a^{-56} \times a^{-40}$
- $32^{-x} \times 32^x$
- $38^2 \div \frac{1}{38^{-9}}$
- $r^{-n} \div \frac{3}{r^{-2}}$

11. $\left(\frac{5}{3}\right)^{-2}$ of the 7th grade class received an A on a recent science test. If there are 25 students in the class, how many students received an A?

Lesson 5.2.3 — Scientific Notation

Write the numbers in Exercises 1–12 in scientific notation.

- 420
- 6,000
- 917,000
- 938,700
- 245,000,000,000
- 93,000,000
- 147,396,000,000
- 53,560,000,000,000
- 0.00032
- 0.00000000819
- 0.000000064
- 0.000000387

Write the numbers in Exercises 13–16 in numerical form.

- 4.35×10^2
- 8.31×10^6
- 4.79×10^{-7}
- 9.101×10^{-12}

- A wealthy businessman is worth 5.28 billion dollars. What is 5.28 billion in scientific notation?
- Pritesh converts the number 16,200 into scientific notation and gets 16.2×10^3 . Explain his mistake.

Lesson 5.2.4 — Comparing Numbers In Scientific Notation

In Exercises 1–6, say which of the two numbers is greater.

- 3.27×10^3 , 3.27×10^6
- 4.9×10^7 , 4.9×10^{-7}
- 7.8×10^9 , 7.8×10^{-9}
- 4.36×10^3 , 8.2×10^2
- $2(1.5 \times 10^5)$, 3.0×10^{10}
- 9.67×10^{12} , 8.412×10^{13}

Order the expressions in Exercises 7–10 from least to greatest.

- 3.2×10^5 , 4.35×10^3 , 9.874×10^2 , 1.4×10^6 , 4.2×10^1
- 9.99×10^6 , 9.9×10^6 , 9.999×10^6 , 9.9999×10^6
- 2.7×10^8 , 3.965×10^6 , 1.982×10^{13} , 8.623×10^8
- 4.3×10^5 , 3.4×10^6 , 5.2×10^5 , 3.5×10^6

A space shuttle has two solid rocket boosters that each provide 1.19402×10^6 kg of thrust; 3 main engines that each provide 154,360 kg of thrust, and 2 orbital maneuvering systems engines which each provide 2.452×10^3 kg of thrust.

- What is the thrust of a single solid rocket booster as a decimal?
- Which type of engine has the greatest amount of thrust?
- Which engine has the least amount of thrust?
- What is the total amount of thrust provided by the engines? Give your answer in scientific notation.

Lesson 5.3.1 — Multiplying Monomials

State whether or not each expression in Exercises 1–3 is a monomial.

- $5y^9$
- $2x + 2y$
- $\frac{y^8}{2}$

Identify the coefficient in Exercises 4–7.

- $17a^3$
- $\frac{x}{2}$
- $31w^4$
- $\frac{51x^6}{9}$

Simplify the expressions in Exercises 8–16 by turning them into a single monomial.

- $17x^3 \times 2x^2$
- $w^3k^2 \times 2wn^6 \times 3k^3n^2$
- $6ab^2 \times b^3c$
- $\frac{40x^3}{9} \times x^2y \times y$
- $18x^2y^2z^2 \times xz^2$
- $\frac{2xz^3}{3} \times \frac{4xy^2}{5} \times \frac{x^6y^2z^2}{3}$

Square each monomial in Exercises 14–16.

- $8y^3$
- $4t^5uv^3$
- $12d^5e^5f^5$

Lesson 5.3.2 — Dividing Monomials

Evaluate each expression in Exercises 1–8.

- $6b^7 \div b$
- $18x^2y^2 \div 3xy$
- $144d^4e^9f^{17} \div 12d^4e^3f^7$
- $0.6t^3u^2v^3 \div 3xy$
- $81d^6 \div 9d^4$
- $27m^{12}n^2 \div 3mn^2$
- $20a^{17}b^{14} \div 5a^8b^{12}$
- $\frac{5}{7}x^3y^2z^4 \div \frac{1}{2}xyz^3$

9. Does $36xy^3 \div 2x^4y$ give a monomial result?

10. If all their chores are done on the weekend, the Anderson children receive $3x^2y^2$ dollars in total, split evenly between the $2x^2$ children. How much does each child receive?

Say whether each division gives a monomial result in Exercises 11–14.

- $5x^2 \div 5x$
- $40x^{34}p^{12} \div 20x^{30}p^{14}$
- $25z^3 \div 5z^4$
- $20z^4x^3 \div 5z^4x^3$

Lesson 5.3.3 — Powers of Monomials

Write the expressions in Exercises 1–9 using a single power.

- $(3^2)^3$
- $(5^4)^4$
- $[(-8)^2]^4$
- $[(-17)^2]^5$
- $(5^{-2})^{-10}$
- $(p^2)^6$
- $(r^a)^b$
- $(s^{-1})^b$
- $[(-t)^{-c}]^{-d}$

Simplify the powers of monomials in Exercises 10–15.

- $(4x^5)^2$
- $(2y^7)^3$
- $(7a^2b^3c)^2$
- $(2n^8o^3p^5)^3$
- $(g^{-2}n^3e^2)^4$
- $(-0.3x^{-2}y^mz^4)^3$

- What is the area of a square room with side lengths of $5x^2y^3z^{10}$ centimeters?
- What is the volume of a cubic container with side lengths of $2a^5b^3$ feet?
- What is the volume of a cylindrical container with a radius of $4x^3y^2$ and a height of $3x^5$?

Lesson 5.3.4 — Square Roots of Monomials

Simplify the expressions in Exercises 1–9.

- $\sqrt{81}$
- $\sqrt{36}$
- $\sqrt{5^2}$
- $\sqrt{z^2}$
- $\sqrt{c^{30}}$
- $\sqrt{a^2}$
- $\sqrt{x^4}$
- $\sqrt{y^{50}}$
- $\sqrt{t^{46}}$

Find the square roots of each monomial in Exercises 10–15.

- $16x^4$
- $100x^4z^8$
- $a^{12}b^{18}c^{20}$
- $49k^2h^4j^8$
- $625r^{12}s^{24}t^{32}$
- $144a^8b^{10}c^{12}d^{14}e^{16}$

- A square painting has an area of $25x^4y^2z^{18}$ square feet. What is the length of its side?

In Exercises 17–19, determine whether each square root will be a monomial.

- $\sqrt{x^{15}y^{12}}$
- $\sqrt{s^{11}t^8}$
- $\sqrt{23x^{16}t^{14}}$

Lesson 5.4.1 — Graphing $y = nx^2$

In Exercises 16–21, find the y -coordinate of the point on the $y = x^2$ graph for each given value of x .

- $x = 5$
- $x = \frac{1}{2}$
- $x = \frac{3}{4}$

Determine which of the points in Exercises 4–9 lie on the graph of $y = 2x^2$.

- $(-1, 2)$
- $(2, 8)$
- $(-5, 12)$
- $(4, 40)$
- $(\frac{1}{2}, \frac{1}{2})$
- $(-3, 18)$

In Exercises 10–15, calculate the two possible x -coordinates of the points on the graph of $y = x^2$ whose y -coordinate is shown.

- 81
- 144
- 4
- 36
- 14
- a

In Exercises 16–18, draw the graph of each of the given equations.

- $y = 2x^2$
- $y = 3x^2$
- $y = \frac{1}{2}x^2$

✓ Lesson 5.4.2 — More Graphs of $y = nx^2$

In Exercises 1–4, plot the graph of the given equation for the values of x between 5 and -5 .

1. $y = -x^2$
2. $y = -2x^2$
3. $-y = x^2$
4. Using your graph from Exercise 2, what is x if $-2x^2 = -18$?
5. Using your graph from Exercise 1, what is x if $-x^2 = -25$?

Answer Exercises 6 and 7 without plotting any points.

6. The point $(5, 5)$ lies on the graph of $y = \frac{1}{5}x^2$. What is the y -coordinate of the point on the graph of $y = -\frac{1}{5}x^2$ with x -coordinate 5?
7. The point $(-2, 8)$ lies on the graph of $y = 2x^2$. What is the y -coordinate of the point on the graph of $-y = 2x^2$ with x -coordinate -2 ?

For each point in Exercises 8–13, say which of the equations shown below it would lie on the graph of.

- | | | |
|------------------|-----------------|--------------|
| $y = x^2$ | $y = -2x^2$ | $y = 0.5x^2$ |
| $y = 3x^2$ | $y = -x^2$ | $y = -4x^2$ |
| 8. $(-9, 81)$ | 9. $(-4, -32)$ | |
| 10. $(6, 18)$ | 11. $(8, 32)$ | |
| 12. $(10, -400)$ | 13. $(-7, -49)$ | |

✓ Lesson 5.4.3 — Graphing $y = nx^3$

1. Make a table of values for $y = 5x^3$ for x between -4 and 4 .
2. Use the points in Exercise 1 to plot the graph.

Use your graph of $y = 5x^3$ from Exercise 2 to get approximate solutions to the equations in Exercises 3–8.

3. $5x^3 = 125$
4. $5x^3 = 305$
5. $5x^3 = -30$
6. $5x^3 = 70$
7. $5x^3 = -50$
8. $5x^3 = -210$

Graph the equations in Exercises 9–11.

9. $y = 2x^3$
10. $y = 3x^3$
11. $y = -x^3$

Make a table of values with x values from -4 to 4 for each of the equations in Exercises 12–15.

12. $y = -2x^3$
13. $y = 3x^3$
14. $y = -3x^3$
15. $y = -\frac{2}{3}x^3$

Answer Exercises 16 and 17 without plotting any points.

16. If the graph of $y = 0.2x^3$ goes through $(4, 12.8)$, what are the coordinates of the point on the graph of $-y = 0.2x^3$ with x -coordinate 4?
17. If the graph of $y = 0.8x^3$ goes through $(15, 2700)$, what are the coordinate of the point on the graph of $y = -1.6x^3$ with x -coordinate 15?

Lesson 6.1.1 — Median and Range

Find the median of each of the data sets in Exercises 1–8.

- {1, 3, 5, 7, 9, 11, 13}
- {9, 23, 48, 7, 100}
- {10, 4, 2, 8, 6}
- {99, 99, 100, 102, 101, 98, 107, 97}
- {30, 33, 30, 33, 33, 33}
- {65, 30, 25, 45, 20, 25}
- {18, 15, 13, 6, 9, 12}
- {20, 60, 80, 80, 20, 60, 60, 60}

Find the range of each of the data sets in Exercises 9–14.

- {71, 50, 32, 55, 90}
- {11, 11, 11, 11, 12, 12}
- {98, 99, 98, 99, 100}
- {500, 550, 575, 600, 625, 675}
- {365, 90, 90, 200, 250}
- {33.5, 6.5, 200.1, 82.3, 66.4}

15. Store X sells class rings with a median price of \$350 and a range of \$50. Store Y sells class rings with a median price of \$350 and a range of \$250. Interpret these statistics.

16. A cell phone company has packages with a median price of \$59.99 per month and a range of \$100. Another company has packages with a median price of \$49.99 and a range of \$50. Interpret these statistics.

Lesson 6.1.2 — Box-and-Whisker Plots

Find the upper and lower quartiles of each data set in Exercises 1–5.

- {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}
- {88, 77, 9, 23, 48, 7, 100, 102, 99}
- {99, 99, 100, 99, 99, 100, 102, 101, 98, 107, 97}
- {20, 22, 24, 25, 30, 33, 30, 33, 33, 33, 50, 53}
- {30, 25, 20, 18, 15, 13, 6, 9, 12}

Create a box-and-whisker plot to illustrate each of the data sets in Exercises 6–11.

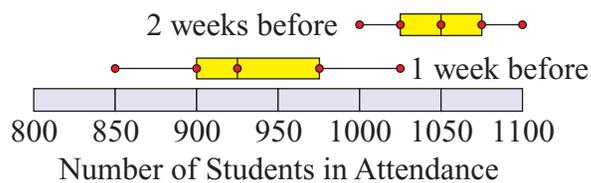
- {11, 11, 11, 11, 12, 12, 13, 13}
- {71, 50, 32, 55, 90}
- {10, 15, 20, 25, 30, 35, 40, 45, 50}
- {150, 150, 200, 365, 90, 90, 200, 250}
- {36, 37, 38, 39, 40, 40, 40, 45}
- {50, 60, 70, 70, 90, 100, 10, 40}

12. Terrell recorded the daily temperature each day for a week and made the data set {45°F, 40°F, 40°F, 53°F, 52°F, 40°F, 33°F}. Draw a box-and-whisker plot to illustrate this data.

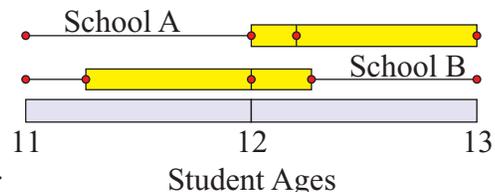
Lesson 6.1.3 — More On Box-and-Whisker Plots

1. Principal Garcia is curious to know whether school attendance drops off before a holiday.

The box-and-whisker plots on the right show school attendance 2 weeks before a holiday and 1 week before a holiday. What conclusions can Principal Garcia draw from the plots?



2. The ages of the band members from two middle schools are shown in the box-and-whisker plots on the right. Which school has a larger percentage of younger students?



3. What is the difference between the range of a box-and-whisker plot and the interquartile range?

4. In a box-and-whisker plot, what is shown by the length of the whiskers?

5. Name the three different areas of a box-and-whisker plot that contain exactly half of the data values.

Lesson 6.1.4 — Stem-and-Leaf Plots

Make stem-and-leaf plots to display the data given in each of Exercises 1–7.

- {11, 13, 24, 29, 33, 35, 37, 39}
- {13, 14, 15, 17, 18, 18, 24, 34, 42}
- {34, 35, 39, 40, 47, 50, 54, 56, 57, 60}
- {2, 3, 6, 6, 10, 12, 14, 19}
- {2, 12, 13, 14, 22, 23, 27, 33, 35}
- {82, 82, 83, 83, 83, 84, 84, 85, 85, 89, 92}
- {3, 4, 4, 4, 12, 15, 23, 25, 27, 33, 34, 34, 34, 36, 42, 44, 54, 57, 60}

Find the median and the range of the stem-and-leaf plots in Exercises 8–9.

8.

4		1 3
5		4 9
6		3 5 7 9

Key: 4|3 represents 43

9.

3		4 5 9
4		0 7
5		0 4 6 7
6		0

Key: 4|0 represents 40

Draw a back-to-back stem-and-leaf plot for each pair of data sets in Exercises 10–15.

- {18, 28, 38, 48, 49, 50}
{23, 23, 23, 35, 35, 41, 41, 43, 52}
- {7, 9, 10, 14, 17, 23, 25, 29}
{13, 13, 14, 14, 21, 22}

Lesson 6.1.5 — Preparing Data to be Analyzed

40 people participated in the tests for a new health food diet. Half ate a regulated 1500 calorie diet while the other half ate as much as they wanted from a selected group of health foods. Their weight loss in pounds after one month is listed in the chart below. Use this data for Exercises 1–10.

1500 Calorie Group	5	9	10	13	11	15	5	6	8	8	7	3	0	9	7	6	5	5	3	10
Health Food Group	7	10	12	15	14	12	10	9	12	10	10	9	15	13	7	8	9	7	8	15

- Find the minimum of each data set.
- Find the maximum of each data set.
- Find the median of the 1500 calorie group.
- Find the median of the health food group.
- What is the lower quartile of the 1500 calorie group?
- What is the lower quartile of the health food group?
- What is the upper quartile of the 1500 calorie group?
- What is the upper quartile of the health food group?
- Create box-and-whisker plots of the two data sets.
- Create a double stem-and-leaf plot of the two sets.

Lesson 6.1.6 — Analyzing Data

A chef was trying to determine which of two daily specials is more popular. He kept track of the number of orders received for each over 22 days. The results are shown below — use this data for Exercises 1–8.

Special 1: {50, 60, 89, 95, 45, 99, 98, 99, 87, 88, 89, 91, 92, 95, 94, 95, 99, 98, 98, 87, 99, 95}

Special 2: {85, 85, 85, 45, 77, 62, 88, 87, 99, 95, 94, 99, 66, 68, 72, 75, 98, 99, 99, 99, 99, 99}

- Find the minimum and maximum of each data set.
- Find the range of each data set.
- Find the median of each data set.
- Find the lower and upper quartile of each data set.
- Find the interquartile range of each data set.
- Draw a box-and-whisker plot of each data set.
- Draw a back-to-back stem-and-leaf plot of the data sets.
- Compare the popularity of the two specials.

Lesson 6.2.1 — Making Scatterplots

1. What data would you need to collect to test the conjecture “the more books read by a student, the higher their grade point average”?
2. Design a table in which to record this data.
3. What data would you need to collect to test the conjecture “the further you live from school, the fewer after-school clubs you’re a member of”?
4. Design a table in which to record this data.
5. Kayla decides to test the conjecture that “the older the child, the taller they are” and collects the data shown below. Draw a scatterplot of this data.

Age (years)	3	5	4	6	5	6	3
Height (inches)	36	42	40	45	40	46	40

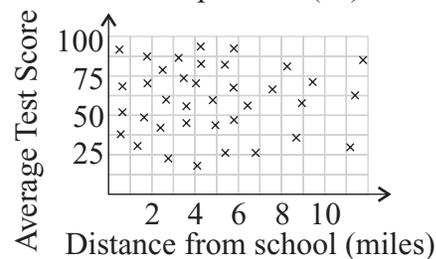
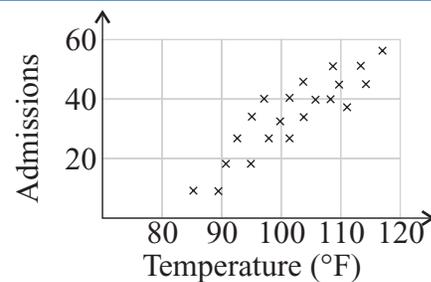
6. The data below was collected to test the conjecture “the more minutes offered by a cell phone contract, the more expensive the contract”. Draw a scatterplot of this data.

Number of minutes	300	300	400	400	500	500	500	500	500	600
Monthly Fee	\$20	\$25	\$25	\$30	\$30	\$35	\$40	\$45	\$50	\$50

Lesson 6.2.2 — Shapes of Scatterplots

A hospital in the desert made the scatterplot on the right to determine how outside temperature is related to hospital admissions due to heat stroke. Use it to answer Exercises 1–2.

1. What sort of correlation does the scatterplot show?
2. Is the correlation strong or weak? Explain your answer.
3. Draw an example scatterplot with no correlation.
4. Draw an example scatterplot with a weak negative correlation.
5. Draw an example scatterplot with a strong positive correlation.
6. The scatterplot on the right shows how average test score is related to the distance that students live from school. What sort of correlation does the scatterplot show?



Lesson 6.2.3 — Using Scatterplots

Roy decides to test his theory that the older a person in the baseball team is, the more professional baseball games they’ve seen. He collects the following data:

Age	11	12	11	13	15	14	14	12	11	12	13
Number of games seen	20	15	10	35	30	29	18	10	5	13	20

1. Create a scatterplot of the data.
2. Draw a line of best fit.
3. Predict how many professional baseball games a 17 year old on the baseball team is likely to have seen.

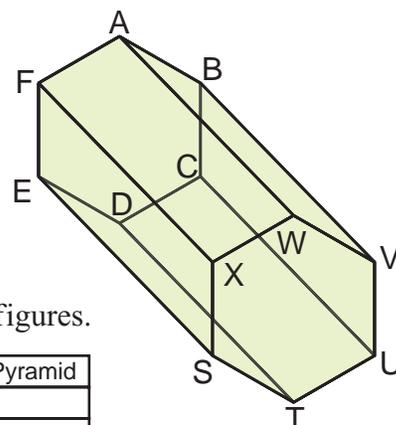
Lesson 7.1.1 — Three Dimensional Figures

In Exercises 1–4, say whether the statements are true or false.

1. A polygon is any shape that is made from straight lines that are joined end-to-end, in a closed shape.
2. Pyramids and prisms have circles for their base.
3. Prisms must have congruent bases at each end.
4. Pyramids have diagonals.

In Exercises 5–7, refer to the figure at the right.

5. Identify the shape as either a cone, cylinder, prism, or pyramid.
6. How many diagonals does this shape have?
7. Name all diagonals by giving the starting and ending vertex.

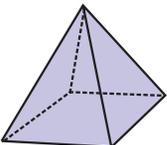
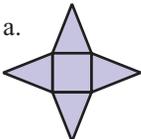
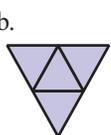
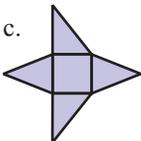


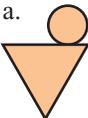
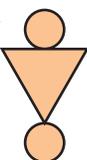
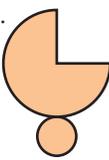
8. Copy the table below and check all statements that are true about the figures.

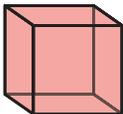
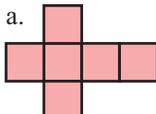
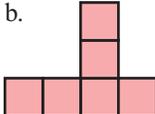
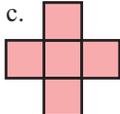
	Cone	Cylinder	Prism	Pyramid
The figure is a polyhedron				
The figure has diagonals.				
The base of the figure has a curved edge.				
The base of the figure is a polygon.				
The bases are congruent.				
The base of the figure is a polygon, and the other faces meet at a single point.				
The base of the figure has a curved edge, and the other end meets at a single point.				

Lesson 7.1.2 — Nets

In Exercises 1–3, say which net could make each three-dimensional figure.

1.  a.  b.  c. 

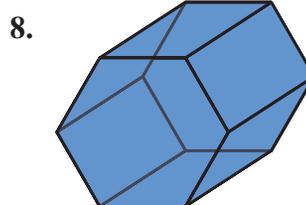
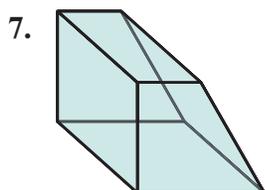
2.  a.  b.  c. 

3.  a.  b.  c. 

In Exercises 4–6, say whether the statements are true or false.

4. Every shape has one unique net.
5. In the net of a cylinder, the length of the rectangle is equal to the circumference of the base circle.
6. The net of a rectangular prism has five rectangles.

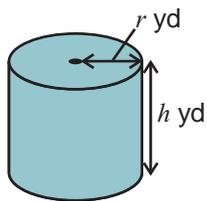
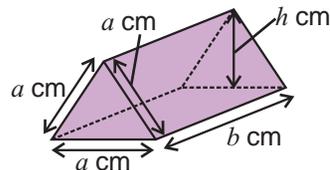
In Exercises 7–8, draw a net for the solid.



Lesson 7.1.3 — Surface Areas of Cylinders and Prisms

In Exercises 1–3, find the surface area of each triangular prism.

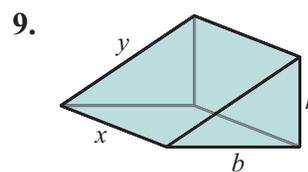
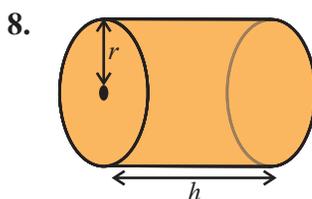
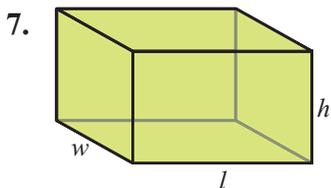
- $a = 5, b = 8, h = 4.3$
- $a = 8, b = 12, h = 6.9$
- $a = 20, b = 35, h = 17.3$



In Exercises 4–6, find the surface area of each cylinder. Use $\pi = 3.14$.

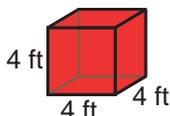
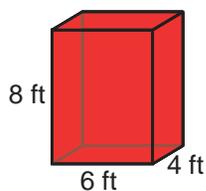
- $r = 6$ and $h = 11$
- $r = 12$ and $h = 17$
- $r = 30$ and $h = 45$

Use each of the diagrams in Exercises 7–9 to write a general formula for finding the surface area of that type of figure.

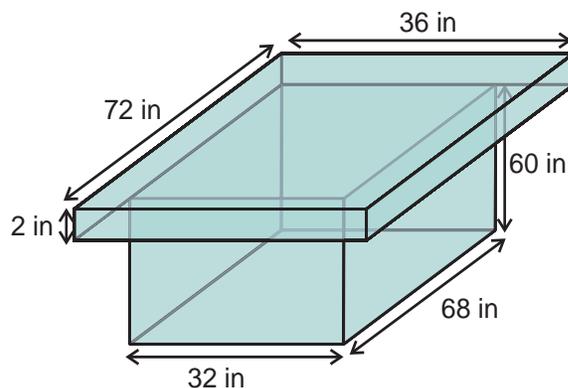
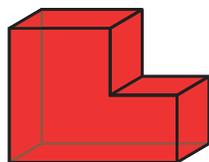


Lesson 7.1.4 — Surface Areas & Perimeters of Complex Shapes

Exercises 1–4 are about the figures shown below:



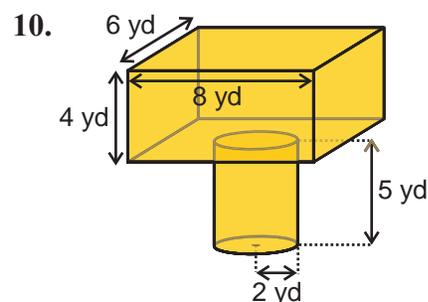
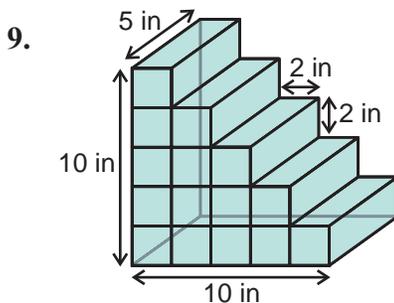
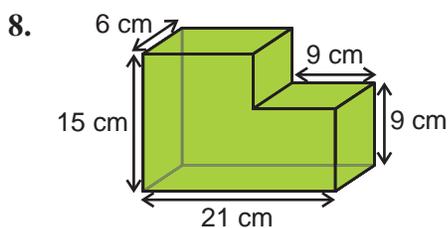
- Find the edge length of the cube.
- Find the edge length of the rectangular prism.
- Find the total edge length of the two figures.
- Find the total edge length once the figures have been joined.



A work table has the shape shown on the right.

- What is the total edge length of the tabletop?
- What is the total edge length of the base?
- Find the total edge length of the work table.

Work out the surface areas of the shapes shown in Exercises 8–10. Use $\pi = 3.14$.



Lesson 7.1.5 — Lines and Planes in Space

In Exercises 1–3, copy the sentences and fill in the missing words.

- When two planes intersect they can meet along a _____ or at a single _____.
- There are _____ ways for a line and a plane to meet.
- Coplanar lines are on the same _____.

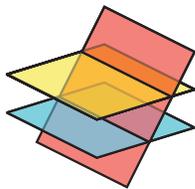
In Exercises 4–6, say whether each statement is true or false. If any are false, explain why.

- Lines that do not intersect are always parallel.
- Perpendicular planes meet at a point.
- Skew lines are not coplanar.

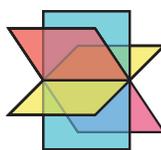
In Exercises 7–9, match each figure to one of the following descriptions:

- The intersecting planes meet along a line.
- The intersecting lines meet at a point.
- One plane intersects two parallel planes.

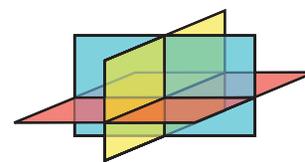
7.



8.



9.

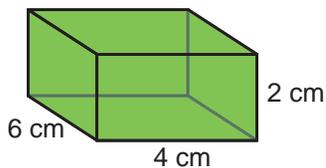


Lesson 7.2.1 — Volumes

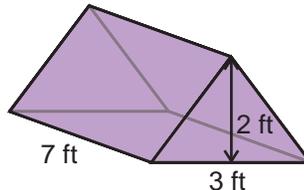
- A prism is 3 meters high. It has volume 12 m^3 . What is the area of the prism's base?

In Exercises 2–6, work out the volume of the figures. Use $\pi = 3.14$.

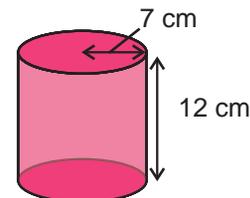
2.



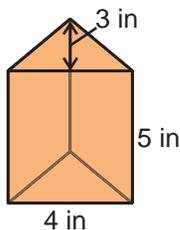
3.



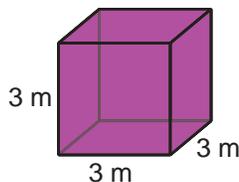
4.



5.



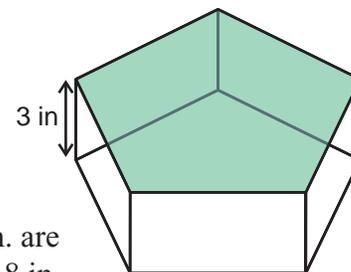
6.



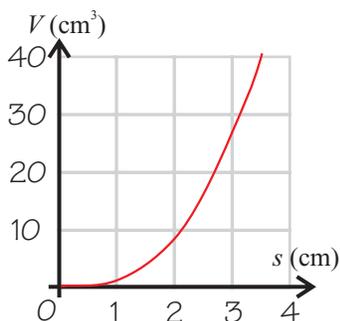
In Exercises 7–8, consider a prism of volume 72 ft^3 .

- What is the height of the prism if the base area is 12 ft^2 ?
- If the prism is cut in half, what is its new volume?
- Find the volume of the figure shown on the right if the area of the shaded base is 26 in^2 .

- The contents of a full baking pan with dimensions 8 in. by 8 in. by 2 in. are poured into a cylindrical container with a diameter of 5 in. and a height of 8 in. Will the cylindrical container hold all the contents of the pan? Explain your answer.



Lesson 7.2.2 — Graphing Volumes

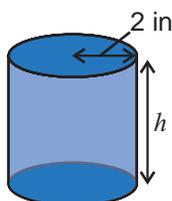
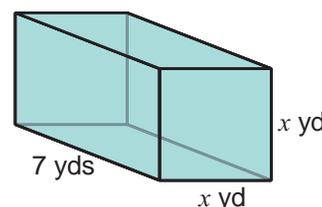


Use the graph of the volume of a cube with side length s , shown on the left, to answer Exercises 1–4.

1. Estimate the volume of a cube with side length 2.1 m.
2. Estimate the side length of a cube with volume 25 m^3 .
3. Estimate the volume of a cube with side length 1.5 m.
4. Estimate the side length of a cube with volume 33 m^3 .

Exercises 5–8 refer to the figure at the right.

5. Write an expression for the volume of the prism.
6. Use your expression to find the volume of the prism when $x = 5$.
7. Graph the volume of the figure against the value of x .
8. Use your graph to estimate the side length x that makes a volume of 28 yd^3 .



Exercises 9–11 refer to the figure on the left.

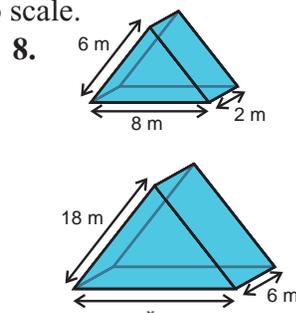
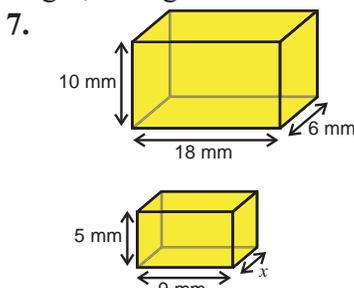
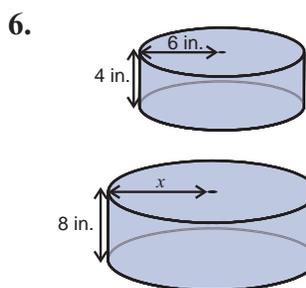
9. Find an expression that gives the volume of the figure. Use $\pi = 3.14$.
10. Graph the volume of the figure.
11. Use your graph to estimate the value of h that makes the volume 55 in^3 .

Lesson 7.3.1 — Similar Solids

In Exercises 1–5, say whether the statements are true or false.

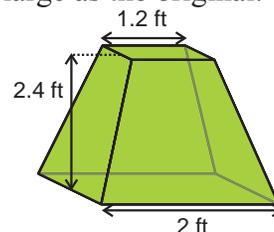
1. When multiplying by a scale factor, the corresponding angles of the image are multiplied by the scale factor.
2. If you multiply a three-dimensional figure by a scale factor you get a similar figure.
3. A scale factor of one produces an image the same size as the original figure.
4. Two figures are similar if one can be multiplied by a scale factor to make a shape that is congruent to the other one.
5. The image will be larger than the original if the scale factor is between 0 and 1.

In Exercises 6–8, find the scale factor that has been used to create the image in the following pairs of similar solids and find the missing length, x . Figures are not drawn to scale.



A class constructed a scale model of the figure below. The model was twice as large as the original.

9. What is the height of the students' model?
10. What is the length of the bottom face of the model?
11. What is the length of the top face of the model?
12. The class decided to construct another model one-quarter the size of the original that will be easier to transport. What is the height of the new model?

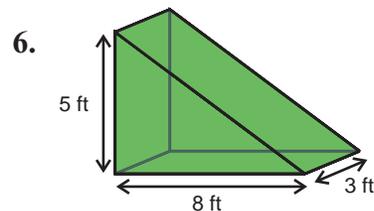
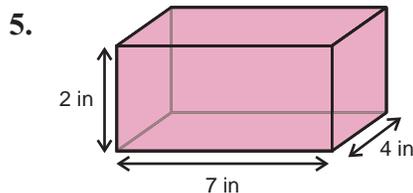
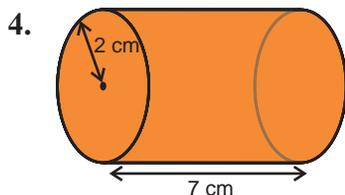


Lesson 7.3.2 — Surface Areas & Volumes of Similar Figures

Luis draws a figure with area 16 cm^2 . Find the area of the image if Luis multiplies his figure by the following scale factors.

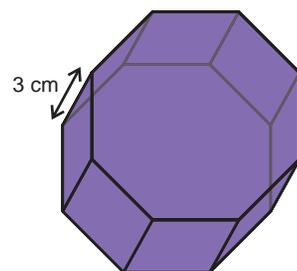
- 3
- 20
- 0.3

In Exercises 4–6, find the surface area and volume of the image if the figure shown is multiplied by a scale factor of 3. Figures are not drawn to scale. Use $\pi = 3.14$.



Exercises 7–8 refer to figure A at the right. The base area of figure A is 24 cm^2 .

- Find the volume of figure A.
- Find the volume of the image if A was enlarged by a scale factor of 4.



- A scale model of a building has a surface area of 37.5 ft^2 . If the actual building has a surface area of $15,000 \text{ ft}^2$, what scale factor was used to make the model?

Lesson 7.3.3 — Changing Units

In Exercises 1–3, convert the following areas to cm^2 .

- 21 m^2
- 0.085 m^2
- 4.5 m^2

In Exercises 4–6, convert the following areas to square feet.

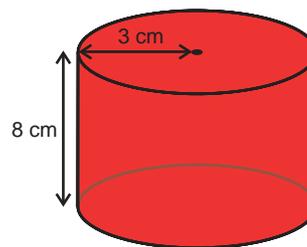
- 864 in^2
- 48 in^2
- 288 in^2

A cube has a surface area of 240 cm^2

- What is the surface area in m^2 ?
- What is the surface area in in^2 ?

Exercises 9–11 refer to the cylinder on the right. Use $\pi = 3.14$

- What is the surface area of the cylinder?
- What is the surface area in m^2 ?
- What is the surface area in in^2 ?



In Exercises 12–14, convert the volumes to cubic inches.

- 3 ft^3
- 0.864 ft^3
- 5.1 ft^3

In Exercises 15–17, convert the volumes to cubic meters. Use the conversion factor $1 \text{ m} = 0.91 \text{ yd}$. Round to the nearest hundredth where appropriate.

- 5 ft^3
- 25 ft^3
- 24.56 yd^3

- Juan's house has a floor area of 4500 square feet. What is this area in square meters?
- An acre is 4840 square yards. What is an acre in square meters?

✓ Lesson 8.2.1 — Discounts and Markups

A stationery store is having a sale. They offer the discounts shown in the table below. Use this information to calculate how much the items in Exercises 1–6 would cost from the sale.

Item	Notebooks	Pencils	Pens	Erasers	Ring Binders	Adhesive Tape
Original Price	\$5.99	\$0.59	\$1.99	\$2.99	\$4.99	\$2.99
Discount	20%	5%	15%	25%	30%	10%

- 2 ring binders
- 3 erasers
- 3 rolls of adhesive tape
- 10 notebooks
- 1 notebook and 1 pen
- 6 pencils and 5 erasers

Find the retail price of each of the items in Exercises 7–11.

- Cherry vanity unit — wholesale price \$150, markup 45%.
- Cherry dresser — wholesale price \$850, markup 60%.
- Florentine mirror — wholesale price \$95, markup 100%.
- Sleigh bed — wholesale price \$530, markup 85%.
- Mattress set — wholesale price \$999, markup 50.5%.

✓ Lesson 8.2.2 — Tips, Tax, and Commission

In Exercises 1–4 use mental math to calculate each tip.

- 10% tip on a \$15 taxi ride.
- 20% tip on a \$30 hair style.
- 15% tip on a \$5 salad.
- 15% tip on a \$34.26 family meal.
- A \$25 concert ticket had a 15% entertainment tax added to the price.

What was the total cost of the ticket?

Work out the total cost after tax of the items in Exercises 6–11.

- 5% added to a \$39.60 purchase.
- 8% tax added to a \$15,000 car.
- 7% tax added to a \$549 laptop.
- 9% tax added to a \$2.99 notebook.
- 6% tax added to a \$0.49 fruit juice.
- 11% tax added to a \$10.34 fruit basket.
- Jarrold bought a bike priced at \$109.98. After taxes he paid \$119.33. What was the rate of tax?

✓ Lesson 8.2.3 — Profit

Find the profit made in Exercises 1–4.

- Expenses: \$800
Revenue: \$3000
- Expenses: \$954
Revenue: \$3975.01
- Expenses: \$777.77; \$5.89
Revenue: \$1,258.34
- Expenses: \$800,034; \$957.45; \$999,381.45
Revenue: \$12,456,901

In Exercises 5–8, find the percent profit. Round each answer to the nearest whole percent.

- Profit: \$245
Total Sales: \$4,875
- Profit: \$300
Total Sales: \$879.20
- Profit: \$9,000,000
Total Sales: \$21,000,000
- Profit: \$4,432,567
Total Sales: \$6,289,437
- The math club sold 500 pencils at 10 cents each as a fundraiser. They paid the supplier 2 cents for each pencil. Emmitt says the profit is \$40 but Lonnie says it is \$50. Who is correct?
- A company increased its profits by 33% over the previous year. If the previous year's profits were \$5,000,000, what are the profits this year?

✓ Lesson 8.2.4 — Simple Interest

1. Jacob put \$3000 in an account with a simple interest rate of 5% per year. How much was in his account after 1 year?
2. Juan invested \$10,000 and received 6.25% simple interest per year. How much did she have after 5 years?
3. Sara borrowed \$500 at a simple interest rate of 5.5% per year. How much did she owe after 5 years?
4. A savings account advertises a simple interest rate of 3.5% per year. If Jan puts \$5000 in the account, how much interest will she have earned after 1 year?
5. A bank advertises a saving scheme which gives 12% simple interest per year if you keep your money in the bank for 8 years with the slogan “double your money”. Is their slogan accurate?
6. Lisa asks her mom to borrow \$20 for new jeans. Her mom says she will charge 3% simple interest per month. If Lisa intends to repay her mom in half a month, how much interest will she owe?
7. Janice has decided to buy a new sofa set on credit. One furniture store offers credit with no interest for 6 months and then 18.5% simple interest per year. Another offers 4 months with no interest and then 18% simple interest per year. Which is the better deal for a \$3000 sofa set which Janice has budgeted to pay off in 2 years?

✓ Lesson 8.2.5 — Compound Interest

1. If you're saving money, is it usually better to receive simple or compound interest?
2. If you're borrowing money, is it usually better to pay simple or compound interest?
3. You put \$250 into an account with a compound interest rate of 6%, compounded annually. What is the account balance after 5 years?
4. Daniel puts \$1500 in a savings account with 6% interest for 8 years compounded quarterly. What is the account balance in 8 years?
5. Cynthia has \$1700 to invest for 8 years. She can choose between either an investment with a simple interest rate of 15% or one with a compound interest rate of 8%, compounded annually. Which investment would leave her better off?
6. Luis puts \$600 into an account with a compound interest rate of 10%, compounded annually. Destiny puts \$600 into an account with a simple interest rate of 12%. What's the difference between the investments after 5 years?

✓ Lesson 8.3.1 — Rounding

In Exercises 1–3, round each number to the nearest percent.

1. 87.5%
2. 45.3%
3. $71\frac{3}{4}\%$

In Exercises 4–9, round each number to the nearest tenth.

4. 103.785
5. 3.265
6. 40.23
7. 75.432
8. 7.9854
9. 41.98

10. Don's favourite group sold 549,873 copies of their last song. How many did they sell to the nearest hundred thousand?

11. Ana and Ava were asked to round 7199.99 to the nearest hundred. Ana said the answer was 7100, Ava said 7200. Who was correct?

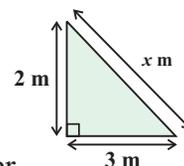
12. 63,495 fans attended a recent football game, which the newspaper rounded to the nearest 10,000. How many did the newspaper report had attended?

✓ Lesson 8.3.2 — Rounding Reasonably

1. Nadia needs 233 yards of ribbon to decorate the hall for the prom. If the ribbon comes on spools 10 yards long how many spools does Nadia need to buy?
2. Raul is buying a window shade for a window that measures 95.2 cm wide. The store has shades of 93 cm, 95 cm and 97 cm wide. What width of shade should Raul buy?
3. The drama club is using its funds to sponsor students acting at the Shakespeare festival by paying their \$25 entrance fee. If they have \$282, how many full entry fees can they pay?
4. Latoyah is baking bread. She needs 0.4 kg of flour for each loaf she makes. If she has 1.8 kg of flour altogether, how many loaves can she bake?
5. Michael's restaurant bill came to \$47.10. He wants to leave a 20% tip, but he only has dollar bills with him. What tip should Michael leave?
6. Anjali is saving \$20 per week towards a new computer. If the computer she wants costs \$728.50, how many weeks will she need to save for?
7. Mr Scott is buying new books for the school library. The publishers will sell him each book for \$12. If he has a budget of \$500, what is the maximum number of books he can buy?

✓ Lesson 8.3.3 — Exact and Approximate Answers

1. Estimate the area of a circle with a radius of 7 cm, using 3.14 as an approximation of π . What is the exact area of the circle?
2. Kiana is buying presents for her family. She has \$85, and wants to spend the same amount on each person. If there are 6 people in her family, what is the most that she can spend on each one?
3. Rico's kitchen scales measure weights in kilograms to two decimal places. He uses the scales to weigh out 1.45 kg of rice. Given that $1.0 \text{ kg} \approx 2.2 \text{ pounds}$, what weight of rice does Rico have in pounds?
4. Find the perimeter of the triangle in the diagram on the right.
5. Emma has a lawn with the same dimensions as the triangle in the diagram. She wants to put edging round it. If edging comes in 0.4 m strips, how many should Emma buy?
6. I measured the side of a square as being 5.6 cm long to the nearest tenth of a centimeter. With round-off error, what are the maximum and minimum possible areas of the square?



✓ Lesson 8.3.4 — Reasonableness and Estimation

1. Tion is shopping at the grocery store, and only has \$35 with him. He puts items costing \$5.99, \$3.98, \$10.57, and \$12.99 in his basket. Use estimation to check whether he has enough money with him to pay.
2. Rachel puts \$102 in a savings account with a yearly rate of 5.2% simple interest. She says that she will earn \$53 in interest each year. Perform your own estimate, and say whether she is likely to be correct.
3. Find the product of 73 and 39. Check your product using estimation.
4. Ms Harris is organising a field trip. 144 students are going on the trip, and the bus hire company can provide up to 5 buses that seat 50 students each. How many buses should Ms Harris hire?
5. James measures the temperature in his kitchen as being 70°F . He converts this to degrees Celsius, and says that the temperature in his kitchen is $21.\bar{1}^\circ\text{C}$. Is this a reasonable answer?
6. Diega is baking scones. Her recipe calls for 0.25 liters of milk. She knows that $1 \text{ liter} \approx 4.2 \text{ cups}$. Diega says that she needs to add about 1 cup of milk to the mixture. Is this a reasonable answer?

Appendixes

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Glossary

Symbols

$<$ is less than	\neq is not equal to
$>$ is greater than	\mathbb{W} the whole numbers
\leq is less than or equal to	\mathbb{Z} the integers
\geq is greater than or equal to	

A

absolute value the absolute value of a number, n , is its distance from zero on the number line, and is written $|n|$. Absolute values are always positive, for example $|-3.6| = 3.6$.

acute triangle a triangle in which all angles are less than 90°

altitude the “height” of a triangle, measured at right angles to its base

arc part of a circle's *circumference*; can be drawn with a compass

associative properties (of addition and multiplication)

for any a, b, c : $a + (b + c) = (a + b) + c$
 $a(bc) = (ab)c$

B

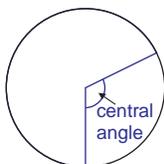
base in the expression b^x , the base is b

bisect divide in half

box-and-whisker plots a diagram showing the *range*, *median*, and *quartiles* of a data set against a number line.

C

central angle the angle between two *radii* of a circle, for example:



chord a straight line joining together two points on the *circumference* of a circle

circumference the distance around the outside of a circle

coefficient the number that a *variable* is multiplied by in an algebraic term. For example, in $6x$, the coefficient of x is 6

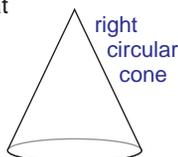
commission money earned by an agent when he or she sells a good or service, usually given as a *percent* of the sale price

common factor a number or expression that is a *factor* of two or more other numbers or expressions

common multiple a multiple of two or more different integers

commutative properties (of addition and multiplication) for any a, b : $a + b = b + a$ and $ab = ba$

cone a three-dimensional figure that has a circle or ellipse as its base, from which a curved surface comes to a point, for example:



conjecture a mathematical statement that is only an informed guess. It seems likely to be true, but hasn't been proved

congruent exactly the same size and shape

constant of proportionality a number, k , which always has the same value in an *equation* of the form $y = kx$ or $y = \frac{k}{x}$

converse of the Pythagorean theorem if a triangle has sides a, b , and c , where $c^2 = a^2 + b^2$, then it is a *right triangle*, and c is its *hypotenuse*

conversion factor the ratio of one unit to another; used for converting between units

coordinate pair an ordered pair of coordinates representing a point on the *coordinate plane*, for example, $(2, -3)$

coordinate plane a flat surface that extends to infinity, on which points are plotted using two *perpendicular axes* (usually x - and y -axes)

coplanar points, lines, or figures are coplanar if they lie in the same plane

correlation a relationship between two *variables*

counterexample an example that disproves a *conjecture*

cube a three-dimensional figure with six identical square *faces*

customary units the system of units that includes: inches, feet, yards, miles, ounces, and pounds

cylinder a three-dimensional figure with two parallel circular or elliptical bases and a constant cross-section

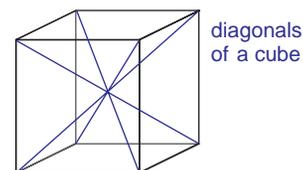
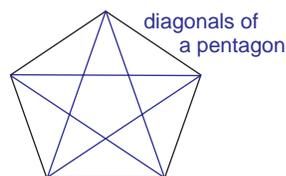
D

data set a collection of information, often numbers

decimal a number including a decimal point; digits to the right of the decimal point show parts of a whole number

denominator the bottom expression of a fraction

diagonal a straight line joining two nonadjacent corners of a two-dimensional figure, or two *vertices* of a three-dimensional figure that aren't on the same *face*, for example:



diameter a straight line from one side of a circle to the other, passing through the center

dimensional analysis a method of checking that a formula is correct by examining units

direct variation a relationship between two variables in which the *ratio* between them is always the same

distributive property (of multiplication over addition)

for any a, b, c : $a(b + c) = ab + ac$

divisor the number by which another number is being divided. For example, in $12 \div 3$, the divisor is 3

E

edge on a three-dimensional figure, an edge is where two *faces* meet

equation a mathematical statement showing that two quantities are equal

equilateral triangle a triangle whose sides are all the same length

Glossary (continued)

equivalent fractions fractions are equivalent if they have the

same value, for example: $\frac{1}{2} = \frac{2}{4}$, $\frac{2}{3} = \frac{4}{6}$

estimate an inexact judgement about the size of a quantity; an "educated guess"

evaluate find the value of an *expression* by substituting actual values for *variables*

exponent in the *expression* b^x , the exponent is x

expression a collection of numbers, *variables*, and symbols that represents a quantity

F

face a flat surface of a three-dimensional figure

factor a number or *expression* that can be multiplied to get another number or expression — for example, 2 is a factor of 6, because $2 \times 3 = 6$

factorization a number written as the product of its *factors*

formula an equation that relates at least two *variables*, usually used for finding the value of one variable when the other values are known. For example, $A = \pi r^2$

G

generalization a statement that describes many cases, rather than just one

greatest common factor (GCF) the largest *expression* that is a *common factor* of two or more other expressions; all other common factors will also be *factors* of the GCF

grouping symbols symbols that show the order in which mathematical operations should be carried out — such as parentheses and brackets

H

hypotenuse the longest side of a *right triangle*

I

identity properties (of addition and multiplication)

for any a , $a + 0 = a$, and $a \times 1 = a$

improper fraction a fraction whose *numerator* is greater than or equal to its *denominator*, for example $\frac{7}{4}$

integers the numbers $0, \pm 1, \pm 2, \pm 3, \dots$; the set of all integers is denoted \mathbb{Z}

interest extra money you pay back when you borrow money, or that you receive when you invest money

inverse (additive) a number's additive inverse is the number that can be added to it to give 0 — for any a , the additive inverse is $(-a)$

inverse (multiplicative)

a number's multiplicative inverse is the number that it can be multiplied by to give 1 — for any a , this is $\frac{1}{a}$

inverse operation an operation that "undoes" another operation — addition and subtraction are inverse operations, as are multiplication and division

isosceles triangle a triangle with two sides of equal length

L

line of best fit a trend line on a *scatterplot* — there will be roughly the same number of points on each side of the line

linear equation an *equation* linking two *variables* that can be written in the form $y = mx + b$, where m and b are constants

least common multiple (LCM) the smallest *integer* that has two or more other integers as *factors*

M

mean a measure of *central tendency*; the sum of a set of values, divided by the number of values in the set

measure of central tendency the value of a "typical" item in a data set. *Mean*, *mode* and *median* are measures of central tendency

median the middle value when a set of values is put in order

metric the system of units that includes: centimeters, meters, kilometers, grams, kilograms, and liters

mixed number a number containing a whole number part and a fraction part

monomial an *expression* with a single *term*

N

net a two-dimensional pattern that can be folded into a three-dimensional figure

numerator the top expression of a fraction

numeric expression a number or an *expression* containing only numbers and operations (and therefore no *variables*)

O

obtuse triangle a triangle in which one angle is greater than 90°

origin on a number line, the origin is at zero; on the *coordinate plane*, the origin is at the point $(0, 0)$

P

parabola a "u-shaped" curve obtained by graphing an *equation* of the form $y = nx^2$

parallelogram a four-sided shape with two pairs of parallel sides

PEMDAS the order of operations — "Parentheses, Exponents, Multiplication and Division, Addition and Subtraction"

percent value followed by the % sign; corresponds to the *numerator* of a fraction with 100 as the *denominator*

perimeter the sum of the side lengths of a polygon

perpendicular at right angles to

power an *expression* of the form b^x , made up of a *base* (b) and an *exponent* (x)

prime factorization a *factorization* of a number where each *factor* is a *prime number*, for example $12 = 2 \times 2 \times 3$

prime number a whole number that has exactly two factors, itself and 1

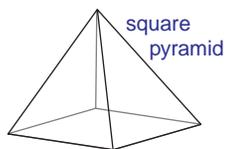
prism a three-dimensional figure with two identical parallel bases and a constant cross-section

product the result of multiplying numbers or *expressions* together

proportion an *equation* showing that two ratios are equivalent

proper fraction a fraction whose *numerator* is less than its *denominator*

pyramid a three-dimensional figure that has a polygon as its base and in which all the other *faces* come to a point, for example:



Pythagorean theorem for a *right triangle* with side lengths a , b , and c , $a^2 + b^2 = c^2$

Pythagorean triple three whole numbers a , b , and c , that satisfy $a^2 + b^2 = c^2$

Q

quadrant a quarter of the *coordinate plane*, bounded on two sides by parts of the x - and y -axes

quadrilateral a two-dimensional figure with four straight sides

quartiles split an ordered data set into four equal groups — the median splits the data in half, and the upper and lower quartiles are the middle values of the upper and lower halves

quotient the result of dividing two numbers or *expressions*

R

radius the distance between a point on a circle and the center of the circle

range the difference between the lowest and highest values in a data set

rate a kind of *ratio* that compares quantities with different units

ratio the amount of one thing compared with the amount of another thing

rational number a number that can be written as a fraction in which the *numerator* and *denominator* are both *integers*, and the *denominator* is not equal to zero

reciprocal the multiplicative *inverse* of an *expression*

regular polygon a two-dimensional figure in which all side-lengths are equal, and all angles are equal, for example a square or an equilateral triangle

repeating decimal a decimal number in which a digit, or sequence of digits, repeats endlessly, for example, 3.33333333...

rhombus a two-dimensional figure with four equal-length sides in two parallel pairs

right triangle a triangle with one right (90°) angle

rounding replacing one number with another number that's easier to work with; used to give an approximation of a solution

S

scale drawing a drawing in which the dimensions of all the features have been reduced by the same *scale factor*

scale factor a *ratio* comparing the lengths of the sides of two *similar* figures

scalene triangle a triangle with three unequal sides

scatterplot a way of displaying ordered data pairs to see if the values in the pairs are related, and if so, how they are related

scientific notation a way of writing numbers (usually very large or small ones) as a product of two factors, where one factor is greater than or equal to 1, but less than 10, and the other is a *power* of 10 — for example, 5.3×10^6 ($= 5,300,000$)

sign of a number whether a number is positive or negative

similar two figures are similar if all of their corresponding sides are in proportion and all of their angles are equal

simplify to reduce an *expression* to the least number of terms, or to reduce a fraction to its lowest terms

skew lines nonparallel, nonintersecting lines in three-dimensional space

slope the “steepness” of a straight line on the coordinate plane, given by the ratio $\frac{\text{change in } y}{\text{change in } x}$

solve to manipulate an *equation* to find out the value of a *variable*

square root a square root of a number, n , is a number, x , that when multiplied by itself, results in n — for example, 3 and -3 are square roots of 9

stem-and-leaf plot a way of displaying numeric data, in order, so that the common values and spread of the data are easy to see

sum the result of adding numbers or *expressions* together

surface area the sum of the areas of all the *faces* of a three-dimensional figure

system of equations two (or more) *equations*, with the same *variables*, which can be solved together

T

terminating decimal a decimal that does not continue forever, for example, 0.378

terms the parts that are added or subtracted to form an *expression*

translation a transformation in which a figure moves around the *coordinate plane* (but its orientation and size stay the same)

trapezoid a four-sided shape with exactly one pair of parallel sides

U

unit rate a comparison of two amounts that have different units, where one of the amounts is “1” — for example, 50 miles per hour.

V

variable a letter that is used to represent an unknown number

vertex the point on an angle where the two rays meet on a two-dimensional figure; the point where three or more *faces* meet on a three-dimensional figure

volume a measure of the amount of space inside a three-dimensional figure

W

whole numbers the set of numbers 0, 1, 2, 3,...; the set of all whole numbers is denoted \mathbb{W}

Formula Sheet

Order of Operations — PEMDAS

Perform operations in the following order:

1. Anything in **parentheses** or other grouping symbols — working from the innermost grouping symbols to the outermost.
2. **Exponents**.
3. **Multiplications** and **divisions**, working from left to right.
4. **Additions** and **subtractions**, again from left to right.

Fractions

Adding and subtracting fractions with the same denominator: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

Adding and subtracting fractions with different denominators: $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$

Multiplying fractions: $a \cdot \frac{c}{d} = \frac{ac}{d}$ and $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

Dividing fractions: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ Reciprocals: $\frac{d}{c}$ is the called reciprocal of $\frac{c}{d}$

Rules for Multiplying and Dividing

positive \times / \div positive = positive negative \times / \div positive = negative
positive \times / \div negative = negative negative \times / \div negative = positive

Axioms of the Real Number System

For any real numbers a , b , and c , the following properties hold:

Property Name	Addition	Multiplication
Commutative Property:	$a + b = b + a$	$a \times b = b \times a$
Associative Property:	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Distributive Property of		
Multiplication over Addition:	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	
Identity Property:	$a + 0 = a$	$a \times 1 = a$
Inverse Property:	$a + (-a) = 0$	$a \times \frac{1}{a} = 1$

Area

Area of a rectangle: $A = bh$ Area of a parallelogram: $A = bh$

Area of a triangle: $A = \frac{1}{2}bh$ Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

where b is the length of the base (for a trapezoid, b_1 and b_2 are the lengths of the bases) and h is the perpendicular height.

Formula Sheet (continued)

Slope of a Line

For a line passing through the points (x_1, y_1) and (x_2, y_2) :

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Circles

Diameter:

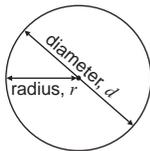
$$d = 2r$$

Circumference:

$$C = \pi d$$

Area:

$$A = \pi r^2$$



Volume of a Prism

$$V = Bh$$

where B stands for the base area, and h stands for the height of the prism.

Powers

For any real numbers, a , m , and n :

Multiplying powers: $a^m \times a^n = a^{(m+n)}$

Dividing powers: $a^m \div a^n = a^{(m-n)}$

And, for any number, $a \neq 0$: Zero exponent: $a^0 = 1$

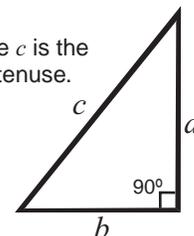
Negative exponent: $a^{-n} = \frac{1}{a^n}$

Pythagorean Theorem

For any right triangle:

where c is the hypotenuse.

$$c^2 = a^2 + b^2$$



Units

Lengths in Customary Units

1 foot (ft) = 12 inches (in.)

1 yard (yd) = 3 feet

1 mile (mi) = 1760 yards = 5280 feet

Capacities in Customary Units

1 cup = 8 fluid ounces (fl oz)

1 pint (pt) = 2 cups

1 quart (qt) = 2 pints

1 gallon (gal) = 4 quarts

Weights in Customary Units

1 pound (lb) = 16 ounces (oz)

1 ton = 2000 pounds

Lengths in Metric Units

1 centimeter (cm) = 10 millimeters (mm)

1 meter (m) = 100 centimeters

1 kilometer (km) = 1000 meters

Capacities in Metric Units

1 liter (l) = 1000 milliliters (ml)

Weights in Metric Units

1 gram (g) = 1000 milligrams (mg)

1 kilogram (kg) = 1000 grams

Customary-to-Metric / Metric-to-Customary Conversions

1 inch = 2.54 centimeters

1 gallon = 3.785 liters

1 kilogram = 2.2 pounds

1 yard = 0.91 meters

1 liter = 1.057 quarts

1 mile = 1.6 kilometers

Converting Between Temperatures in Fahrenheit and Celsius

$$F = \frac{9}{5}C + 32$$

$$C = \frac{5}{9}(F - 32)$$

Applications Formulas

Speed

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Simple Interest

The interest (I) earned in t years when p is invested at a simple interest rate of r (as a fraction or decimal) is given by: $I = prt$

Compound Interest

The amount in an account (A) when P is invested at a compound interest rate of r is given by: $A = P(1 + rt)^n$

where t is the time (in years) between each interest payment, and n is the total number of interest payments made.

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